A SIMPLE CODE-LIKE FORMULA FOR ESTIMATING THE TORSIONAL EFFECTS ON STRUCTURES SUBJECTED TO EARTHQUAKE GROUND MOTION EXCITATION

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ABSTRACT :

Plan asymmetric (eccentric) structures, characterized by non coincident centre of mass and centre of stiffness, when subjected to dynamic excitation, develop a coupled lateral-torsional response that may increase their local peak dynamic response.
In order to effectively apply the performance-based design approach to seismic design, there is a growing need for code oriented methodologies aimed at predicting deformation parameter. In this respect, for plan asymmetric structures, estimating maximum displacements at different locations in plan, especially at the perimeter, requires an evaluation of the floor rotations. The ability to predict floor rotations can be also useful to extend simplified procedures of seismic design, such as push-over analyses, to plan irregular structures.
In this paper, starting from a closed-form formulation identified in previous re-search works by the authors, an estimation of the maximum rotational response of one-storey asymmetric systems under seismic excitation is obtained and developed with respect to different applications. In detail: (1) a corrective eccentricity for the evaluation of the dynamic response of asymmetric systems through “equivalent” static procedures is identified, (2) a sensitivity analysis is carried out upon the accidental eccentricity, (3) the increase in the peak local displacements due to the eccentricity is evaluated at the corner-point of the side of the system. The results provide useful insight into understanding the torsional behavior of asymmetric systems and may directly used for preliminary design and/or check of results obtained through three-dimensional finite-element modeling of the structural system.

KEYWORDS: eccentric structures, torsional effects, floor rotations, structural parameters, simple code-like formula

1. INTRODUCTION

Eccentric structures, characterized by non coincident centre of mass and centre of stiffness, when subjected to dynamic excitation, develop a coupled lateral-torsional response that may increase their local peak dynamic response. This behavior has been investigated by many researchers since the late 1970s (Rutenberg 1998, Peruš and Fajfar 2005). In previous research works, the authors (Trombetti and Conte 2005, Trombetti et al. 2008) have widely investigated the torsional behavior of one-storey (both linear and non-linear) asymmetric systems, such as the three-dimensional system idealization displayed in Fig. 1. This system is characterized by non-coincident centre of mass (C_M) and centre of stiffness (C_K). It is assumed that the diaphragm is infinitely rigid in its own plane, and that all lateral-resisting elements (e.g. columns, shear walls, …) are massless and axially inextensible. The three degrees of freedom (the two displacements, $u_x(t)$ and $u_y(t)$, along the *x*- and *y*-directions, respectively, and the rotation, $u_\theta(t)$, around the *z*-axis) are supposed to be attached to C_M.

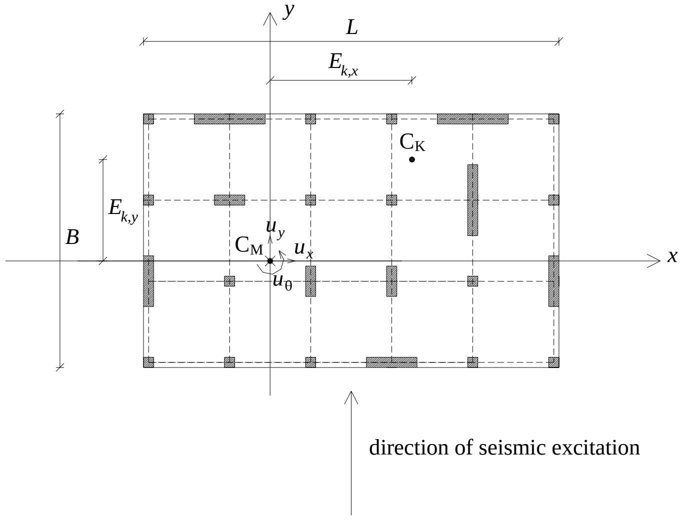


Figure 1: Three-degrees-of-freedom one-storey system idealization and reference coordinate system with origin located at the centre of mass C_M

The parameters describing the linear and the non-linear systems are defined by the authors in a previous research work (Silvestri et al. 2008). The system is considered as subjected to uni-axial seismic excitation only, so that, with respect to the direction of excitation, a longitudinal and a transversal eccentricity can be defined. With reference to Fig. 1, the seismic input is supposed to be applied along the *y*-direction, so that the eccentricity along the *x*-direction, $E_{k,x}$, is the transversal eccentricity, while the eccentricity along the *y*-direction, $E_{k,y}$, is the longitudinal one.

2. THE ALPHA PARAMETER

The maximum rotational response developed by the asymmetric system, $\left|u_\theta\right|_{\max}$, under dynamic loading (e.g. seismic excitation) can be described through the following response parameter:

$$R = \rho_m \cdot \frac{\left|u_\theta\right|_{\max}}{\left|u_y\right|_{\max}} \tag{2.1}$$

where $\left|u_y\right|_{\max}$ represents the maximum absolute values of the longitudinal response and ρ_m is the mass radius of gyration of the system as computed with respect to the *z*-axis (which passes through C_M).

In previous research works (Trombetti and Conte 2005, Trombetti et al. 2008), the authors have identified a structural parameter (called α), being defined as "the maximum rotational to maximum longitudinal displacement response ratio in free vibrations, adimensionalised using the mass radius of gyration of the structure":

$$\alpha \overset{def}{=} \rho_m \cdot \frac{\left|u_{\theta,\text{ free vibration}}\right|_{\max}}{\left|u_{y,\text{ free vibration}}\right|_{\max}} \tag{2.2}$$

where $\left|u_{\theta,\text{ free vibration}}\right|_{\max}$ and $\left|u_{y,\text{ free vibration}}\right|_{\max}$ represent, respectively, $\left|u_{\theta}\right|_{\max}$ and $\left|u_{y}\right|_{\max}$ as obtained in free vibration conditions starting from a given displacement along the *y*-direction.
For undamped eccentric structures, in the special case of null longitudinal eccentricity (in (Trombetti and Conte 2005, Trombetti et al. 2008) the authors have shown that, for uni-axial excitation, this special case represents the one that maximises the rotational response of the system) and of same total lateral stiffnesses of the system along the *x*- and the *y*-directions, the α parameter (hereafter named α_u, where subscript *u* stands for "undamped") has the following closed-form expression:

$$\alpha_u = \frac{4e_k\sqrt{3}}{\sqrt{\left(\gamma_{C_M}^2 - 1\right)^2 + 48e_k^2}} \tag{2.3}$$

where $e_k = E_k / D_e$ is the relative eccentricity of the system; $\gamma_{C_M} = \rho_k / \rho_m$; E_k is the stiffness eccentricity (distance between C_M and C_K); $D_e = \rho_m \cdot \sqrt{12}$ is a reference length; and ρ_k is the stiffness radius of gyration of the system as computed with respect to the *z*-axis (which passes through C_M).
The authors have shown (Trombetti and Conte 2005, Trombetti et al. 2008, Silvestri et al. 2008), by means of a wide range of numerical simulations carried out with reference to one-storey asymmetric systems subjected to seismic inputs, that, in general $R \le \alpha_u$ and $R \cong \psi \cdot \alpha_u$ with (Silvestri et al. 2008):

$$\psi = \begin{cases} 0.55 & \text{for } \mu \le 3 \text{ (or elastic systems with } \xi = 0.05\text{)} \\ 0.63 - 0.025\mu & \text{for } \mu > 3 \end{cases} \tag{2.4}$$

(coefficients obtained with a 50% degree of confidence (Silvestri et al. 2008)) where ξ indicates, as usual, the ratio to critical damping and μ the kinematic ductility demand (ratio of maximum displacement to yield displacement, as computed at C_M).
Eq. 2.1 and 2.4 together with the observation that the maximum longitudinal displacement of an eccentric system, $\left|u_y\right|_{\max}$, is in general very close to that of an "equivalent" non-eccentric system, $\left|u_y\right|_{\max-ne}$, (i.e. $\left|u_y\right|_{\max} \cong \left|u_y\right|_{\max-ne}$) lead to the following formula for maximum rotational response estimation:

$$\left|u_\theta\right|_{\max} \cong \psi \cdot \alpha_u \cdot \frac{\left|u_y\right|_{\max-ne}}{\rho_m} \tag{2.5}$$

3. CORRECTIVE ECCENTRICITY FOR "EQUIVALENT" STATIC ANALYSIS

Eq. 2.5 clearly links the maximum rotational response (under dynamic excitation) of an eccentric system to its maximum longitudinal deformation. This suggests the identification of a "corrective" eccentricity, E_c, to which static forces, F_{static}, (representative of the seismic action) can be applied in order to obtain a simultaneous estimation of the maximum longitudinal and rotational deformation of the system (Bosco 2008).
Indeed, the torque (moment M_{static}) generated by the application of F_{static} at a distance E_c from the centre of stiffness C_K is equal to $M_{static} = F_{static} \cdot E_c$. On the other hand, the torque (moment M_θ) which generates

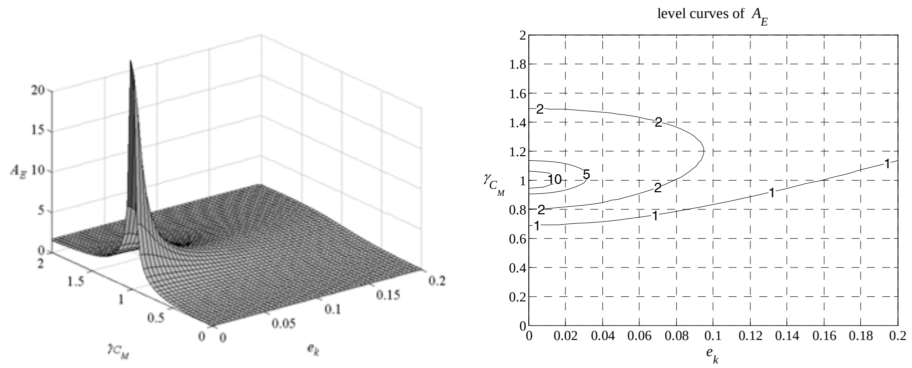
(statically) a rotation equal to $\left|u_\theta\right|_{\max}$ can be expressed as $M_\theta = I_{p,k,C_M} \cdot \left|u_\theta\right|_{\max}$ with I_{p,k,C_M} being the polar moment of inertia of the stiffness as computed with respect to the *z*-axis (which passes through C_M).
Recalling that $I_{p,k,CM} = \rho_k^2 K$ (*K* is the total lateral stiffness of the elastic system) (Silvestri et al. 2008) and substituting $\left|u_\theta\right|_{\max}$ with its estimation given by Eq. 2.5, it can be written:

$$M_\theta \cong \psi \cdot \gamma_{C_M}^2 \cdot \rho_m \cdot K \cdot \alpha_u \cdot \left|u_y\right|_{\max - ne} \tag{3.1}$$

Imposing that M_{static} is equal to M_θ, and recalling that $F_{static} = K \cdot \left|u_y\right|_{\max - ne}$, it is thus possible to obtain the following estimation for $E_c \cong \psi \cdot \gamma_{C_M}^2 \cdot \rho_m \cdot \alpha_u$ .Finally, this allows to identify a virtual “amplification”, A_E, of E_k due to the dynamic response of the system, as:

$$A_E = \frac{E_c}{E_k} = \psi \frac{\gamma_{C_M}^2 \cdot \rho_m \cdot \alpha_u}{E_k} = \psi \frac{2\gamma_{C_M}^2}{\sqrt{\left(\gamma_{C_M}^2 - 1\right)^2 + 48 e_k^2}} \tag{3.2}$$

Figs. 2a and b show the three-dimensional surface and the contour plot (isolines), respectively, of A_E as a function of e_k and γ_{C_M} (for the common case of $\psi = 0.55$). These plots indicate that A_E is very close to (even if not lower than) 1 for torsionally-flexible systems characterised by $\gamma_{C_M} < 0.8$; whilst it assumes the maximum values for structures characterised by $\gamma_{C_M} \cong 1$ and small values of e_k. In general, torsionally-stiff structures amplify the eccentricity E_k, with A_E values in the range of 1.5 (for all large values of γ_{C_M} and for small values of γ_{C_M} coupled with large values of e_k) and 5 (for γ_{C_M} approaching 1 coupled with e_k values smaller than 0.03).



(a) (b)

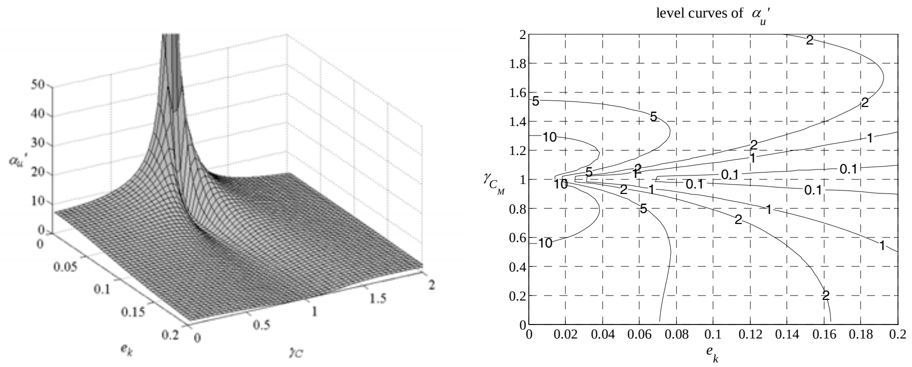
Figure 2: (a) 3D surface and (b) contour plot of A_E as a function of e_k and γ_{C_M}

4. ACCIDENTAL ECCENTRICITY

In seismic analysis of building structures, the stiffness eccentricity of the system, E_k, is typically defined as the distance between the centre of stiffness C_K and the centre of mass C_M. The position of C_K can be estimated with a fair degree of confidence, whilst the determination of C_M may prove to be somewhat more difficult due to the random distribution in space of live loads.
A common practice, often suggested by seismic codes, is that of determining E_k as the sum of the two contributions: an intrinsic eccentricity E_i ($e_i = E_i / D_e$) corresponding to the distance between C_K and $\overline{C_M}$ ($\overline{C_M}$ being the centre of mass as obtained considering a uniform distribution of the live loads) and an

accidental eccentricity E_a ($e_a = E_a / D_e$), typically estimated equal to 5% of the side of the building structure. This sum being symbolically expressed as $E_k = E_i + E_a$ or, equivalently $e_k = e_i + e_a$.
Eq. 2.5 provides an estimation of the maximum rotational response of an eccentric system, through a number of parameters, among which α_u (through e_k and γ_{C_M}) and ρ_m depend upon the location of C_M. Given the relatively small values of e_a, it can be assumed that $\gamma_{C_M} \cong \gamma_{\overline{C_M}}$ and $\rho_m \cong \rho_{\bar{m}}$. Thus, the unknown position of C_M influences α_u only through e_k, i.e. $\alpha_u = \alpha_u(e_k) = \alpha_u(e_i + e_a)$. Thus, for a given system, it is possible to perform a sensitivity analysis of $|u_\theta|_{\max}$ upon e_a, which clearly indicates that the structures which are the most sensitive to the accidental eccentricity are those characterised by null intrinsic eccentricity and $\gamma_{C_M} \cong 1$.

Figs. 3a and b show the three-dimensional surface and the contour plot (isolines), respectively, of $\dfrac{\partial \alpha_u}{\partial e_k}$ as a function of e_k and γ_{C_M} (for the common case of $\psi = 0.55$). These plots indicate that (i) torsionally-stiff and torsionally-flexible systems show a substantial symmetric behaviour with respect to the $\gamma_{C_M} = 1$ axis; (ii) the structures which are the most sensitive to the eccentricity are those characterised by $0.6 \le \gamma_{C_M} \le 1.3$ and $e_k < 0.03$; and (iii) the structures which are less sensitive to the eccentricity are those characterised by $\gamma_{C_M} \cong 1$ and $e_k > 0.07$.



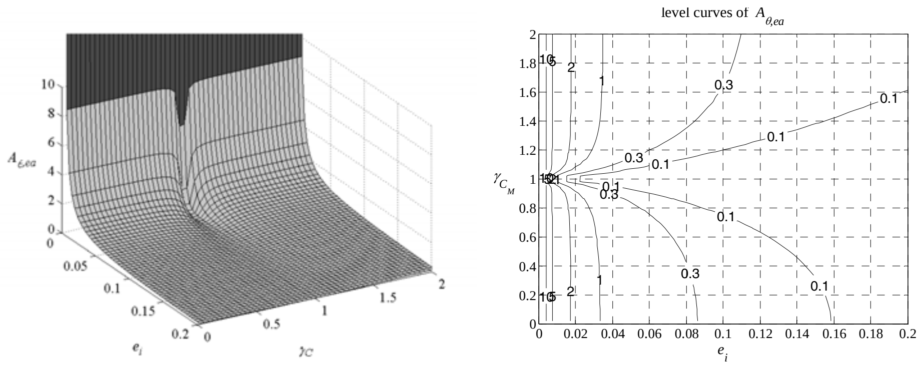
(a) (b)

Fig. 3: (a) 3D surface and (b) contour plot of $\dfrac{\partial \alpha_u}{\partial e_k}$ as a function of e_k and γ_{C_M}

The amplification of the maximum rotational response, A_{θ,e_a} due to the accidental eccentricity can be then estimated as:

$$A_{\theta,e_a} = \frac{|\Delta u_\theta|_{\max}\big|_{e_a}}{|u_\theta|_{\max}\big|_{e_i}} = \frac{\left.\dfrac{\partial \alpha_u}{\partial e_k}\right|_{e_i} \cdot e_a}{\alpha_u(e_i)} = \frac{(\gamma_{cm}^2 - 1)^2}{(\gamma_{cm}^2 - 1)^2 + 48 e_i^2} \cdot \frac{e_a}{e_i} \qquad (4.1)$$

Figs. 4a and b show the three-dimensional surface and the contour plot (isolines), respectively, of A_{θ,e_a} as a function of e_i and γ_{C_M}, for the common case of $\psi = 0.55$ and for $e_a = 0.035$ which corresponds to $E_a = 0.05 \cdot L$ for a square-plan system. These plots indicate that (i) torsionally-stiff and torsionally-flexible systems show a substantial symmetric behaviour with respect to the $\gamma_{C_M} = 1$ axis; (ii) provided that γ_{C_M} values around the unity are still associated with a low-sensitive zone, in general, A_{θ,e_a} seems to be independent from γ_{C_M}, but strongly dependent on e_i; (iii) the structures which are the most sensitive to the accidental eccentricity are those characterised by $e_i < 0.03$; and (iv) the structures which are less sensitive to the accidental eccentricity are those characterised by $e_i > 0.03$.



(a) (b)

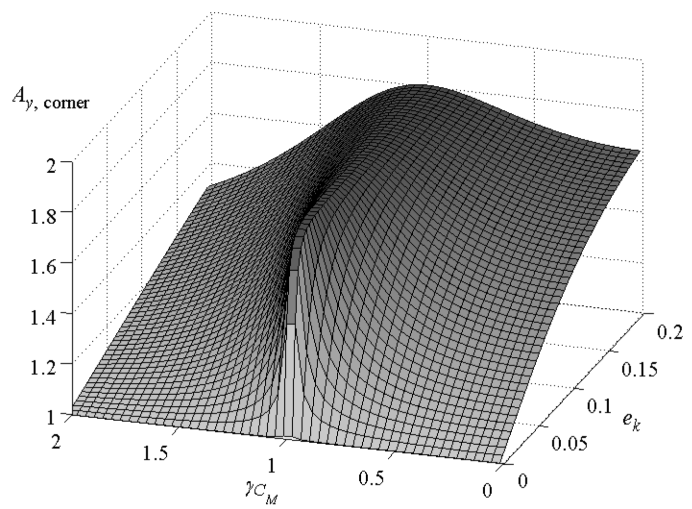
Fig. 4: (a) 3D surface and (b) contour plot of A_{θ,e_a} as a function of e_i and γ_{C_M}

5. INCREASE OF THE PEAK DISPLACEMENT AT THE CORNER POINT OF THE SYSTEM DUE TO ECCENTRICITY

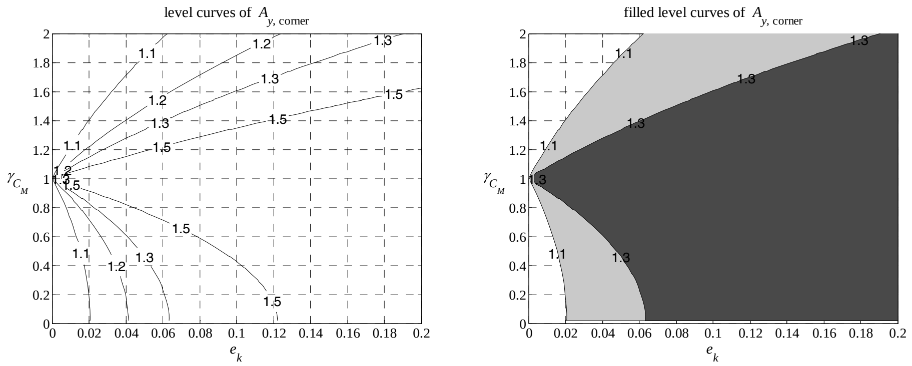
Referring to a square-plan system and under the conservative hypothesis of simultaneous maximum rotation and displacement, it is possible to estimate the amplification, $A_{y,\text{ corner}}$ of the maximum displacement at the corner point of the system, $|u|_{\max, y+\theta(e_k),\text{ corner}}$, due to eccentricity e_k as:

$$A_{y,\text{ corner}} = \frac{|u|_{\max, y+\theta(e_k),\text{ corner}}}{|u_y|_{\max}} = \sqrt{1+\sqrt{6}\cdot\psi\cdot\alpha_u(e_k)+3\cdot\psi^2\cdot\alpha_u^2(e_k)} \tag{5.1}$$

Figs. 5a, b and c show the three-dimensional surface, the contour plot (isolines) and the filled contour plot, respectively, of $A_{y,\text{ corner}}$ as a function of e_k and γ_{C_M} for an elastic (5% damped) system. The maximum values of $A_{y,\text{ corner}}$ are about 1.7 ÷ 1.8. Note that A_y can be used to estimate the error introduced neglecting the system eccentricity (i.e. performing a plane analysis instead of a full three-dimensional one).



(a)



(b) (c)

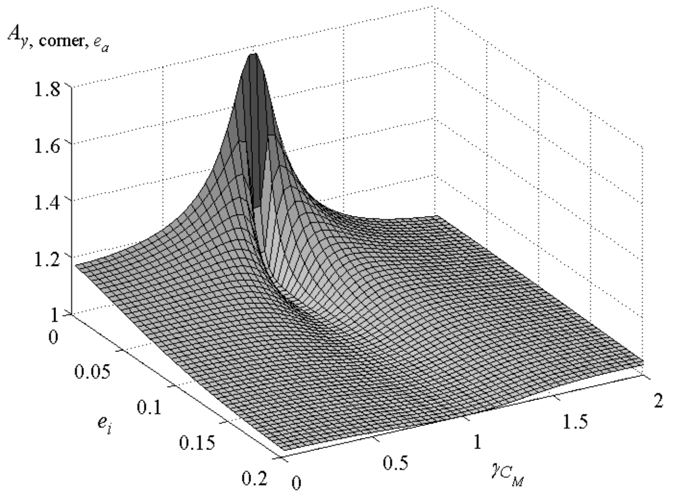
Fig. 5: (a) 3D surface, (b) contour plot and (c) filled contour plot of $A_{y,\,\text{corner}}$ as a function of e_k and γ_{C_M}

6. INCREASE OF THE PEAK DISPLACEMENT AT THE CORNER POINT OF THE SYSTEM DUE TO ACCIDENTAL ECCENTRICITY

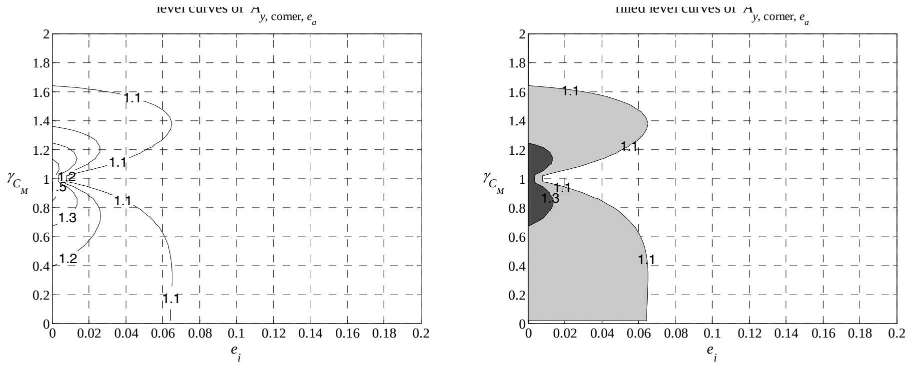
It is possible to estimate the amplification of the maximum displacement at the corner point of the system due to the accidental eccentricity e_a as:

$$A_{y,\,\text{corner},\,e_a} = \frac{|u|_{\max,y+\theta(e_k),\,\text{corner}}}{|u|_{\max,y+\theta(e_i),\,\text{corner}}} = \frac{\sqrt{1+\sqrt{6}\cdot\psi\cdot\alpha_u\left(e_i+e_a\right)+3\cdot\psi^2\cdot\alpha_u^2\left(e_i+e_a\right)}}{\sqrt{1+\sqrt{6}\cdot\psi\cdot\alpha_u\left(e_i\right)+3\cdot\psi^2\cdot\alpha_u^2\left(e_i\right)}} \tag{6.1}$$

Figs. 6a, b and c show the three-dimensional surface, the contour plot (isolines) and the filled contour plot, respectively, of $A_{y,\,\text{corner},\,e_a}$ as a function of e_i and γ_{C_M} for an elastic (5% damped) system and considering $e_a = 0.035$ (which corresponds to $E_a = 0.05 \cdot L$). The same observations made for the mid-point can also be drawn for the corner point. Note that $A_{y,\,e_a}$ can be used to estimate the error introduced neglecting the system accidental eccentricity (i.e. performing a three-dimensional analysis considering a uniform distribution of the live loads).



(a)



(b) (c)

Figure 6: (a) 3D surface, (b) contour plot and (c) filled contour plot of $A_{y,\,\mathrm{corner},\,e_a}$ as a function of e_i and γ_{C_M}

7. CONCLUSIONS

Starting from a closed-form formulation identified in previous research works, the authors here identify (i) a corrective eccentricity at which static loads (representative of the shear loads induced by seismic action) can be applied in order to account for the dynamic torsional response of asymmetric structures, (ii) the sensitivity of the maximum dynamic torsional response to accidental eccentricity, and (iii) the increase in peak local displacements due to both structural and accidental eccentricity.
All the above results are obtained in closed-form, provide useful insight into understanding the torsional behaviour of asymmetric systems and may directly used for preliminary design and/or check of results obtained through three-dimensional finite-element modeling of the structural system.
Also the results obtained can be used to estimate the error introduced neglecting the system eccentricity (i.e. performing a plane analysis instead of a full three-dimensional one) and the error introduced neglecting the system accidental eccentricity (i.e. performing a three-dimensional analysis considering a uniform distribution of the live loads).

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