DUCTILITY CAPACITY MODELS FOR BUCKLING-RESTRAINED BRACES

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ABSTRACT:

Buckling-restrained braces (BRBs) have recently become popular for use in the primary lateral-force-resisting systems of structures located in high seismic regions of the United States. A BRB is a steel brace that does not buckle in compression but instead yields in both tension and compression. While testing has shown that BRBs possess high ductility capacity, no generally accepted method yet exists to predict the cumulative plastic ductility (CPD) capacity of BRBs, where capacity is defined as deformation undergone before fatigue fracture of the brace. To address this lack of knowledge, this study investigated ductility capacity modeling of BRBs. A database of 76 BRB tests was compiled, and parameters from the test database were considered as potential predictive parameters in a CPD capacity model for BRBs. Two types of capacity models were considered in this research: end-capacity models, which predict a static total CPD capacity and remaining capacity models, which predict available (or remaining) CPD capacity after a given deformation history is imposed. The maximum likelihood estimation method was used to calibrate the capacity model parameters to maximize the probabilities that predicted values match test results, providing an unbiased model that relates predictive parameters to CPD capacity. Results show that the end-capacity formulation has limitations and may lead to results that contradict behavior, whereas the remaining capacity formulation is more intuitive and moderately precise.

KEYWORDS: Ductility capacity, buckling-restrained braces, maximum likelihood estimation method

1. INTRODUCTION

Buckling-restrained braces (BRBs) have recently become popular for use in the primary lateral-force-resisting systems of structures located in high seismic regions of the United States. A BRB is a steel brace that does not buckle in compression but instead yields in both tension and compression. Although BRB cumulative ductility demands under seismic excitation can be reasonably estimated from nonlinear dynamic analysis (e.g. [1]), no generally accepted method exists for predicting the cumulative plastic ductility (CPD) capacity of BRBs, where CPD capacity is defined by the cumulative plastic deformation sustained before fracture of the steel core. In addition, CPD capacity has been shown to be dependent on loading history. Carden [2] and Fahnestock [1] have observed that braces which undergo large maximum deformations exhibit lower CPD capacity than those braces which undergo relatively smaller maximum deformations. Furthermore, other important parameters affecting capacity have not been clearly identified yet.

As one of the few CPD capacity models available, Takeuchi et al. [3] developed a deterministic fatigue model. This model accounts for the effect of loading history on BRB CPD capacity by decomposing the imposed brace deformations into skeleton and Bauschinger parts as described by Benavent-Climent [4]. By contrast to the deterministic approach by Takeuchi et al., this research effort developed probabilistic capacity models. In addition, unlike the models by Takeuchi et al., which require knowledge of the force-deformation histories of BRBs, the models described in this paper require knowledge of only the BRB imposed deformation histories.

Initially, a database of 76 BRB tests was compiled. Data analysis was performed using this database to extract potential predictive parameters for the CPD capacity models. Using the predictive parameters, the maximum likelihood estimation (MLE) method was employed to construct models that predict the CPD capacity of BRBs. Two types of capacity models were considered in this research: end-capacity models, which predict a static total CPD capacity and remaining capacity models, which predict available (or remaining) CPD capacity after a given deformation history is imposed. Results and conclusions for each model type are discussed.

2. TEST DATABASE

A BRB test database was compiled through literature review of brace tests performed by researchers from around the world, with the majority of testing performed in the U.S. and Japan (see references in [5]). Of the 76 specimens in the database, 34 failed due to fracture during testing, and 42 did not fail. For each specimen, the test database contains brace geometrical properties (core shape, core area, and core length), the steel yield strength, and the imposed deformation history. The imposed deformation history is either a regular cyclic history (67 specimens) or simulated seismic loading (9 specimens). In general, the test database does not contain brace axial force data.

3. PREDICTIVE PARAMETERS

In order to evaluate the factors affecting BRB CPD capacity and to produce the best BRB CPD capacity models, a wide variety of predictive parameters were investigated. The predictive parameters used in this research (denoted by *h*) can be divided into three groups: (1) brace geometric properties, (2) brace material properties, and (3) descriptors of the imposed deformation history.

Table 3.1 Predictive Parameters

Brace Geometric Properties	**Brace Material Properties**	**Deformation Descriptors**	
$h_1 = A_c/(A_c)_{\max}$	$h_3 = \varepsilon_{yc}$	$h_5 = (\mu_{\max})_t$	PE Distribution: $h_{7-10} = N_{PE}, \mu_{PE}, \sigma_{PE}, \upsilon_{PE}$
$h_2 = L_c/(L_c)_{\max}$	$h_4 = F_u/F_y$	$h_6 = (\mu_{\max})_c$	RF Distribution: $h_{11-14} = N_{RF}, \mu_{RF}, \sigma_{RF}, \upsilon_{RF}$

Table 3.1 defines all the predictive parameters, and the variables are further defined as follows: A_c is the cross sectional area of the BRB core; $(A_c)_{\max}$ is the largest core cross sectional area of all BRBs in the test database; L_c is the length of the yielding core region of the BRB; $(L_c)_{\max}$ is the maximum core length of all BRBs in the test database; ε_{yc} is the yield strain of the BRB core ; F_u is the ultimate tensile strength of the BRB core (from coupon tests); and F_y is the yield stress of the BRB core (from coupon tests); $(\mu_{\max})_t$ is the maximum tensile ductility demand in the deformation history; and $(\mu_{\max})_c$ is the maximum compressive ductility demand in the deformation history. $\mu_{\max}$ is defined as $\max\{(\mu_{\max})_t, (\mu_{\max})_c\}$, and a ductility demand μ is defined as Δ_c/Δ_{yc}, where Δ_c is the instantaneous deformation of the BRB core (measured across L_c), and Δ_{yc} is the core deformation at incipient yielding of the core. Cumulative plastic ductility (CPD) demand is the summation of all plastic core deformation ($\sum\Delta_p$) occurring up to a specific deformation increment, normalized

by the yield deformation, i.e. $\mu_c = \frac{\sum \Delta_p}{\Delta_{yc}}$. Both A_c and L_c are normalized by database maximum values since unit-less predictive parameters are desirable during the model construction.

The "PE distribution" terms are related to the plastic excursion (PE) distribution: count (N_{PE}), mean value (μ_{PE}), standard deviation (σ_{PE}), and skewness (υ_{PE}). A single PE is defined as the sum of all deformation (expressed as ductility) occurring consecutively in the plastic domain. A PE begins at the yield point and ends when unloading commences. Many such single PEs occur during a typical load history, and the aggregation of these single PEs forms the PE distribution.

The "RF distribution" terms are related to the Rainflow (RF) distribution: count (N_{RF}), mean value (μ_{RF}), standard deviation (σ_{RF}), and skewness (υ_{RF}). The RF distribution is a distribution of cycle amplitudes (plastic deformation only) calculated from the deformation history using the Rainflow Method [6]. This method converts an irregular deformation history into a cyclic deformation history composed of full and half cycles.

The CPD capacity of the BRB test specimens as well as the deformation descriptor predictive parameters were determined by assuming that the steel BRB core behaves in an elastic-perfectly plastic manner when subjected to the imposed deformation history. The CPD capacity was calculated by summing all plastic deformations throughout the imposed deformation history to the point of failure of the BRB.

4. CAPACITY MODELING OVERVIEW

To study the CPD capacity of BRBs in greater depth, the MLE method [7] was employed to construct capacity models. In the MLE method, model parameters were calibrated to maximize the probabilities that the model would predict the observed data. The procedure for developing BRB CPD capacity models consisted of the following four steps: (1) model form definition, (2) model parameter calibration, (3) model reduction, and (4) error analysis. These steps are discussed in this section in detail in reference to end-capacity model formulations. Remaining capacity models are discussed in a subsequent section.

The following form is chosen for the end-capacity models

$$C = \gamma(\theta, \mathbf{h}) + \sigma\varepsilon \tag{4.1}$$

where C is the predicted CPD end-capacity; $\gamma(\theta, \mathbf{h})$ is the model form; θ is a vector of model parameters (used to fit the model to test data); **h** is a vector of predictive parameters (defined in the previous section); σ is the model error standard deviation; and ε is the standard normal random variable (zero mean and unit variance). Together, the quantity $\sigma\varepsilon$ represents the error in the model.

The likelihood function was used to fit the model to test data, where the likelihood function is proportional to the conditional probability that the capacity model agrees with the test results. The residual (the difference between the predicted capacity and measured values) is defined as

$$r = C_{measure} - \gamma(\theta, \mathbf{h}) = \sigma\varepsilon \tag{4.2}$$

where $C_{measure}$ is the CPD capacity from test results. Thus the likelihood function is given as

$$L(\theta, \sigma) \propto \prod_{\text{failure data}} P(\varepsilon = r_i / \sigma) \times \prod_{\text{non-failure data}} P(\varepsilon > r_i / \sigma) \tag{4.3}$$

$$L(\theta,\sigma) \propto \prod_{\text{failure data}} \varphi(r_i/\sigma)/\sigma \times \prod_{\text{non-failure data}} \Phi(-r_i/\sigma) \tag{4.4}$$

in which $\varphi(\cdot)$ and $\Phi(\cdot)$ respectively denote the probability density function and cumulative distribution function of the standard normal distribution. The model was fit to test results by varying θ and σ such that the likelihood function was maximized. This was accomplished through a standard iterative non-linear minimization algorithm using MATLAB®.

Following the parameter calibration, model reduction was performed. In this process, predictive parameters in **h** were removed in an iterative fashion such that the number of predictor terms was minimized with model error (which is proportional to σ) maintained at a level judged to be reasonably low. The goal of model reduction was to find the simplest model that was still accurate, and the process of model reduction allowed identification of the most influential predictive parameters. Following model reduction steps, error analysis was performed.

Error analysis was accomplished in a typical fashion, where the distribution of $Z = C_{predict}/C_{measure}$ was constructed using all specimens in the test database that failed, where $C_{predict}$ is the predicted CPD capacity from the capacity model, and $C_{measure}$ is the measured CPD capacity from testing. A mean value of Z is greater than 1 because the model is constructed using both failure and non-failure data. The coefficient of variation (COV) of Z is an indicator of the precision of a particular model.

5 END-CAPACITY MODELS

Through the process of parameter exploration, model creation, and model reduction for the end-capacity models, the following conclusions may be stated [5]:

1) A variety of predictive parameters and model forms were explored. Predictive parameters included BRB material properties, BRB geometric properties, and parameters which characterize the imposed deformation histories. Model forms explored included linear and nonlinear.
2) Of the parameters investigated, it was found that deformation history predictive parameters were more important and contributed more substantially to model accuracy than BRB property parameters. Those models without deformation history predictive parameters performed very poorly.
3) Although the Rainflow deformation history predictive parameters and the plastic excursion predictive parameters attempt to characterize the same behavior (size and shape of the imposed plastic deformation demand distribution), the Rainflow parameters were found to perform better than the plastic excursion parameters.
4) Overall, no high-fidelity model capable of predicting the end-CPD capacity of BRBs was found.
5) When using deformation history predictive parameters, the end-capacity model may lead to counter-intuitive results that are artifacts of the distribution of the parameters in the test database and are not representative of behavior. For example, it is thought that those BRBs subjected to higher ultimate demands, i.e. higher $\mu_{\max}$, should have relatively lower CPD capacity. However, some capacity models, as formulated in terms of predicting end capacity, indicated that larger ultimate demands cause larger CPD capacity. This appeared to occur because those specimens with larger ultimate demands simply tended to be tested to higher CPD capacities, but this in general does not mean that higher ultimate demands lead to relatively higher CPD capacities.
6) It is possible to err with the end-capacity formulation, and include in the predictive terms the value that the model seeks to predict.

6. REMAINING CAPACITY MODELS

In an attempt to overcome the disadvantages of the end-capacity models described above, remaining capacity (RC) models were developed. The basic form of the remaining capacity models is given by

$$RC = TC - UC = \prod h_i^{\theta_i} - \sum h_j \theta_j \tag{6.1}$$

where RC is the remaining capacity; TC is the total capacity (the capacity of the brace in an undamaged state); and UC is the used capacity (all in terms of CPD). RC varies with the applied deformation history, from a value of TC at the beginning of the applied deformation history to a value of 0 when the brace fractures. The form of equation 6.1 is a combination of the end-capacity models and a damage evolution-type model. The total capacity component, i.e., $TC = \prod h_i^{\theta_i}$, is an end-capacity-type formulation that utilizes only static predictive parameters (those that do not change with the imposed deformation). Conversely, the used capacity component, i.e., $UC = \sum h_j \theta_j$, is a damage-evolution-type model and utilizes deformation history predictive parameters (those that vary with the imposed deformation).

The parameters used in the RC models were the same as in the end-capacity models, summarized in Table 3.1. Two additional parameters used not described previously are μ_{ult} and $\mu_{\max loc}$. μ_{ult} is the ultimate ductility capacity, which is assumed to be equal to the value of ductility at the ultimate tensile strain of the steel. This is given by $\mu_{ult} = \varepsilon_{uc} / \varepsilon_{yc}$, where ε_{uc} is the ultimate tensile strain of the core, assumed to be 35% for all specimens. $\mu_{\max loc}$ is defined as $\mu_{\max loc} = \dfrac{\mu_c @ \mu_{\max}}{\mu_c @ end}$, i.e. the value of μ_c that occurs at the location of $\mu_{\max}$ divided by the value of μ_c at the end of the deformation history. Thus $\mu_{\max loc}$ may be thought of as the relative location of the maximum ductility demand in the deformation history in terms of CPD. This parameter was created to potentially characterize the effects of the location of maximum ductility demands on the CPD capacity.

The MLE method, as discussed previously, was used to calibrate the model parameters using the complete model form:

$$RC = TC - UC = \prod h_i^{\theta_i} - \sum h_j \theta_j + \sigma\varepsilon \tag{6.2}$$

As before, the model parameters θ and σ were calibrated to maximize the likelihood function, which, for the remaining capacity models, is given by

$$L(\theta, \sigma) \propto \prod_{l=1}^{n_{specimens}} \left\{ \prod_{m=1}^{n_{increments}} P\left[\left(RC_{predict} \right)_{l,m} = \left(RC_{measure} \right)_{l,m} \right] \right\} \tag{6.3}$$

in which $\left(RC_{predict} \right)_{l,m}$ is the predicted remaining capacity given by equation 6.1 for BRB specimen l at deformation increment m. Similarly, $\left(RC_{measure} \right)_{l,m}$ is the measured remaining capacity from testing for BRB specimen l at deformation increment m, which is given as

$$(RC_{measure})_{l,m} = (\mu_c)_{l,end} - (\mu_c)_{l,m} \tag{6.4}$$

where $(\mu_c)_{l,end}$ is the CPD demand from testing for BRB specimen l at the end of testing, i.e. the total CPD demand, and $(\mu_c)_{l,m}$ is the CPD demand from testing for BRB specimen l at deformation increment m, i.e. all plastic deformation (in terms of ductility), accumulated from the start of the imposed deformation history up to point m.

Since the RC models were fit to test data at various intervals in the deformation histories (and not just at the beginning and/or end points), it was difficult to quantify the overall model precision. The metric used to do this was the distribution of $(RC_{predict})_{failure,end}$, which is the distribution of predicted remaining capacities at the end of the imposed deformation histories for failure specimens. The mean of this distribution is denoted by $\mu_{RCpredict}$ and measures the model accuracy. The standard deviation of the distribution is denoted by $\sigma_{RCpredict}$ and measures the model precision. Similar metrics could be derived for different points in the deformation history, but the distribution at the end of the history is most informative.

Various RC models were investigated by implementing the terms mentioned above in a variety of combinations. Through trial and error, two best models were identified, RC 1 and RC 2. Modeling results are presented in Table 6.1 and Table 6.2, which list, for each model, the equation for predicted RC, values of σ, model accuracy, and model precision.

Table 6.1 Remaining Capacity Modeling Equations

Model	Equation
RC 1	$RC = 2^{-3.434} \cdot \left(\frac{A_c}{(A_c)_{\max}}\right)^{0.2019} \cdot \left(\frac{L_c}{(L_c)_{\max}}\right)^{0.0466} \cdot \varepsilon_{yc}^{-1.319} \cdot \left(\frac{F_u}{F_y}\right)^{0.2181} \cdot \mu_{\max loc}^{-0.9883} - 0.8677\mu_c + 185.8\frac{\mu_{\max}}{\mu_c}$
RC 2	$R = 2^{-21.20} \cdot \left(\frac{A_c}{(A_c)_{\max}}\right)^{0.425} \cdot \left(\frac{L_c}{(L_c)_{\max}}\right)^{0.044} \cdot \varepsilon_{yc}^{-3.45} \cdot \left(\frac{F_u}{F_y}\right)^{-1.46} - 152.9\frac{\mu_c}{\mu_{ult}} - 1.12\frac{\mu_{\max}}{\mu_{ult}}$
RC Avg	$R = 2^{10.27} - \mu_c$

Table 6.2 Remaining Capacity Modeling Accuracy and Precision

Model	Value of σ	$\mu_{RCpredict}$	$\sigma_{RCpredcit}$
RC 1	193	-30	217
RC 2	434	243	368
RC Avg	870	0	891

Figure 6.1 shows the comparison of the predicted remaining capacity versus the measured remaining capacity over the entire deformation histories as predicted by model RC2 (the plot for RC 1, though not shown, is similar). In the figure, the measured remaining capacity is represented by the mean plus and minus one standard deviation envelope of the distribution of measured remaining capacities (for failure specimens only).

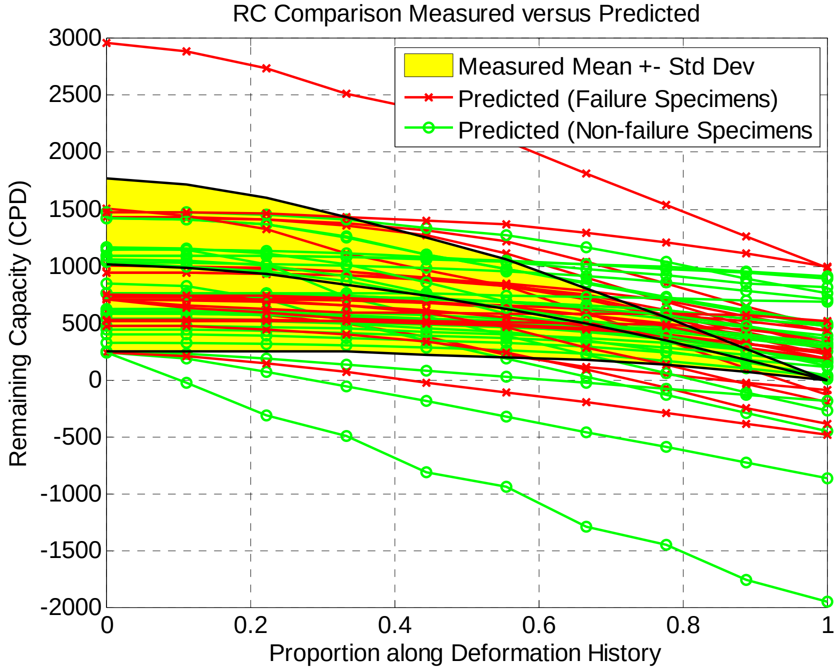

Figure 6.1 Predicted versus Measured RC Comparison for RC 2 Model

The following conclusions apply to models RC 1 and RC 2:

- For both models, the behavior of the predicted remaining capacity over the deformation history (i.e. the shape of the plots) is monotonically decreasing.
- The majority of the predicted values fall within the measured distribution envelopes for both models.
- The RC 1 model is more accurate and precise than RC 2, because $\mu_{RCpredict}$ is nearer to 0 for RC 1 than RC 2, and because $\sigma_{RCpredict}$ is smaller for RC 1 than RC 2.
- RC 2, in general, overestimates remaining capacity, as $\mu_{RCpredict}$ is significantly greater than 0.
- Both models RC 1 and RC 2 are significantly more precise than the model RC Avg, which is a model based on quantifying the average brace remaining capacity and which uses no additional predictor terms. Thus, the use of RC models is warranted instead of using just the average brace capacity from tests.

While RC 1 appears to be a better model than RC 2 in all respects, the inclusion of the $\mu_{\max loc}$ term in RC 1 presents problems. The overall effect of the $\mu_{\max loc}^{-0.9883}$ term is that BRBs subjected to seismic loading with relatively high early demands (a smaller value of $\mu_{\max loc}$) are predicted to have larger CPD capacity. This conflicts with the observations made by Carden [2] and Fahnestock [1]. To avoid the problems discussed above, the RC 2 model was developed without the use of $\mu_{\max loc}$.

7 SUMMARY AND CONCLUSIONS

In this research, CPD capacity models for BRBs were created. The approach was empirical, and was based on a test database of 76 BRB specimens, of which 34 failed via tensile fracture and 42 did not. Predictive parameters were extracted from the test database to be used as inputs to the capacity models. The maximum likelihood estimation method, in which the parameter models are calibrated to maximize the probabilities that the observed data will be predicted by the model, were applied to develop probabilistic models that relate predictive parameters to BRB CPD capacity. Two types of capacity models were considered in this research: end-capacity models, which predict a static total CPD capacity and remaining capacity models, which predict available (or remaining) CPD capacity after a given deformation history is imposed. Remaining capacity models proved most applicable, and while they may not conform to engineering-level accuracy or precision expectations, they may be used in a performance-based engineering framework to predict BRB failure, where the CPD capacity model error is taken explicitly into account. Developing both accurate and precise models is challenging when using only basic BRB properties and the imposed deformation histories, since the variability in the imposed deformation histories (regular cyclic, to irregular cyclic, to simulated seismic) prevents detailed understanding of the factors that affect performance. There are three recommended actions to create better BRB CPD capacity models:

1) Obtain knowledge about more BRB properties, particularly those related to ductility (such as ultimate stress and ultimate strain capacity).
2) Implement a more uniform testing program, similar to the procedures used to create high-cycle fatigue curves, where the imposed deformation histories are similar and systematically planned to study CPD capacities at various constant strain ranges (i.e. regular cyclic and simulated seismic loadings should not be mixed).
3) Measure both the force and deformation histories of the BRBs and use the information to build capacity models (as in Takeuchi et al. [3]), but using a probabilistic framework as described above.

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