

# DYNAMIC RESPONSE OF UTILITY TUNNEL DURING THE PASSAGE OF RAYLEIGH WAVES

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### ABSTRACT :

The dynamic response of the utility tunnel is studied when the Rayleigh waves is considered. First the free field response under the Rayleigh wave passage is analyzed and the numerical results agree well with the analytical results. Based on the research above, the approximate Rayleigh earthquake waves are put forward. Then using the fast transfer technique, the approximate Rayleigh earthquake wave field is obtained. The dynamic response of utility tunnel during the passage of Rayleigh waves is researched with commercial finite element software ABAQUS as a platform. The results reveal that the utility tunnel shows global bending deformation pattern and the amplitude at the top of the structure is about 1 multiply greater than the corresponding point at the bottom of the structure. To the shallow buried underground structures, Rayleigh waves can be the key factors to control the damage of the structures.

**KEYWORDS:** utility tunnel, approximate Rayleigh earthquake wave field, dynamic response, fast Fourier transform technique

#### **1. INTRODUCTION**

The utility tunnel is one kind of lifeline facilities which can hold many kinds of lifelines, such as water pipelines, gas pipelines, and communication lines. The utility tunnel has many advantages over the traditional method of burying the pipe directly. To the author's knowledge, however, there is still limited available research on the seismic response of the utility tunnel.

The majority of research conducted on the dynamic response of underground structures concentrated on the study of soil-structure interaction. Most of this research assumes vertically propagating shear waves(S-waves) (Wang, 1993; Penzien and Wu, 1998; Penzien, 2000; Hashash, 2001; Matsui etc. 2001; Huo, 2005). Despite the significant progress in understanding the seismic behavior of underground structures, very little is known about their response, when the soil is excited from the passage of Rayleigh waves. Makris (1994, 1995) studied the response of pile during the passage of Rayleigh wave. It is well established that surface waves have a significant contribution to response of the structures that shallow buried. Furthermore, surface waves induce motions and stresses in the soil and structure which are substantially different from those calculated based on the assumption of vertical S-waves. The above reasons motivated the study reported herein.

This paper studies the dynamic response of the utility tunnel during the passage of the Rayleigh waves. Firstly some free field analysis is conducted using the steady wave and the results are compared with the analytical solutions. Based on the FFT, the approximate earthquake Rayleigh wave field is obtained. Then the dynamic response of utility tunnel during the Rayleigh waves is calculated.



#### 2. SOIL DISPLACEMENTS DUE TO RAYLEIGH WAVES

From the theory of wave propagation(Achenbach,1973), one can show that the horizontal displacement  $u_x$ , and the vertical displacement  $u_y$ , of a soil particle because of the passage of Rayleigh wave can be expressed in the form

$$u_{x} = Aik \left[ \exp\left(-aky\right) - \frac{(1+b^{2})}{2} \exp\left(-bky\right) \right] \exp\left[ik\left(x-ct\right)\right]$$
(2.1a)

$$u_{y} = Ak \left[ -a \exp\left(-aky\right) + \frac{(1+b^{2})}{2b} \exp\left(-bky\right) \right] \exp\left[ik\left(x-ct\right)\right]$$
(2.1b)

If only the real part is considered

$$u_x = f_1(y) \sin[k(x-ct)]$$
(2.2a)

$$u_{y} = f_{2}(y)\cos[k(x-ct)]$$
(2.2b)

Where

$$f_1(y) = -Ak \left[ \exp(-aky) - \frac{(1+b^2)}{2} \exp(-bky) \right]$$
(2.3a)

$$f_2(y) = Ak \left[ -a \exp(-aky) + \frac{(1+b^2)}{2b} \exp(-bky) \right]$$
 (2.3b)

$$a = \sqrt{1 - \frac{c^2}{V_p^2}}$$
  $b = \sqrt{1 - \frac{c^2}{V_s^2}}$  (2.4)

With k being the wave number which can get from the formulation  $k = \omega/c$ ,  $V_p$ ,  $V_s$  and c being the dilatational wave velocity, shear wave velocity and Rayleigh wave velocity respectively.

#### 3. APPROXIMATE RAYLEIGH WAVES INPUT UNDER THE EARTHQUAKE

The above formulas are the steady wave. In reality most of the wave type is transient waves, such as earthquake waves. But in the linear medium, the transient wave can be seen as the superposition of the steady wave. Based on the Fourier transform, the transient wave can be expressed as the superposition of harmonic waves. The method will be described in detail as follows.

The approximate earthquake Rayleigh waves can be expressed as the superimposition of the

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harmonious waves. The horizontal and vertical displacement time history of the earthquake can be expressed respectively as:

$$u_{x} = \sum_{n=0}^{\infty} f_{1}^{n} (y) \sin[k_{n} (x - ct)]$$
(3.1a)

$$u_{y} = \sum_{n=0}^{\infty} f_{2}^{n}(y) \cos[k_{n}(x-ct)]$$
(3.1b)

In which

$$f_{1}^{n}(y) = -A_{n}\left[\exp(-ak_{n}y) - \frac{(1+b^{2})}{2}\exp(-bk_{n}y)\right]$$
(3.2a)

$$f_{2}^{n}(y) = A_{n}k_{n}\left[-a\exp(-ak_{n}y) + \frac{(1+b^{2})}{2b}\exp(-bk_{n}y)\right]$$
(3.2b)

 $k_n = \frac{\omega_n}{c}$ ,  $\omega_n = \frac{\pi n}{T}$ , *T* represents the period, and  $A_n$  is the amplitude parameter, which can be derived from the wave motion. The parameter *a*, *b* and *c* can be got from the aforementioned formula.

Assume one of the horizontal earthquake time histories is known as h(t), which can be considered as incident wave at the origin of the coordinate, that is x = 0, y = 0. Substituting x = 0, y = 0 into Eqn. 3.1a and Eqn. 3.2a, h(t) can be expressed as:

$$h(t) = \frac{1 - b^2}{2} \sum_{n=1}^{\infty} A_n \sin(\omega_n t)$$
(3.3)

Considering the orthotropic of the trigonometric functions, the amplitude parameter  $A_n$  can be demonstrated as:

$$A_n = \frac{2}{(1-b^2)} \left[ \frac{2}{T} \int_0^T \sin\left(\omega_n t\right) h(t) dt \right]$$
(3.4)

By substituting the amplitude parameter  $A_i$  for Eqn.3.2, the two dimensional Rayleigh waves field can be obtained. But the calculation will be time consuming using the method mentioned above. And the stability of numerical integral is not easy to guarantee, so the using of the fast Fourier transfer (FFT) method will be the better choice.

Let us consider there is a sampling signal which can be expressed as the discrete one:

$$h_k \equiv h(t_k), \quad t_k \equiv k\Delta, \quad k = 0, 1, 2, \dots, N-1$$
 (3.5)

Where  $\Delta$  is the sampling space,  $t_k$  is the sampling time, the corresponding sampling frequency is  $1/\Delta$ .

By substituting Eqn.3.5 for Eqn.3.4, the amplitude parameter can be obtained as follows:



$$A_{n} = \frac{2}{1-b^{2}} \left[ \frac{2}{T} \int_{0}^{T} \sin(\omega_{n}t) h(t) dt \right] = \frac{2}{1-b^{2}} \frac{2}{N} \left[ \sum_{k=0}^{N-1} h_{k} \sin(\pi i k n / N) \right]$$
(3.6)

The part in the bracket can be obtained using the fast discrete sinusoidal transfer (FDST). By substituting Eqn.3.6 for Eqn.3.2, the functions that describe the variation along the depth can be obtained:

$$f_1^n(y) = -\frac{2}{1-b^2} \frac{2}{N} \left[ \sum_{k=0}^{N-1} h_k \sin(\pi i k n / N) \right] \left[ \exp\left(-\frac{ay}{c} \omega_n\right) - \frac{(1+b^2)}{2} \exp\left(-\frac{by}{c} \omega_n\right) \right]$$
(3.7a)

$$f_2^n(y) = \frac{2}{1-b^2} \frac{2}{N} \left[ \sum_{k=0}^{N-1} h_k \sin\left(\pi i k n / N\right) \right] \left[ -a \exp\left(-\frac{ay}{c}\omega_n\right) + \frac{(1+b^2)}{2b} \exp\left(-\frac{by}{c}\omega_n\right) \right] \frac{\omega_n}{c}$$
(3.7b)

To the arbitrary node (x, y), the displacement can be expressed as

$$u_{x} = \sum_{n=0}^{N-1} f_{1}^{n} \sin(\omega_{n} \frac{x}{c}) \cos(\omega_{n} t) - \sum_{n=0}^{N-1} f_{1}^{n} \cos(\omega_{n} \frac{x}{c}) \sin(\omega_{n} t)$$
(3.8a)

$$u_{y} = \sum_{n=0}^{N-1} f_{2}^{n} \cos(\omega_{n} \frac{x}{c}) \cos(\omega_{n} t) + \sum_{n=0}^{N-1} f_{2}^{n} \sin(\omega_{n} \frac{x}{c}) \sin(\omega_{n} t)$$
(3.8b)

As to now, the whole Rayleigh wave field can be gotten to approximate the earthquake.

#### 4. COMPARISIONS BETWEEN THE ANALYTICAL SOLUTIONS AND NUMERICAL SOLUTIONS

In order to verify the Rayleigh wave excitation method we suggested, some simple but convictive example is conducted. We consider a free field and sin input after Hanning window function. The input is as figure 1.



Fig. 1 The input time history at the free surface

Fig. 2 Sketch map of analysis model

Two dimensional plane strain elements are chosen in the finite element analysis. Figure2 is the sketch map of the finite element model. Considering the character of the finite element formulation of wave



motion, the mesh size is  $1m \times 1m$ .

The horizontal and vertical Rayleigh wave's displacements are input from the left side. In order to reduce the reflection and dispersion of the wave on the boundary, the infinite elements are adopted (Yue etc. 2006). The soil parameters that use in the calculation are as follows: the mass density is  $1800 \text{Kg/m}^3$ , elastic modulus is  $20 \times 10^6$ Pa and Poisson's ratio is 0.3. From the parameters above, we can get that the Rayleigh wave velocity  $c = V_R = 0.926V_S = 60.5604$ ,  $\lambda_R = V_R / f = 20.1868$ m, a = 0.86886, b = 0.37715, k = 0.3112.

The analytical solutions are compared with the numerical solutions to verify the precision of the simulation. Take node B as example. Node B is at the center of the free surface as shown in figure 2.



(a) Horizontal displacement input (b) Vertical displacement input

Fig. 3 Displacement time history of node B

From figure 3, it can be seen that the analytical solution and numerical solution agree well. And the wave propagation can also be seen.

#### 5. THE UTILITY TUNNEL RESPONSE DURING THE PASSAGE OF THE RAYLEIGH WAVES

Based on the above analysis, we did some research on the utility tunnel structure. The calculation model is  $60 \times 30 \times 100$  m in size. The size of structure is  $3 \times 3 \times 100$  m and the thickness of the bottom plate is 0.4m and the top and side plate is 0.3m. The buried depth is 3 m.

The Drucker-Prager model is adopted in the simulation of the soil. And the structure is assumed as elastic, which is reasonable for the underground structure. Table 1 is the material parameters of soil and structure. The slippage and the separation between the soil and structure are not considered here for the time being.



	soil	structure
Unit weight(KN/m <sup>3</sup> )	19.6	25
Poisson's ratio	0.35	0.15
Yong modulus(MPa)	300	24,000
friction angel(degree)	30	
dilation angel(degree)	3	
$k^*$	0.8	
yielding stress(KPa)	500	

Table 1. Material parameters of the soil and structure

\*k is the ratio of the flow stress in tri-axial tension to the flow stress in tri-axial compression.

According to the above technique about the generation of the approximate earthquake Rayleigh waves field, the horizontal and vertical input displacement at the top boundary is as in figure 4. And the input displacement along the depth is obtained according to the character of the Rayleigh wave, which is the functions  $f_1^n(y)$ ,  $f_2^n(y)$ . The approximate Rayleigh earthquake wave excitation is input from the left side.



Fig. 4 The input at the top of the model



Fig. 5 Typical Output point on the utility tunnel

In order to research the structural response systematically, the strain time history of the utility tunnel is studied. Figure 5 is the output point on the utility tunnel.

Figure 6 is the strain time history of the corresponding top and bottom point of the utility tunnel. From the figure, it shows that the structural deformation is mainly the whole bending deformation. And on the other hand, the amplitude of the strain on the top of the structure is about 1 multiple greater than the bottom corresponding one, which agrees with the character of the Rayleigh waves. Figure 7 is the stain time history of the different point A1 and A5. The time delay is displayed clearly. And from the

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figure, the time delay is about 1.15 second and the distance between the point A1 and A5 is 65 meters. So the wave group velocity is about 56.5m/s. On the other hand, from the soil parameters, the shear wave velocity can be got from the formula  $V_s = \sqrt{G/\rho}$  and then using the relation between the shear velocity and Rayleigh wave velocity, the Rayleigh wave velocity can be approximate estimate as 60.5m/s. The two velocities are adjacent which indicate that the wave is stable during the propagation.



Fig. 7 The strain time history

#### 6. CONCLUSIONS

In the research of wave motion theory, the input motion due to the earthquake is a problem that is not solved well. But it is a very important problem to the analysis of the soil and underground structures. The utility tunnel is shallow buried underground structures; the effect of the Rayleigh wave can not be ignored. This paper studied the soil displacement caused by the passage of the Rayleigh waves. And gets the whole approximate Rayleigh earthquake waves field via the FFT. The results of free field indicate that the wave excitation method is proper for the Rayleigh wave. The wave can propagate in the soil steady. The results of the structural analysis show that the deformation of the structure usually



greater under the passage of Rayleigh waves. And from the deformation of the structure, the bending deformation is the greatest. On the other hand, the deformation at the top of the structure is about 1 multiply greater than the corresponding bottom point, which agrees with the character of the Rayleigh waves.

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