A MEASURE OF DRIFT DEMAND FOR EARTHQUAKE GROUND MOTIONS BASED ON TIMOSHENKO BEAM MODEL

Junju XIE[1] and Zenping WEN[2]

[1] *PH. D. Student, Institute of Geophysics, China Earthquake Administration, Beijing China*
[2] *Professor, Institute of Geophysics, China Earthquake Administration, Beijing China*
E-mail: wenzp@cea-igp.ac.cn, xiejunjv05@mails.gucas.ac.cn

ABSTRACT:
A new measure of demands for earthquake ground motion is presented. It is base on a continuous Timoshenko beam model. While considering the effect of nonuniform distribution of drift demands along the building height and taking into account the coupling effect of bending and shear distortion on the vibration of cantilever beam structures, the drift spectrum leads to more accurate estimations of drift demand for earthquake ground motions. This continuous model is compared with the combined flexural-shear beam model proposed by Miranda. Similar to the model proposed by Miranda, the dynamic properties of the cantilever Timoshenko beam model are highly influenced by a parameter called the height-width ratio. A formula is given to indicate the relation of height-width ratio with lateral stiffness ratio. The effects of height-width ratio, damping and higher modes on drift demands are examined.

KEYWORDS: earthquake, drift demands, ground motion, Timoshenko theory, cantilever beam.

1. INTRODUCTION

The controlling of displacement during the design process of building structures has been given more concern since the 1971 San Fernando earthquake. It has also been shown that the use of lateral displacements as demand parameters could permit a more direct way to control the damage in the building structures during the design process. Among all of the displacement related parameter, the lateral story drift (defined as the ratio of story drift between two consecutive floors to story height) are considered as a significant cause that leads to the damage of building structures when subjected to earthquake ground motion. (Moehle 1984 1992; Wallace 1994; Wallace and Moehle 1992).

Examination of damage patterns during the earthquake showed that buildings in the near field might suffer large localized ductile displacement and story drifts, it could not be associated with the peak accelerations alone. The near-field ground motions often contain coherence waveforms that appear as distinct velocity or displacement pulses with high intensity. The coherent pulse-like nature of the near-field ground motion time history may cause the maximum response of the structure to occur before a resonant mode-like response can build up. These ground pulses may be associated with substantial inelastic response as well as higher-mode effects that generally cannot be represented adequately by response spectrum. In this case, Iwan（1997） introduced a simple measure of drift demands for earthquake ground motion called the drift spectrum. It is based on the wave analysis of a continuous shear beam model. The drift spectrum has become an increasingly important topic since then. Following Iwan, many researchers studied the drift spectrum in different ways. Chopra and Chintanapakdee (2001) show that drift spectra based on a shear beam model could also be computed using conventional modal analysis. Gulkan and Akkar (2002) introduced a simple replacement method for the drift spectrum. Kim and Collins's study (2002) indicated that Iwan's model could result in residual drifts for certain ground motion. Miranda (2006) used a combined flexural-shear beam model to obtain drift demands for earthquake ground motions, he introduced the generalized interstory drift spectrum which is applicable for different kinds of building structures which deform in the way of a combined flexural and shear type to estimate the maximum interstory drift demands.

The elastic drift spectrum directly shows the story drift that a ground motion record would cause in a multistory shear building. It is a convenient tool that displays the maximum local drift demands at chosen elevation in buildings, which is very important during the building design process. Despite its revolutionary concept, the drift spectrum introduced in Iwan's model also has some disadvantages. It's based on the wave analysis of a continuum shear beam, the effect of bending is not considered in the shear beam model. Furthermore, the wave analysis is not familiar with structural engineers and could result in residual drifts for certain ground motion as identified by Kim and Collins.

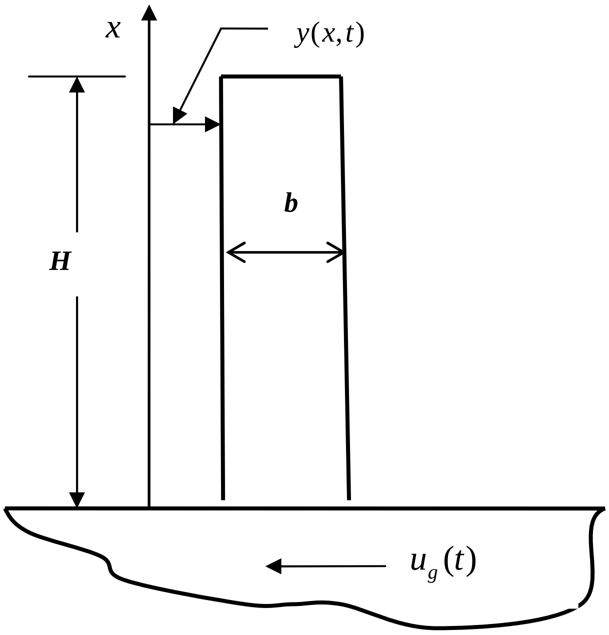

Figure 1 Simplified model for cantilever structures

In this paper, an alternative method to estimate the drift demands for ground motions is developed and presented. Continuous cantilever beam model based on Timoshenko beam theory is utilized to estimate drift spectrum which is a measure of demand for earthquake ground motions on structures (see Fig.1). It has been commonly recognized that the interstory drift demands are not uniformly distributed along the building height. While considering this kind of nonuniform effect and taking into account the coupling effect of bending and shear distortion on the vibration of cantilever beam structures, the drift spectrum leads to more accurate estimations of drift demand for earthquake ground motions. This study can also avoid the residual drifts problem by using the conventional modal analysis techniques. The maximum drift spectra can supply as a rapid evaluation for the drift demands of earthquake ground motions, which is very important for the design of buildings to resist the seismic load.

2. CONTINUOUS BEAM MODELS

Many researchers have used the continuous beams to model the behavior of cantilever structures when subjected to static and dynamic loads since the last century. Westergaard (1933) introduced a continuous shear beam model with uniform-mass and stiffness to estimate lateral deformations of buildings subjected to earthquake ground motion. Rosenblueth et al. (1968) used shear beams and response spectrum analysis to estimate story shears and overturning moments in buildings structures. Montes and Rosenblueth (1968) used flexural beam model to estimate shear and overturning moment demands in chimneys. Based on a continuum model consisting of combination of a flexural beam and a shear beam, Miranda (1999, 2002) evaluated drift and acceleration response for different kinds of building structures.

The Timoshenko beam theory was first introduced by Timoshenko in 1921. In this continuous beam theory, Timoshenko (1921; 1922) extended to include the effect of transverse shear deformation on the basis of Rayleigh beam theory. Following Timoshenko, many researchers studied the dynamics and vibrations of Timoshenko beams using traveling wave method or model vibration analysis (Miklowitz 1953; Anderson 1953). More recently, Geist and McLaughlin (1997) discussed the phenomena of double eigenvalues in Timoshenko beams. Kausel (2002) studied the effect of rotational motion on the normal modes of unrestrained shear beams.

Aristizabal-Ochoa (2004) investigated the vibration of Timoshenko beam-column with generalized support conditions and subjected to a constant axial load along its span. Challamel (2006) compared Timoshenko and shear models in beam dynamics.

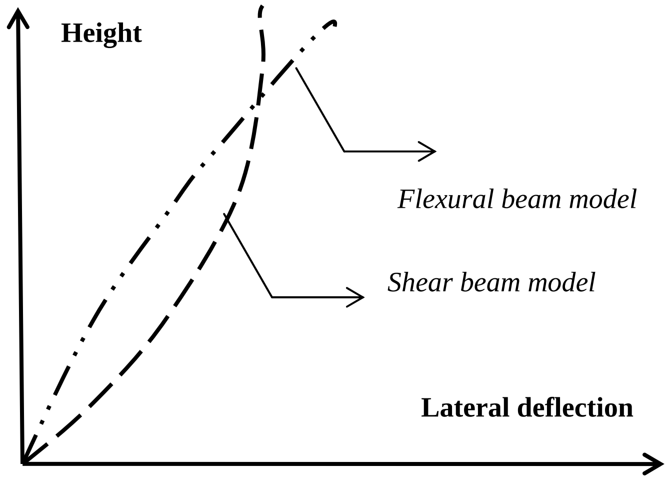

Figure 2 Deflection shapes of flexural beam model and shear beam model

Many studies have shown that the structures may deform in a flexural type or a shear type when subjected to lateral loads (see Fig.2), but more commonly, the structures deform in the form of a combined flexural-shear shape, and the overall lateral deformation is denoted by a superposition of shear deformation and flexural deformation (Khan and Sbarounis 1964; Heidebrecht and Stafford Smith 1973; Miranda 1999; Miranda and Carlos J 2002). In this paper, the coupling effects of bending and shear distortion on the vibration of cantilever beam structures are considered according to Timoshenko theory. It is shown that the joint effects of bending and shear deformations in Timoshenko beam structures are highly influenced by the height-width ratio. It is interesting to note that the deflection shapes of cantilever Timoshenko beam in the fundamental mode exhibit the form of a flexural type or a shear type when a certain value of beam height-width ratio is given.

3. DYNAMIC PROPERTIES OF TIMOSHENKO BEAMS

The governing equations for the free vibration of Timoshenko beam structures (see Fig.1 and Fig.3) are

$$EI\frac{\partial^2\theta}{\partial x^2}+k^{'}GA(\frac{\partial y}{\partial x}-\theta)-\rho I\frac{\partial^2\theta}{\partial t^2}=0 \tag{3.1}$$

$$k^{'}GA(\frac{\partial^2 y}{\partial x^2}-\frac{\partial\theta}{\partial x})-\rho A\frac{\partial^2 y}{\partial t^2}=0 \tag{3.2}$$

The Timoshenko beam in the model has the following material property: density ρ, Young modulus E and shear modulus G. Its transverse cross section is denoted by total area A and moment of inertia $I = Ar^2$ ($r =$radius of gyration of the cross section). k' is the shear correction coefficient depending on the shape of the cross section and material's Poisson ratio υ; y is the transverse deflection; $\frac{\partial y}{\partial x}$ is the slope of the centerline of the beam; θ is the rotation of the cross section. The transverse deflection y of Timoshenko beam is denoted by a superposition of shear deformation and bending deformation. The relation is denoted by

$$\frac{\partial y}{\partial x}=\gamma+\theta \tag{3.3}$$

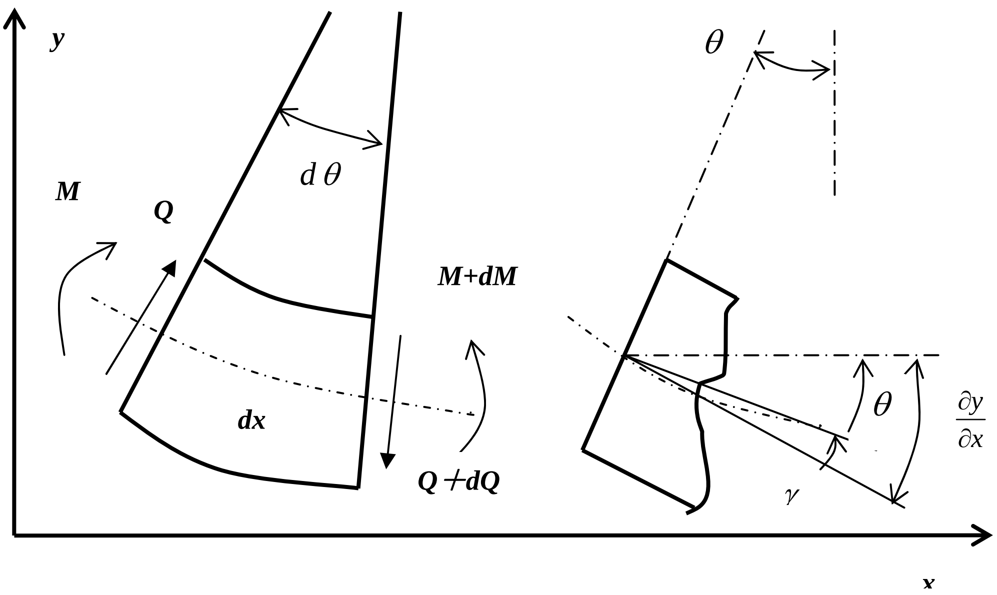

Figure 3 Differential analysis of Timoshenko beams

The solutions to Eqn. 3.1 and 3.2 are of the form

$$y(x,t) = \phi(x)\sin(\omega_n t) \tag{3.4}$$

$$\theta(x,t) = \psi(x)\sin(\omega_n t) \tag{3.5}$$

Substituting Eqn. 3.4 and Eqn. 3.5 into Eqn. 3.1 and Eqn. 3.2, and eliminating all functions of time, the following pair equations in terms of the shape functions $\phi(x)$ and $\psi(x)$ are obtained:

$$EI\left(\frac{d^2\phi}{dx^2}\right) + k'GA\left(\frac{d\phi}{dx} - \psi\right) + \rho I\omega_n^2\psi = 0 \tag{3.6}$$

$$k'GA\left(\frac{d^2\phi}{dx^2} - \frac{d\psi}{dx}\right) + \rho A\omega_n^2\phi = 0 \tag{3.7}$$

Eqn. 3.1 and 3.2 describe coupled rotation and lateral displacement of the uniform beam, where θ can be eliminated to form a singular equation

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} - \rho I\left(1 + \frac{E}{k'G}\right)\frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{\rho^2 I}{k'G}\frac{\partial^4 y}{\partial t^4} = 0 \tag{3.8}$$

For a continuous cantilever beam with total height of H , introducing the following nondimensional variables:

$$\bar{x} = \frac{x}{H}$$

and

$$\bar{\phi} = \frac{\phi}{H}$$

Using the nondimensional variables just listed above, substituting Eqn. 3.4 into Eqn. 3.8, and eliminating the functions of time, then the following differential equation is obtained:

$$\frac{d^4\bar{\phi}}{d\bar{x}^4} + \rho\omega_n^2 H^2\left(\frac{1}{E} + \frac{1}{k'G}\right)\frac{d^2\bar{\phi}}{d\bar{x}^2} - \frac{\rho A\omega_n^2 H^4}{EI}\left(1 - \frac{\rho r^2\omega_n^2}{k'G}\right)\bar{\phi} = 0 \tag{3.9}$$

Introducing the following parameters:

$$\mu = \frac{E}{k'G} \tag{3.10}$$

$$\alpha = H\sqrt{\frac{\rho}{k'G}} = H\sqrt{\frac{m}{k'GA}} \tag{3.11}$$

$$\beta = H^2\sqrt{\frac{\rho A}{EI}} = H^2\sqrt{\frac{m}{EI}} \tag{3.12}$$

where $m = \rho A$ is the uniform mass per unit length of the continuum beam, then Eqn. 3.9 can be written as follows:

$$\bar{\phi}^{(4)} + \alpha^2\omega_n^2\left(1+\frac{1}{\mu}\right)\bar{\phi}'' - \left(\beta^2\omega_n^2 - \frac{1}{\mu}\alpha^4\omega_n^4\right)\bar{\phi} = 0 \tag{3.13}$$

Similar to the dimensionless parameter in the combined flexural-shear beam model proposed by Miranda, the lateral stiffness ratio η of the Timoshenko beam is defined as

$$\eta = \frac{\beta}{\alpha} = H\sqrt{\frac{k'GA}{EI}} \tag{3.14}$$

Substituting $I = Ar^2$ and $\mu = \frac{E}{k'G}$ into Eqn. 3.14, we have

$$\eta = \frac{H}{r}\sqrt{\frac{k'G}{E}} = \frac{H}{r}\sqrt{\frac{1}{\mu}} \tag{3.15}$$

The solution of Eqn. 3.13 is of the form

$$\bar{\phi} = Ce^{\lambda\bar{x}} \tag{3.16}$$

The following polynomial is obtained after substituting the above equation into Eqn. 3.13

$$\lambda^4 + \alpha^2\omega_n^2\left(1+\frac{1}{\mu}\right)\lambda^2 - \left(\beta^2\omega_n^2 - \frac{1}{\mu}\alpha^4\omega_n^4\right) = 0 \tag{3.17}$$

The solution of the above equation is of the form

$$\begin{aligned}\lambda_1^2 &= \frac{-\alpha^2\omega_n^2}{2}\left[\left(1+\frac{1}{\mu}\right) + \sqrt{\left(1-\frac{1}{\mu}\right)^2 + \frac{4\eta^2}{\alpha^2\omega_n^2}}\right]\\ \lambda_2^2 &= \frac{-\alpha^2\omega_n^2}{2}\left[\left(1+\frac{1}{\mu}\right) - \sqrt{\left(1-\frac{1}{\mu}\right)^2 + \frac{4\eta^2}{\alpha^2\omega_n^2}}\right]\end{aligned} \tag{3.18}$$

The sign of λ_1^2 is clearly negative. However, the sign of λ_2^2 is indefinite, it is controlled by the sign of

$$\Delta = \frac{1}{\mu}\frac{\alpha^2\omega_n^2}{\eta^2} - 1 \tag{3.19}$$

Case 1: when $\Delta < 0$, the sign of λ_2^2 is positive.

The four roots of Eqn. 3.17 are

$$\lambda_1 = \pm j\delta_n, \qquad \lambda_2 = \pm\varepsilon_n$$

where

$$\begin{aligned}\delta_n &= \frac{\sqrt{2}\alpha\omega_n}{2}\sqrt{\left(1+\frac{1}{\mu}\right) + \sqrt{\left(1-\frac{1}{\mu}\right)^2 + \frac{4\eta^2}{\alpha^2\omega_n^2}}}\\ \varepsilon_n &= \frac{\sqrt{2}\alpha\omega_n}{2}\sqrt{-\left(1+\frac{1}{\mu}\right) + \sqrt{\left(1-\frac{1}{\mu}\right)^2 + \frac{4\eta^2}{\alpha^2\omega_n^2}}}\end{aligned} \tag{3.20}$$

The above relation of Eqn. 3.20 can also be expressed as follows:

$$\begin{aligned}\delta_n^2 - \varepsilon_n^2 &= (1+\frac{1}{\mu})\alpha^2\omega_n^2\\ \delta_n^2\varepsilon_n^2 &= \beta^2\omega_n^2 - \frac{1}{\mu}\alpha^4\omega_n^4\end{aligned} \tag{3.21}$$

The function of the lateral deflection $\bar{\phi} = \phi / H$ is expressed as follows

$$\bar{\phi} = C_1\sin(\delta_n\bar{x}) + C_2\cos(\delta_n\bar{x}) + C_3\sin h(\varepsilon_n\bar{x}) + C_4\cos h(\varepsilon_n\bar{x}) \tag{3.22}$$

Substituting $\phi = H\bar{\phi}$ and $x = H\bar{x}$ into Eqn. 3.7, the shape function of the rotation ψ is obtained in the form of

$$\psi = \chi\left[C_1\cos(\delta_n\bar{x}) - C_2\sin(\delta_n\bar{x})\right] + \vartheta\left[C_3\cosh(\varepsilon_n\bar{x}) + C_4\sinh(\varepsilon_n\bar{x})\right] \tag{3.23}$$

where

$$\chi = \delta_n - \frac{\alpha^2\omega_n{}^2}{\delta_n}$$

and

$$\vartheta = \varepsilon_n + \frac{\alpha^2\omega_n{}^2}{\varepsilon_n}$$

C_1, C_2, C_3, and C_4 are coefficients that depend on the following boundary conditions:

At the base where $\bar{x} = 0$:

$$\bar{\phi}(0) = 0 \quad ; \quad \psi(0) = 0$$

At the roof where $\bar{x} = 1$:

$$\frac{d\psi}{d\bar{x}}(1) = 0 \quad ; \quad \frac{d\bar{\phi}(1)}{d\bar{x}} - \psi(1) = 0$$

Four homogeneous equations in terms of coefficients C_1, C_2, C_3 and C_4 is obtained according to the boundary conditions. Setting the determinant of this set of equations equal to zero, and eliminating the parameters α and β, the characteristic equation can be expressed as follows:

$$\begin{aligned}&[(\delta_n^2 + \mu\varepsilon_n^2)^2 + (\mu\delta_n^2 + \varepsilon_n^2)^2]\cos\delta_n\cosh\varepsilon_n - (\delta_n^2 + \mu\varepsilon_n^2)(\varepsilon_n^2 + \mu\delta_n^2)\\ &(-2 + \frac{\delta_n^2 - \varepsilon_n^2}{\delta_n\varepsilon_n}\sin\delta_n\sinh\varepsilon_n) = 0\end{aligned} \tag{3.24}$$

Using the relation $\eta = \dfrac{\beta}{\alpha}$, eliminating α, β and ω_n from Eqn. 3.21, the following equation in terms of eigenvalues δ_n and ε_n is obtained:

$$(\delta_n^2 - \varepsilon_n^2)\left[\eta^2 - \frac{1}{\mu+1}(\delta_n^2 - \varepsilon_n^2)\right] - \left(1 + \frac{1}{\mu}\right)\delta_n^2\varepsilon_n^2 = 0 \tag{3.25}$$

The eigenvalues δ_n and ε_n of the nth mode can be easily obtained from Eqn. 3.24 and 3.25. As it is shown above, double eigenvalues δ_n and ε_n depend on the lateral stiffness ratio η and the dimensionless ratio μ. The mode shape of lateral deflection normalized by $C_2 = 1$ is expressed as follows:

$$\bar{\phi}_n = \zeta_n\sin(\delta_n\bar{x}) + \cos(\delta_n\bar{x}) - \zeta_n\frac{\left(\delta_n + \dfrac{\mu\varepsilon_n^2}{\delta_n}\right)}{\left(\varepsilon_n + \dfrac{\mu\delta_n^2}{\varepsilon_n}\right)}\sinh(\varepsilon_n\bar{x}) - \cosh(\varepsilon_n\bar{x}) \tag{3.26}$$

Where ζ_n is nondimensional parameter for the nth mode of vibration and given by

$$\zeta_n = \frac{\cos\delta_n + \dfrac{\left(\varepsilon_n^2 + \mu\delta_n^2\right)}{\left(\delta_n^2 + \mu\varepsilon_n^2\right)}\cosh\varepsilon_n}{-\left(\sin\delta_n + \dfrac{\varepsilon_n}{\delta_n}\sinh\varepsilon_n\right)} \tag{3.27}$$

Taking the derivative of the shape function of lateral deflection $\bar{\phi}_n$, we obtain

$$\frac{d\bar{\phi}_n}{d\bar{x}} = \zeta_n\delta_n\cos(\delta_n\bar{x}) - \delta_n\sin(\delta_n\bar{x}) - \zeta_n\frac{\left(\delta_n + \dfrac{\mu\varepsilon_n^2}{\delta_n}\right)}{\left(1 + \dfrac{\mu\delta_n^2}{\varepsilon_n^2}\right)}\cosh(\varepsilon_n\bar{x}) - \varepsilon_n\sinh(\varepsilon_n\bar{x}) \tag{3.28}$$

According to Eqn. 3.21, the circular frequency ω_n of the nth mode can be computed by

$$\omega_n^2 = \frac{\mu}{(\mu+1)} \frac{\left(\delta_n^2 - \varepsilon_n^2\right)}{\alpha^2} \tag{3.29}$$

where δ_n and ε_n are eigenvalues of the nth mode of vibration corresponding to the nth roots of the characteristic equations 3.24 and 3.25.

Case 2: when $\Delta > 0$, the sign of λ_2^2 is negative.
For higher frequency modes with larger value of ω_n, the four roots of Eqn. 3.17 are

$$\lambda_1 = \pm j\delta_n, \qquad \lambda_2 = \pm j\sigma_n$$

where

$$\delta_n = \frac{\sqrt{2}\alpha\omega_n}{2}\sqrt{\left(1+\frac{1}{\mu}\right) + \sqrt{\left(1-\frac{1}{\mu}\right)^2 + \frac{4\eta^2}{\alpha^2\omega_n^2}}}$$

and

$$\sigma_n = \frac{\sqrt{2}\alpha\omega_n}{2}\sqrt{\left(1+\frac{1}{\mu}\right) - \sqrt{\left(1-\frac{1}{\mu}\right)^2 + \frac{4\eta^2}{\alpha^2\omega_n^2}}}$$

Using the boundary condition as case 1, the transverse deflection $\bar{\phi}$ in the mode shape are obtained as follows:

$$\bar{\phi}_n = \zeta_n' \sin(\delta_n \bar{x}) + \cos(\delta_n \bar{x}) - \zeta_n' \frac{\left(\delta_n - \frac{\mu\sigma_n^2}{\delta_n}\right)}{\left(\sigma_n - \frac{\mu\delta_n^2}{\sigma_n}\right)} \sin(\sigma_n \bar{x}) - \cos(\sigma_n \bar{x}) \tag{3.30}$$

where

$$\zeta_n' = \frac{\cos\delta_n - \frac{\left(\sigma_n^2 - \mu\delta_n^2\right)}{\left(\delta_n^2 - \mu\sigma_n^2\right)}\cos\sigma_n}{-\left(\sin\delta_n - \frac{\sigma_n}{\delta_n}\sin\sigma_n\right)} \tag{3.31}$$

The circular frequency ω_n of the nth mode at this case is computed by

$$\omega_n^2 = \frac{\mu}{(\mu+1)} \frac{\left(\delta_n^2 + \sigma_n^2\right)}{\alpha^2} \tag{3.32}$$

4. INFLUENCE OF HEIGHT-WIDTH RATIO ON DYNAMIC PROPERTIES OF CONTINUOUS TIMOSHENKO MODEL

Considering a uniform beam with rectangular cross section, and that the beam is constructed of a homogeneous isotropic material with Poisson's ratio υ_0. Common values of Poisson's ratio υ_0 are 0.25 to 0.30 for steel, approximately 0.33 for most other metals, and 0.20 for concrete. The shear correction coefficient k' can be calculated using the following equation（Cowper, 1966）

$$k' = \frac{10(1+\upsilon_0)}{12+11\upsilon_0} \tag{4.1}$$

The relation of Young modulus E and shear modulus G can also be written in terms of υ_0 using the well-known formula (see for instance Popov 1968)

$$\frac{E}{G} = 2(1+\upsilon_0) \tag{4.2}$$

Substituting Eqn. 4.1 and 4.2 into Eqn. 3.15, the dimensionless ratio μ can be expressed as follows:

$$\mu = \frac{12+11\upsilon_0}{5} \tag{4.3}$$

Let $\upsilon_0 = \dfrac{3}{11}$, then $\mu = 3$, and according to Eqn. 3.15 the lateral stiffness ratio

$$\eta = \frac{\sqrt{3}}{3}\frac{H}{r} \tag{4.4}$$

For a uniform beam with rectangular cross section, the radius of gyration $r = \dfrac{\sqrt{3}b}{6}$, where b =width of the cross section along the vibration direction (see Fig.1); Introducing the parameter called the height-width ratio

$$R = \frac{H}{b} \tag{4.5}$$

Then the lateral stiffness ratio η is computed by:

$$\eta = 2R \tag{4.6}$$

According to Eqn. 3.24-3.25 and Eqn. 4.4-4.6, when the dimensionless ratio μ has a constant value of 3, the eigenvalues δ_n and ε_n is determined by the height-width ratio R. That is to say, the mode shapes of lateral deflection, rotation and shear distortion in the continuous model depend only on a single parameter, the height-width ratio R.

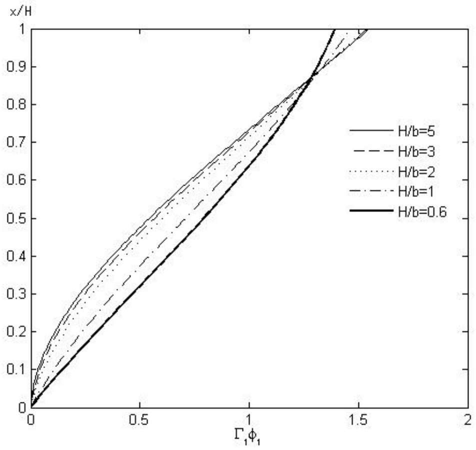

Figure 4 Effects of height- width ratio on deflection of the fundamental mode

Figure 4 shows lateral deflection shape of continuous Timoshenko beam in the fundamental mode. Products corresponding to five values of height-width ratio are shown for to examine the influence of this ratio on the deflection shape. It can be seen that the dynamic properties of the continuous Timoshenko beam model are highly influenced by the height-width ratio, the deflection shows a shape of a flexural type for bigger values of height-width ratio and it changes to a shear type when this ratio becomes smaller.

5. EARTHQUAKE RESPONSE HISTORY MODAL ANALYSIS OF THE CONTINUOUS MODEL

The response of the uniform Timoshenko beam model shown in Fig.1 when subjected to a ground motion at the base $\ddot{u}_g(t)$ is given by the following partial differential equation:

$$\frac{EI}{H^4}\frac{\partial^4 y}{\partial \bar{x}^4} + \rho A\frac{\partial^2 y}{\partial t^2} - \frac{\rho I}{H^2}\left(1 + \frac{E}{k^{'}G}\right)\frac{\partial^4 y}{\partial \bar{x}^2 \partial t^2} + \frac{\rho^2 I}{k^{'}G}\frac{\partial^4 y}{\partial t^4} = -\rho A\frac{\partial^2 u_g(t)}{\partial t^2} \tag{5.1}$$

The total lateral displacement response of the system can be calculated by superposition of all modes.

$$y(\bar{x},t) = \sum_{i=1}^{n} y_i(\bar{x},t) \tag{5.2}$$

where $y_i(\bar{x},t)$ =contribution of the ith mode with classic damping ratio, and it is given by

$$y_i(\bar{x},t) = \Gamma_i \phi_i(\bar{x}) D_i(t) \tag{5.3}$$

Where Γ_i is modal participation factor of the ith mode of vibration; $\phi_i(\bar{x})$ =amplitude of the ith deflection shape of vibration at nondimensional height $\bar{x}$; and $D_i(t)$ =deformation response of a single-degree-of-freedom (SDOF) system corresponding to the ith mode, whose response is computed with the following equation:

$$\ddot{D}_i(t)+2\xi_i\omega_i\dot{D}_i(t)+\omega_i^2D_i(t)=\ddot{u}_g(t) \tag{5.4}$$

For a continuum model with uniform mass, the modal participation factor of the ith mode is given by

$$\Gamma_i=\frac{\int_0^1\phi_i\left(\bar{x}\right)d\bar{x}}{\int_0^1\phi_i^2\left(\bar{x}\right)d\bar{x}} \tag{5.5}$$

The lateral deflection $\phi(\bar{x})$ consists of shear deformation γ and bending deformation ψ. Lateral drift ratio of the system is defined as the derivative of total deflection $y(\bar{x},t)$ with respect to x as follows:

$$IDR=\frac{\partial y(\bar{x},t)}{\partial x} \tag{5.6}$$

From Eqn. 5.2 and 5.3, we have

$$\frac{\partial y(\bar{x},t)}{\partial x}=\frac{1}{H}\sum_{i=1}^{\infty}\Gamma_i\phi_i'(\bar{x})D_i(t) \tag{5.7}$$

The ordinates of drift demand spectrum are defined as maximum lateral drift ratio over the height of the building structures and are given by

$$IDR_{\max}=\max_{\forall t,x}\left(\frac{\partial y(\bar{x},t)}{\partial x}\right) \tag{5.8}$$

6. INFLUENCE OF HEIGHT-WIDTH RATIO ON INTERSTORY DRIFT DEMANDS

Giving the material properties as Young's modulus E =30Gpa, mass density ρ =2500 $\mathrm{kg/m^3}$, the circular frequency ω_n of cantilever structures with height H can be computed using Eqn. 3.29 and 3.32 according to solution case 1 and 2 respectively, and the periods of the vibration modes can be computed by $T_n=\frac{2\pi}{\omega_n}$. The maximum interstory drifts are computed according to Eqn. 5.7 and 5.8, and only the first three modes are considered.

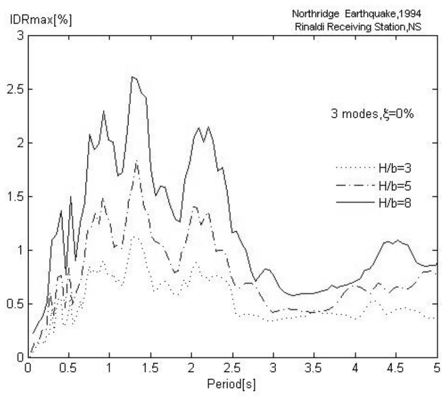

Figure 5 Influence of height-width ratio on drift demand spectra

Figure 5 presents interstory drift demand spectrum for undamped models subjected to the NS component of the ground motion recorded at Rinaldi Receiving Station during the 1994 Northridge earthquake when the height-width ratio is equal to 3, 5 and 8 respectively. It can be seen that the ordinates of the drift spectra augment with increasing height-width ratio. That is to say, the cantilever structures with bigger height-width ratio will suffer larger drifts and tend to be more fragile when subjected to the earthquake ground motions.

7. INFLUENCE OF DAMPING ON INTERSTORY DRIFT DEMANDS

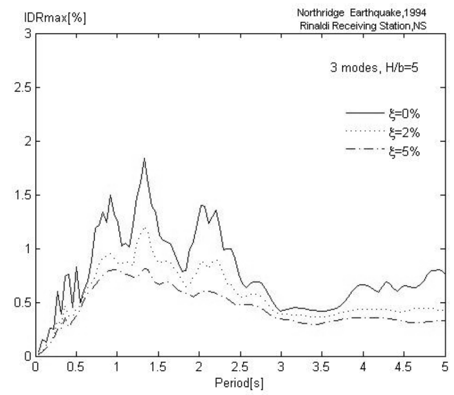

Figure 6 Influence of damping on interstory drift demands

Figure 6 shows changes in interstory drift demands owing to difference in damping when subjected to the Rinaldi Receiving Station ground motion during the 1994 Northridge earthquake. The drift spectrum is obtained by giving height-width ratio a constant value of 5. The interstory drift demands are computed using the first three modes and the damping ratio is assumed to be the same in all modes. For three different levels of damping 0, 2% and 5%, it can be seen that the interstory drift demands decrease with increasing damping and the influence varies in different vibration period range.

8. INFLUENCE OF HIGHER MODES ON INTERSTORY DRIFT DEMANDS

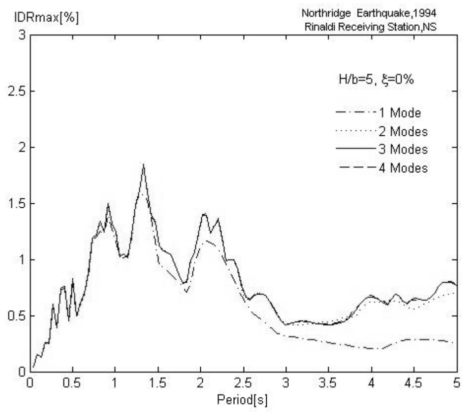

Figure 7 Influence of higher modes on interstory drift demands

Figure 7 presents drift spectra of the NS component of the Rinaldi Receiving Station record using 1, 2, 3 and 4 modes respectively. In all cases, the damping ratio is assumed to be zero. It can be seen that demands computed with 3 modes are practically the same as those computed with 4modes. It is concluded that only a small number of modes is enough to obtain maximum interstory drift demands. For this record, only 3 modes can provide adequate interstory drift estimates. Using only the fundamental mode is enough to obtain drift estimates when the fundamental periods are smaller than 2s. However, when the periods are longer than 2s, using only the first mode will result in underestimation of maximum interstory drift ratio.

9. COMPARISON WITH PREVIOUS STUDY

The vibration properties of this continuous Timoshenko beam model are compared with the combined flexural-shear beam model proposed by Miranda in Fig. 8. In the combined flexural-shear beam model proposed by Miranda, the vibration modes in the continuous model depend on the parameter called lateral stiffness ratio α. As shown in Fig. 8, a value of $\alpha = 0$ corresponds to a flexural beam while a value of $\alpha = 650$ corresponds approximately to a shear beam. Intermediate values of α, represent structural behaviors that combine flexural and shear deformation. Similar to the model proposed by Miranda, the mode shapes in the cantilever Timoshenko beam model are controlled by the height-width ratio. Fig.8 also shows lateral deflection shape of continuous Timoshenko beam in the fundamental mode. Products corresponding to five values of height-width ratio are shown for to examine the influence of this ratio on the deflection shape in the fundamental mode. It can be seen that the deflection shows a shape of a flexural type for bigger values of height-width ratio and it changes to a shear type when this ratio becomes smaller. Similar to the generalized interstory drift spectrum proposed by Miranda, the drift spectrum in this study is associated with a parameter called the height-width ratio. In this study, it is showed that the lateral stiffness ratio is determined by the height-width ratio based on the continuum model composed of a single Timoshenko beam, and this study indicated that the drift spectrum is highly influenced by the height-width ratio.

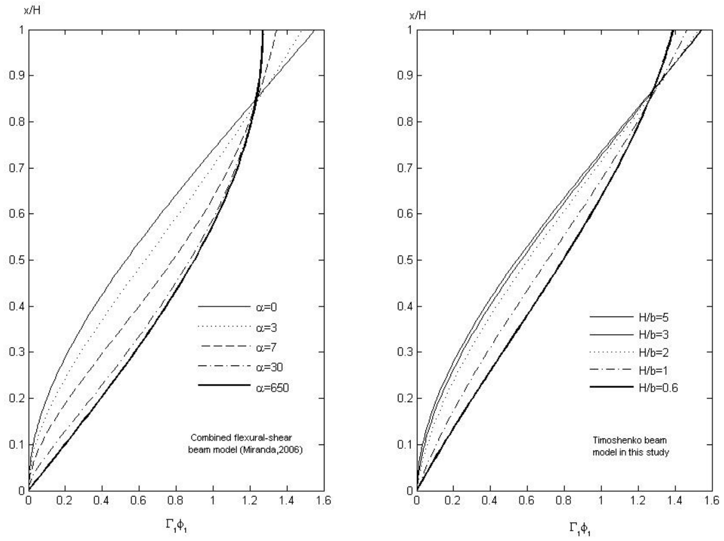

Figure 8 Comparison of Timoshenko beam model in this study with the combined flexural-shear beam model proposed by Miranda

In former study of Iwan(1997) and Miranda (2006), the fundamental periods of the continuous model were computed using experimental relationship suggested for steel-moment-resisting frames in the 1997 UBC code, namely, $T_1 = 0.0853H^{0.75}$. As pointed out by Miranda (2006) in his study, there is a significant uncertainty in the estimation of fundamental periods for buildings when using the experimental period relationship, and this uncertainty will cause uncertainty in the estimation of interstory drift demands. In this study, the periods of the continuous model are computed using the theoretical formula based on Timoshenko beam theory, so the model in this study can avoid the uncertainty in the estimation of interstory drift demands due to period uncertainty. However, the author thinks that this theoretical equation could only be accurate enough when used to evaluate the vibration periods of RC shear wall structures, but it should not be used to compute the periods for all kinds of building structures.

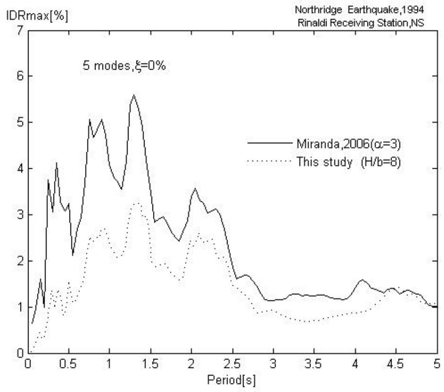

Figure 9 Comparison of interstory estimates using Timoshenko beam model in this study with the model proposed by Miranda subjected to Rinaldi record

Figure 9 shows the Comparison of interstory drift estimates using Timoshenko beam model in this study with the model proposed by Miranda when subjected to Rinaldi record. As is shown by this figure, the interstory drift demands obtained in this study based on the Timoshenko beam model are smaller than the results obtained using Miranda's model. This is accordant with the knowledge that the lateral resisting system such as the shear walls has the effects of reducing interstory drift demands, which has been confirmed by Miranda in his study. It is concluded that the drift spectrum based on the continuous Timoshenko beam model in this study is restricted for the evaluation of lateral drifts for shear wall structures. It might not be adequate to estimate lateral drift demands for other building structures when using the simplified cantilever beam model.

10. SUMMARY AND CONCLUSIONS

A new method to estimate the seismic demand of ground motions is developed and presented in this paper. Providing an estimate of maximum interstory drift demands for earthquake ground motions based on the continuous Timoshenko beam model, the interstory drift spectrum in this study can directly show us damage potential for the cantilever structures with different fundamental periods when subjected to earthquake ground motions.

The drift demands of this cantilever Timoshenko beam model are highly influenced by the height-width ratio. It was shown that the cantilever beam structures with bigger height-width ratio will suffer larger drifts. It was also conclude that drift spectrum is much correlated with damping, and the influence of higher modes can not be neglected in the estimation of drift demands for cantilever structures when the fundamental periods are longer than 2s.

REFERENCES

Anderson, R. A. (1953). Flexural vibration in uniform beams according to the Timoshenko theory. ASME, *J. Appl. Mech*., **28,** 579-584.
Akkar, S., Yazgan, U., and Gülkan, P. (2005). Drift estimates in frame buildings subjected to near-fault ground motions. *J. Struct. Eng.,* **131:7,** 1014-1024.
Aristizabal-Ochoa, J. Dario. (2004). Timoshenko beam-column with generalized end conditions and nonclassical modes of vibration of shear beams. *J. Eng. Mech*., **130:10,** 1151-1159.
Challamel, N. (2006). On the comparison of Timoshenko and shear models in beam dynamics. *J. Eng. Mech*., **132：10**，1141-1145.
Chopra,A.K., and Chintanapakdee,C. (2001). Drift spectrum vs. modal analysis of structural response to near-fault ground motions. *Earthquake Spectra*, **17:2,** 221-234.
Cowper, G. R. (1966). The shear coefficient in Timoshenko's beam theory. ASME, *J. Appl. Mech.*, **33,** 335-340.
Geist, B., and McLauglin, J. R. (1997). Double eigenvalues for the uniform Timoshenko beam. *Appl. Math. Lett.*, **10:3,** 129-134.
Gulkan,P., and Akkar, S. (2002). A simple replacement for the drift spectrum. *Engineering Structures*, **24:11,**

1477-1484.
Heidebrecht, A. C., and Stanford Smith, B. (1973). Approximate analysis of tall wall-frame structures. *J. Struct. Div.*, ASCE, **99:2,** 199-221.
Iwan, W. D., and Chen.X . (1994). Important near-field ground motion data from the Landers earthquake. *Proc., 10th European Conf. on Earthquake Engineering*, A. A. Balkema, Rotterdam. The Nether-lands, 229-234.
Iwan, W. D. (1997). Drift spectrum: Measure of demand for earthquake ground motions. *J. Struct. Eng*., **123:4,** 397-404.
Kausel, E. (2002). Nonclassical modes of unrestrained shear beams. *J. Eng. Mech*., **128:6,** 663-667.
Khan, F.R., and Sbarounis , J. A. (1964). Interaction of shear walls and frames. *J. Struct. Div*., ASCE, **90:3,** 285-335.
Miklowitz, L. (1953). Flexural wave solutions of coupled equations representing the more exact theory of bending. ASME, *J. Appl. Mech.*, **28,** 511-514.
Miranda, E. (1999). Approximate lateral deformation demands in multistory buildings. *J. Struct. Eng*., **125:4,** 417-425.
Miranda, E., and Carlos J.R. (2002). Approximate lateral drift demands in multistory buildings with nonuniform stiffness. *J. Struct. Eng*., **128:7,** 840-849.
Miranda, E., and Taghavi, S. (2005). Approximate floor acceleration demands in multistory buildings. *J. Struct. Eng*., **131:2,** 203-211.
Miranda, E., and Akkar, S. (2006). Generalized interstory drift spectrum. *J. Struct. Eng*., **132:6,** 840-852.
Moehle, J. P. (1984). Strong motion drift estimates for R/C structures. *J. Struct. Eng.*, ASCE, **110:9,** 1988-2001.
Moehle, J. P. (1992). Displacement-based design of RC structures subjected to earthquakes. *Earthquake Spectra*, **8:3,** 403-428.
Montes, R., and Rosenblueth, E. (1968). Shears and over turning moments in chimneys. *Proc., 2nd Mexican Conf. on Earthquake Engineering*, Mexican Society of Earthquake Engineering, Veracruz, Mexico.
Somerville, P. (1995). Characterization of ground motion at the sites of subjected buildings. SAC *Joint Venture Rep*. SAC 95-03, Sacramento, Calif.,March.
Timoshenko, S. P. (1921). On the correction for shear of the differential equation for transverse vibrations of prismatic bars. *Philos. Mag*., **41,** 744-746.
Timoshenko, S. P. (1922). On the transverse vibration of bars with uniform cross-section. *Philos. Mag*., **43,** 125-131.
Popov, E. P. (1968). Introduction to mechanics of solids, Prentice-Hall, Englewood Cliff, New York.
Rosenblueth, E., Elorduy, J., and Mendoza, E.(1968). Shears and over turning moments in shear buildings during earthquakes. *Proc., 2nd Mexican Conf. on Earthquake Engineering*, Mexican Society of Earthquake Engineering, Veracruz, Mexico.
Wallace, J. W. (1994). New methodology for seismic design of RC shear walls. *J. Struct. Eng*., **120:3,** 863-884.
Wallace, J. W., and Moehle, J. P. (1992). Ductility and detailing requirements of bearing wall buildings. *J. Struct. Eng*., ASCE, **118:6,** 1625-1644.
Wallace, J. W., and Moehle, J. P. (1992). An evaluation of ductility and detailing requirements of bearing wall buildings using data from the March 3, 1985 Chile earthquake. *Earthquake Spectra*, **9:1,** 137-156.
Westergaard, H.M. (1933). Earthquake-shock transmission in tall buildings. *Eng. News-Rec*., **111,** 654-656.