# MODELLING OF THE HORIZONTAL SLAB OF A 3D IRREGULAR BUILDING FOR NONLINEAR STATIC ASSESSMENT

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## ABSTRACT

The manner in which the horizontal slab in 3D buildings, or better the rigid diaphragm effect, is modeled may influence the response of the structure when subjected to seismic action. In this work different ways of modelling the rigid diaphragm effect are tested and compared, through the application to the irregular 3D SPEAR building, a full-scale specimen tested under pseudo-dynamic conditions at JRC Ispra, representing typical old three-storey buildings built in the Mediterranean region in the early 1970s. The dynamic properties of the different models (periods, modes of vibration and effective modal mass percentages) are analysed, and the corresponding interaction with seismic action is studied by means of nonlinear static and dynamic analyses; different response measures, such as capacity curves, interstorey drifts and displacements are evaluated for the two orthogonal directions. The results of this study show that the most accurate and reliable way of modelling the floor's behaviour is the Rigid Diaphragm with Lagrange Multipliers nodal constraints model.

**KEYWORDS:** numerical simulation, diaphragm effect, nodal constrains, 3D irregular RC building, seismic response

## 1. INTRODUCTION

Typically, in reinforced concrete buildings floors usually exhibit a very large in-plane stiffness when compared with the lateral stiffness of vertical elements. Hence, such slabs may be modelled assuming a rigid diaphragm behaviour in plan, i.e. a rigid body motion of the floor without deformation in their own plane. There are many techniques that can be used to model such rigid diaphragm effect. In this study, several solutions are therefore tested, and their results are then compared, in terms of floors displacements, interstory drifts and capacity curves in both directions.

The structure under analysis is the SPEAR building, designed and tested under the framework of the European project SPEAR - Seismic Performance Assessment and Rehabilitation (Fardis and Negro, 2006). This structure is intended to represent typical existing three-storey buildings in the Mediterranean region, and as such was designed for gravity loads only, following the concrete design code implemented in Greece between 1954 and 1995, with the construction practice and materials used in Greece in the early 1970s, including smooth reinforcement bars. Plan and elevation views are shown in Figure 1, whilst further details on the structure and its pseudo-dynamic testing can be found in Fardis (2002) and Fardis and Negro (2006).

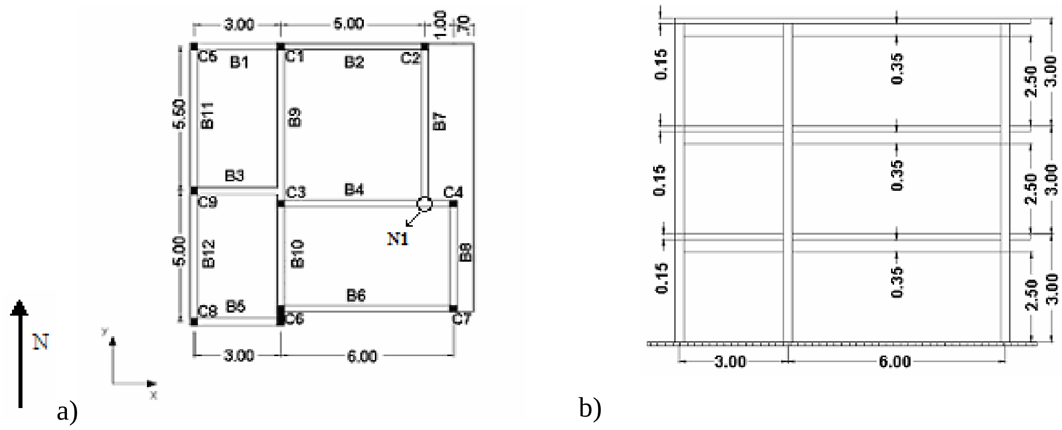


Figure 1 Building configuration: a) in plan; b) at the south west facade (units meters)

## 2. NUMERICAL MODELLING

### *2.1. Geometry*

This structure has been modeled by other researchers in the past (e.g. Jeong and Elnashai, 2005; Franchin *et al.*, 2005; Fajfar *et al.*, 2006), who have however employed different software solutions and did not discuss the issue of diaphragm modelling. The structural analysis software used in this study was SeismoStruct (SeismSoft, 2008), freely downloadable from the internet and capable of predicting the large displacement behaviour of space frames under static or dynamic loading, taking into account the inelastic behaviour of the materials as well as the geometric nonlinearities of the elements.

The 3D building was represented with a space frame model assuming the centrelines dimensions (Figure 2). Rigid offsets on the column-beam end connections were not considered; it was pragmatically assumed that the resulting additional deformation/flexibility somehow compensates for the lack of bar-slippage modeling in the beam-column joints. Due to its larger dimensions, however, column C6 was modelled as a wall element. The storey heights amount to 2.75m for the first and 3.00m for the upper storeys. To model the inelastic behaviour of the structural elements, a fibre element model is adopted, with each fibre being characterised by the material relationships described below.

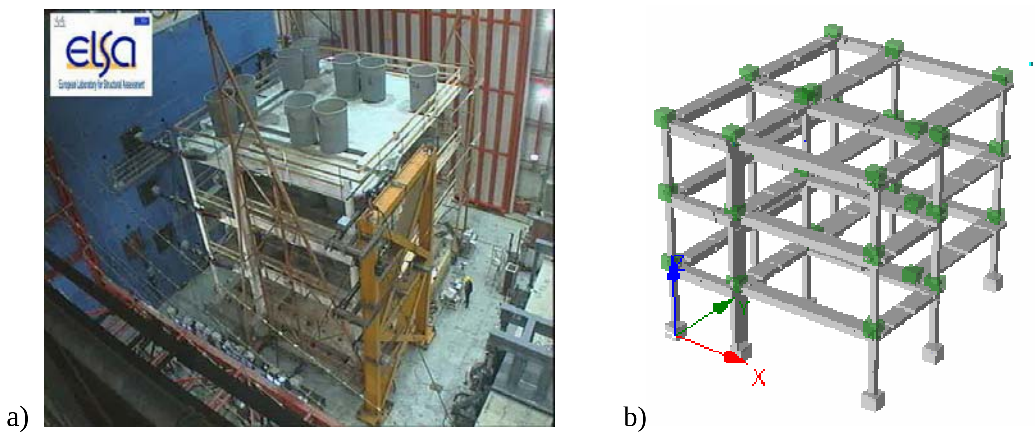


Figure 2 SPEAR building: (a) Experiment specimen, (b) computer model

### *2.2. Materials*

A uniaxial model that follows the constitutive relationship proposed by Mander *et al.* (1988) and the cyclic rules proposed by Martinez-Rueda and Elnashai (1997) was adopted for the concrete. The confinement effects provided by the lateral transverse reinforcement are incorporated through the rules proposed by Mander *et al.* (1988) whereby constant confining pressure is assumed throughout the entire stress-strain range. A compressive strength of 25 MPa was considered.

The constitutive model used for the steel was that proposed by Menegotto and Pinto (1973) coupled with the isotropic hardening rules proposed by Filippou *et al.* (1983). An average yield strength of 360 MPa and ultimate strength of 450 MPa was assumed.

### *2.3. Mass and loading*

A mass of 67.3 tonnes for floors 1 and 2 and 62.8 tonnes for the roof was considered. The coordinates of the CM (centre of mass) and of the CR (centre of stiffness) are the same for the floors 1 and 2 and vary slightly for the roof. The CR is eccentric with respect to the CM by (1.3; 1.0) m, which effective renders the structure are irregular, according to the criteria set by Eurocode 8 (CEN, 2004). Loads were automatically computed by the software, using the defined masses.

## 3. DIAPHRAGM MODELLING

The slab of the SPEAR specimen was very rigid, in order to warrant a correct distribution of inertia forces throughout the structure and also allow an appropriate control of the pseudo-dynamic test. Herein, different manners in which such rigid diaphragm can be modeled are thus scrutinized.

### *3.1. Horizontal Truss System*

Lanese *et al.* (2008) carried out a sensitivity study to determine the best possible configuration of horizontal trusses (see Figure 3), which may not be a straightforward decision when plan-irregular buildings with beams framing on beams are modeled. By comparing the experimental response with the numerical results, it was concluded that the truss configuration shown in Figure 3(d) was the most effective. The axial stiffness of the trusses was computed so as to match the membrane stiffness of a given floor “panel” (the slab was sub-divided into four of these).

### *3.2. Nodal Constraints*

Both *rigid diaphragm* and *rigid link* options were considered; they differ only in the fact that in the latter several rigid constraints are explicitly defined by the user per each floor level whilst in the former such multiple constraint approach is internally set by the computer code. Evidently, displacements along the horizontal $x$ and $y$ axes as well as rotation around the vertical $z$ axis are restrained.

Nodal constraints are typically implemented in structural analysis programs through the use of (i) geometrical transformations, (ii) penalty functions, or (iii) Lagrange multipliers. In nonlinear analysis, however, the first of these three tends to lead to difficulties in numerical convergence, for which reason only the latter two are commonly employed, and have been considered in this study. Whilst readers are advised to refer to existing literature (e.g. Cook *et al.*, 1989; Felippa, 2004) for further information on this topic, herein it is simply noted that whilst penalty functions have the advantage of introducing no new variables (and hence the stiffness matrix does not increase and remains positive definite), the penalty matrix may significantly increase the bandwidth of the structural equations (Cook *et al.*, 1989). In addition, penalty functions have the disadvantage that penalty numbers must be chosen in an allowable range (large enough to be effective but not so large as to provoke numerical difficulties), and this is not necessarily straightforward (Cook *et al.*, 1989).

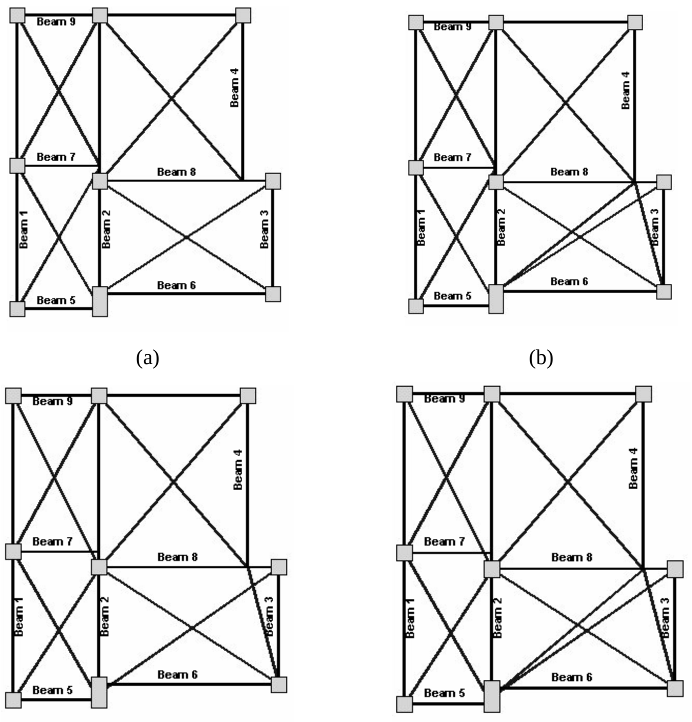


Figure 3 Horizontal truss system configurations tested by Lanese *et al.* (2008)

## 4. MODAL ANALYSIS

Modal analyses were performed in order to make a preliminary assessment of the different rigid diaphragm modelling strategies used in this work. The periods, the modes of vibration and the modal participation mass percentage were compared for all the slab modelling options studied. In addition, the results obtained with the fibre model were also compared with an elastic model developed in SAP2000 (Computers and Stuctures, 2004).

In the results that follow, the *RDLM* terminology stands for Rigid Diaphragm Lagrange Multipliers, *RDPF* for Rigid Diaphragm Penalty Functions, *RLLM* for Rigid Links Lagrange Multipliers and *RLPF* for Rigid Links Penalty Functions. Group 1 includes the following modeling options: Truss elements; RDLM; RDPF10^1, 2, -1,-3,-6,-10; RLLM; RLPF10^1, -1,-3,-6,-10. Group 2, instead, features: Truss elements; RDLM; RDPF10^1, 2, 3; RDPF10^-1,-3,-6,-10; RLLM; RLPF10^1,3,-1,-3,-6,-10.

The results obtained are graphically summarized in Figures 4 to 6, from which one can observe that:

- the penalty functions 10^3, 10^4 and 10^6 options lead to results much different from the other models. Therefore, these models were not employed in the subsequent nonlinear analyses;
- the results obtained from the remaining models (i.e. Group 1 for first modes and Group 2 for higher modes) were all very similar;
- the periods obtained with SAP2000 are slightly higher than those given by the fibre model. This is because in the latter the reinforcement bars are included, rendering the structure slightly stiffer.

It is noted that small penalty coefficients (even lower than one, i.e. reductive) lead to good results, which could seem paradoxal. Penalty factors are computed as the product of the penalty coefficients indicated above and the highest value found in the stiffness matrix. Since an equivalent column with very rigid beam elements was used to model the large column/wall element, there were indeed already very large elements in the stiffness matrix, which when multiplied with modest or small penalty coefficients warrant nonetheless penalty coefficients that are large enough to simulate the rigid diaphragm effect in this given structure.

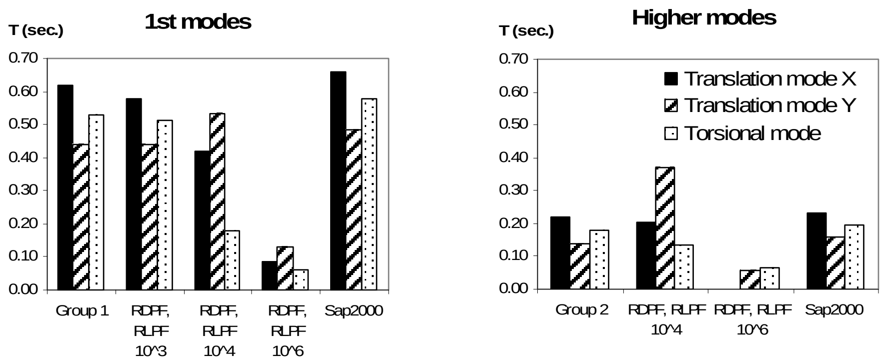


Figure 4 Periods for the 1st and higher modes

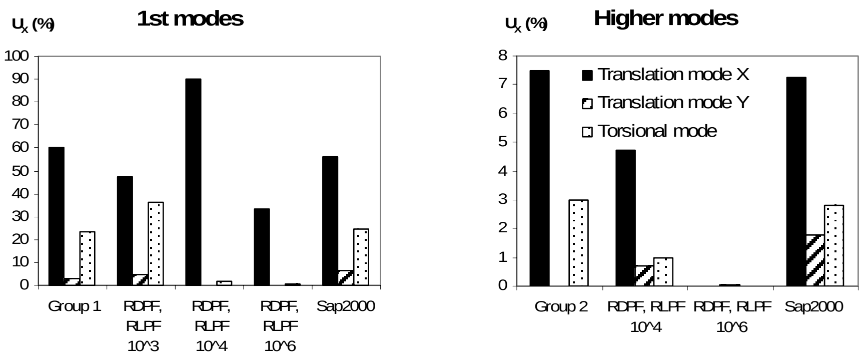


Figure 5 Modal participation mass percentage along X

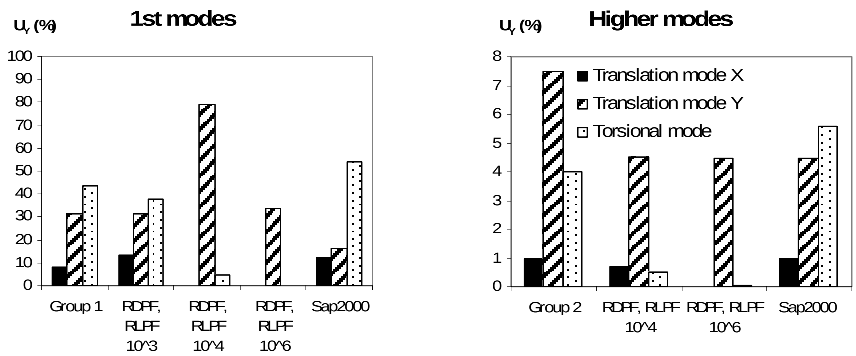


Figure 6 Modal participation mass percentage along Y

## 5. NONLINEAR STATIC ANALYSIS

The main objective of this section is to check the performance of the different slab modelling strategies when nonlinear static analysis is considered. For this purpose, conventional pushover analyses were performed using the Group 1 models (those that were shown to be “reliable” in the modal analysis described above).

A pushover force distribution proportional to the fundamental mode shapes, in *x* and *y* directions, was employed. At each storey, and for each direction, the forces are distributed throughout the structure in proportion to the mass allocated to each beam-column joint (where the loads are applied). The loading was applied both simultaneously in *x* and *y* directions, as well as in separate individual fashion.

The pushover analysis (e.g. Figure 7) results laid evident the following:

- the use of penalty coefficients 10^-10 leads to capacity curves that were clearly diverse from the others, and indication that the slab was exaggeratedly flexible;

- the use of multiple rigid links lead to capacity curve that typically ended prematurely, especially when penalty factors were employed, indicating numerical difficulties even at early stages of the analysis;
- the use of rigid diaphragm with penalty coefficients larger than 10^-10 does not allow accurate modelling of the post-peak softening response of the structure;
- instead, the use of rigid diaphragm with Lagrange Multipliers allowed for capacity curves with complete softening branches to be obtained, be it when *x* and *y* loads were applied independently, be it when they were applied in simultaneous fashion.

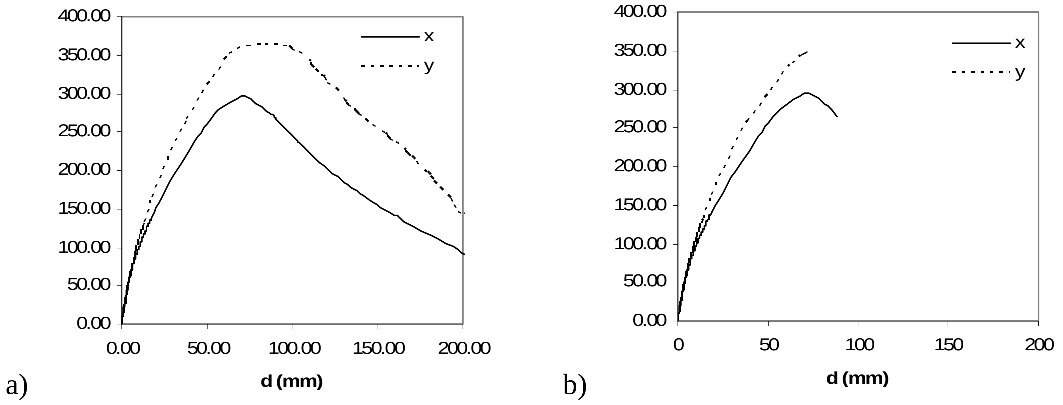


Figure 7 Example of capacity curves obtained with: (a) rigid diaphragm with Lagrange multipliers and (b) all other modelling approaches

## 6. NONLINEAR DYNAMIC ANALYSIS

From the scrutiny described above, slab modeling through the employment of rigid diaphragm with Lagrange Multipliers appeared to be the most numerically stable modelling strategy among all those considered. To complete this brief study, a final verification under nonlinear dynamic analysis conditions was undertaken.

The SPEAR building was pseudo-dynamically tested with a bi-directional loading based on a record obtained at Hercegnovi station during the 1979 Montenegro earthquake and fitted to the EC8 spectrum (type I, Soil C). This bi-directional accelerogram was applied to the structure in three runs of increasing amplitude (0.02g, 0.15g and 0.2g). The very same input load was used in the nonlinear dynamic analyses carried out.

It is recalled that the objective is not one of duplicating the experimental results recorded at the ELSA Lab (JRC Ispra), but rather assessing the numerical stability of the adopted slab modeling strategy. Indeed, without the explicit modeling of rebar slippage, identified during the test as one of the main factors influencing the response of the test specimen, it would be unrealistic to think that the model assembled within the scope of this work could have any chance of fully and accurately reproducing the pseudo-dynamic experiment.

The dynamic simulations were carried out without any particular numerical difficulties, including those at the third and highest intensity level, which had forced the structure to enter its post-peak softening range. The rigid floor behaviour was fully reproduced, and, as shown in Figure 8, the top displacement response history did somehow resemble that observed during the experiment (it is once again recalled that the numerical model does feature all the characteristics of the test specimen).

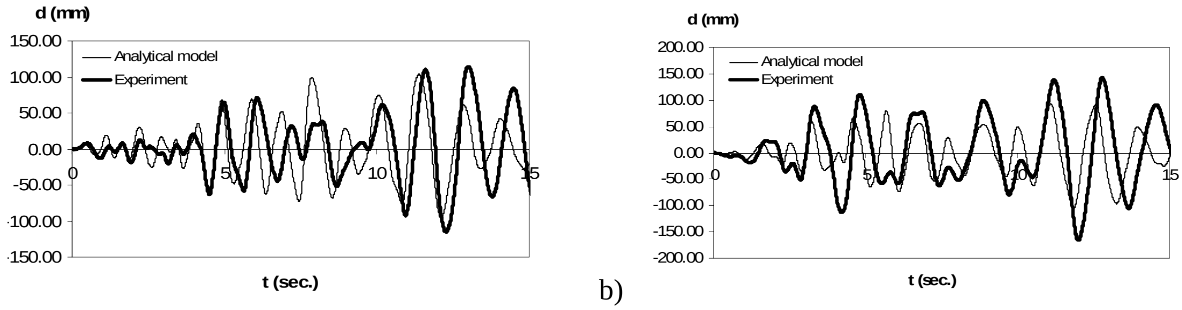


Figure 8 a) Top displacement in node C7, *x*-direction; b) Top displacement in node N1, *y*-direction

## 7. CONCLUSIONS

In this study, the manner in which the horizontal slab in 3D buildings subjected to seismic action should be modeled was scrutinized, through the (simplified) modeling of the SPEAR building, a full-scale specimen tested under pseudo-dynamic conditions at JRC Ispra, representing typical old three-storey buildings built in the Mediterranean region in the early 1970s.

The dynamic properties of the different models (periods, modes of vibration and effective modal mass percentages) were analysed, and the corresponding interaction with seismic action was studied by means of nonlinear static and dynamic analyses; different response measures, such as capacity curves, interstorey drifts and displacements were evaluated for the two orthogonal directions.

The results of this study seemed to indicate that the most accurate and reliable way of modelling the presence of very stiff floors in this type of irregular structures, which featured misaligned beams supported on beams, is through a rigid diaphragm nodal constraints approach that makes used of Lagrange Multipliers.

On the contrary, the employment of a horizontal bracing system, as elaborate as it might be, did not seem to lead to an ideal numerical stability. Similar difficulties were observed with the introduction use of multiple rigid constraints per floor, and with the employment of penalty functions to numerically introduce such nodal constraints.

Finally, the study also evidenced how the choice of a correct value of penalty coefficient is far from being straightforward. Indeed, and for the case of the structured studied here, uncharacteristically low values of penalty coefficients (which are then multiplied by the highest value in the stiffness matrix to define the penalty) had to be employed, due to the fact that very stiff beam elements had been introduced to model the large column/wall section with an equivalent column approach.

The above does constitute another good reason for favouring the employment of Lagrange Multipliers to introduce nodal constraints in nonlinear models of structures subjected to seismic action. Having said this, it is also true that Lagrange Multipliers may not work adequately on some structural models (e.g. when one wishes to constrain a degree-of-freedom with no stiffness), in which case an informed and well-judged use of penalty functions will be inevitably required.

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