STIFFNESS IDENTIFICATION OF FRAMED MODELS UNDER CONTROLLED DAMAGE

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ABSTRACT :

This paper presents an original method of estimating the location and severity of damage in a framed building based on experimental measurements of its fundamental vibration modes. The procedure requires prior knowledge of the fundamental vibration modes of the undamaged structure and mass matrix, either through experimental tests or through an accurate analytical model. An important advantage of this method is that very little experimental data is needed. To study the accuracy of damage identification under noisy conditions, we conducted numerical simulations of a multi-story, framed building. The new method was also tested by measuring the physical vibrations of a scale model under controlled damage conditions. Both variations of the method proved to be suitable, successfully identifying the location of damage and quantifying the stiffness reductions under various conditions.

KEYWORDS: experimental testing, structural systems, evaluation and retrofit, structural response, modal analysis

1. INTRODUCTION

Accurate structural analysis requires evaluation of the differences between theoretical models and real structures. Real structural stiffness, for example, varies over time due to building modifications, damage, overload, seismic effects, and other sources of degradation. It is important to realize that structural damage and defects in the original construction are not always visible. The damage may be too slight to detect easily, or the structural elements may be too difficult to access. It is nonetheless often necessary to evaluate the stiffness and resistant capacity of a structure, in order to decide whether it should be repaired or demolished (**Numayr et al.**). This is especially true after natural disasters such as strong earthquakes.

Several stiffness identification methods have been developed taking as data modal experimental results and structural typology, leading to stiffness changes and damage evaluation (**Baruch**, **Kabe**, **Papadoupulus et al.,, Lieven et al, Teughels et al., Pandey et al.**). There are specialized algorithms that allow one to locate and quantify damage in cantilevers, shear buildings, and other types of structures (**Garcés et al, Li et al)**. If the topology of the stiffness matrix is known, one can greatly reduce the number of modes and measurement points required for accurate structural estimation. The method presented in this paper falls into this category.

The goal of this study is to propose a new method of locating and quantifying changes in the stiffness and/or mass matrix of a framed structure. We demonstrate that damage can be efficiently estimated using only one or two modes. This approach takes advantage of the banded matrix shape typical of framed buildings, which greatly simplifies the procedure. Although the algorithms proposed here are limited to framed structures, they have the advantage of requiring few experimental tests. The displacement of only one floor need be measured (we do not consider rotations, which are more difficult to measure), and only one or two modes of the undamaged structure need be known.

The methodology was applied to a simplified scale model of a framed building in addition to numerical simulations of an appropriate finite element model. Controlled stiffness modifications were imposed on the structure, and free vibration tests were applied. The results of these tests show that structural damage can be accurately quantified and localized.

2. FRAMED BUILDING STIFFNESS ASSESSMENT

2.1 Stiffness variation evaluation with known masses

Let **K** and **M** be the stiffness and mass matrices respectively, and n the number of degrees of freedom. In this method, an experiment must be carried out on the undamaged structure to determine its initial dynamic parameters, including the frequency and shape of at least one eigenpair (i.e., one frequency and its corresponding vibration mode).

The structure is assumed to be undamped. In an experiment, it will be subjected to free vibration tests. The system complies with the equation

$$\mathbf{K}\Phi = \mathbf{M}\Phi\Delta, \qquad (2.1)$$

where Δ is the diagonal matrix of eigenvalues ($\lambda_i = \omega_i^2$, ω_i being the ith frequency) and Φ is the matrix of eigenvectors (defining the shape of each vibration mode).

Framed buildings with shear behaviour can be modelled using undeformable slabs with lumped mass values and columns of infinite axial stiffness. In this case **K** is a banded matrix, and **M** is a diagonal matrix:

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & -k_2 & \cdots & \cdots & 0 \\ -k_2 & k_2+k_3 & -k_3 & & \vdots \\ \vdots & & \ddots & & -k_n \\ 0 & \cdots & \cdots & -k_n & k_n \end{bmatrix} (2.2) \quad \mathbf{M} = \begin{bmatrix} m_1 & & \cdots & & 0 \\ & m_2 & & & \\ \vdots & & \ddots & & \vdots \\ & & & m_{n-1} & \\ 0 & & \cdots & & m_n \end{bmatrix} \qquad (2.3)$$

where m_i is the lumped mass value of the ith floor and k_i is the lateral stiffness of the columns supporting the ith floor.

To represent stiffness variations due to damage, each k_i is multiplied by a reduction factor α. Then **K** becomes

$$\mathbf{K} = \begin{bmatrix} \alpha_1 k_1+\alpha_2 k_2 & -\alpha_2 k_2 & \cdots & \cdots & 0 \\ -\alpha_2 k_2 & \alpha_2 k_2+\alpha_3 k_3 & -\alpha_3 k_3 & & \vdots \\ \vdots & & \ddots & & -\alpha_n k_n \\ 0 & \cdots & \cdots & -\alpha_n k_n & \alpha_n k_n \end{bmatrix} \qquad (2.4)$$

K and **M** from Eqs. (2.3) and (2.4) can be introduced into Eq. (2.1) for a particular mode a:

$$\left(\begin{bmatrix} \alpha_1 k_1+\alpha_2 k_2 & -\alpha_2 k_2 & \cdots & & 0 \\ -\alpha_2 k_2 & \alpha_2 k_2+\alpha_2 k_3 & & & \\ \vdots & & \ddots & & \vdots \\ & & & \alpha_{n-1}k_{n-1}+\alpha_n k_n & -\alpha_n k_n \\ & & \cdots & -\alpha_n k_n & \alpha_n k_n \end{bmatrix} - \lambda_a \begin{bmatrix} m_1 & & \cdots & & 0 \\ & m_2 & & & \\ \vdots & & \ddots & & \\ & & & m_{n-1} & \\ 0 & & \cdots & & m_n \end{bmatrix} \right) \begin{Bmatrix} \phi_a^1 \\ \phi_a^2 \\ \vdots \\ \phi_a^{n-1} \\ \phi_a^n \end{Bmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \qquad (2.5)$$

The individual equations from system (2.5) are:

$$
\begin{gathered}
(\alpha_1 k_1 + \alpha_2 k_2)\phi_a^1 - \alpha_2 k_2 \phi_a^2 - \lambda_a m_1 \phi_a^1 = 0 \\
-\alpha_2 k_2 \phi_a^1 + (\alpha_2 k_2 + \alpha_3 k_3)\phi_a^2 - \alpha_3 k_3 \phi_a^3 - \lambda_a m_2 \phi_a^2 = 0 \\
\vdots \\
-\alpha_{n-1} k_{n-1} \phi_a^{n-2} + (\alpha_{n-1} k_{n-1} + \alpha_n k_n)\phi_a^{n-1} - \alpha_n k_n \phi_a^n - \lambda_a m_{n-1} \phi_a^{n-1} = 0 \\
-\alpha_n k_n \phi_a^{n-1} + \alpha_n k_n \phi_a^n - \lambda_a m_n \phi_a^n = 0
\end{gathered}
\tag{2.6}
$$

As there are α_i unknown values, Eq. (2.6) can be rewritten as follows:

$$
\begin{bmatrix}
k_1\phi_a^1 & k_2(\phi_a^1 - \phi_a^2) & \cdots & & 0 \\
 & k_2(\phi_a^2 - \phi_a^1) & k_3(\phi_a^2 - \phi_a^3) & & \\
\vdots & & \ddots & & \\
 & & & k_{n-1}(\phi_a^{n-1} - \phi_a^{n-2}) & k_n(\phi_a^{n-1} - \phi_a^n) \\
0 & & \cdots & & k_n(\phi_a^n - \phi_a^{n-1})
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n-1} \\ \alpha_n
\end{bmatrix}
=
\begin{bmatrix}
\lambda_a m_1 \phi_a^1 \\ \lambda_a m_2 \phi_a^2 \\ \vdots \\ \lambda_a m_{n-1} \phi_a^{n-1} \\ \lambda_a m_n \phi_a^n
\end{bmatrix}
\tag{2.7}
$$

or $$[A]\,(x) = (c), \tag{2.8}$$

Where (x) is the vector of unknown reduction factors α_i. Thus, the solution to Eq (2.8) for a particular mode shape *a* will provide the new stiffness values of the structure.

3. NUMERICAL STUDY

To demonstrate the effectiveness of the procedures developed in section 2, we now conducted a numerical study of a reinforced concrete, multi-story, framed building. The dynamic parameters of the model were determined under two different conditions: a) undamaged, and b) damaged in two stories. The influence of noise in the modal data was also examined.

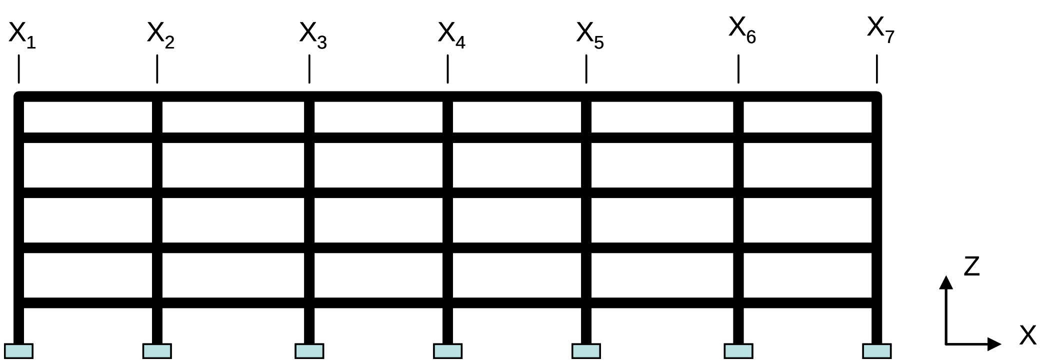

Figure 1 Numerical model of the structural axis.

3.1 Numerical Model

Figure 1 shows the geometry of a two-dimensional finite element structural model. It has five stories, each with six 8 m bays, and its columns have a diameter of 0.80 m. The floor system is a slab with column capitals and drop panels. The thickness of each slab is 0.20 m. The modulus of elasticity is E=25,000 MN/m^2.

Natural frequencies and mode shapes along the structural axis were calculated by finite element analysis using **SAP2000**. The slab masses were lumped into the nodes associated with each floor. The building was clamped at the ground level.

3.2 Study cases

Two cases were calculated: a) an undamaged structure; and b) a structure with stiffness reduction factors of 40% and 20% at the first and third levels respectively.

3.3 Effects of errors in dynamics measurements

Uncertainties in modal data can affect the quality of damage estimation. To address this issue, a normally distributed random perturbation is added in the values of frequencies and mode shapes calculated by the finite element model. The noise was simulated by generating a random number between 0 and 1. Three cases were used to study the effect of measurement noise on damage identification:

Case a: The frequency was corrupted and the mode shape is uncorrupted, Case b: The frequency was uncorrupted and the mode shape is corrupted, Case c: Both frequency and mode shape were corrupted at the same signal-to-noise level. Three values of the noise level were considered: 2%, 5% and 10%.

3.4 Results

Tables 1, 2 and 3 list the structural damage inferred from mode shape measurements for the cases defined in section 3.3. In addition, we performed the calculation for several mode shapes of the damaged structure. The following observations can be made:

1. The quality of damage identification is much more sensitive to noise in the mode shapes (i.e., the eigenvectors) than noise in the frequencies.

2. The quality of damage identification is independent of the mode utilized.

3. In all cases, the methods described here accurately identify the location of the stiffness changes. The accuracy of the stiffness reduction factors depends on the noise level, but is usually quite good.

Note that in this study using mode shapes 1 to 4, all the relative errors were smaller than 12% Only cases (b) and (c), sometimes produced a large error in the inferred reduction factors. When the damage identification was made using mode shape 5, the relative errors for cases b and c were 32.6% and 32.8% respectively.

4. EXPERIMENTAL ASSESSMENT

4.1 Simplified scaled frame model

To verify the effectiveness of this damage estimation method, we also tested the dynamic modes of a physical model. The dynamical parameters of the model were determined from the experimental data.
The model (Figure 2) is a simplified frame building, with three levels and only one span per side. The 4 columns are steel bars, and the floors are rigid acrylic plastic slabs. To simplify the structural model we assumed that only lateral floor displacements are important, which depend only on the flexural behaviour of the columns. A single lateral stiffness coefficient can then be determined for each level. Table 4 presents the geometric and mechanical properties of the model.

Table 1 Relative errors Case a

	Noise Level (%)														
	Mode shape 1			Mode shape 2			Mode shape 3			Mode shape 4			Mode shape 5		
Storey	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10
1	0.08	0.15	0.26	0.11	0.17	0.29	0.06	0.13	0.24	0.09	0.16	0.27	0.37	0.45	0.56
2	0.22	0.16	0.04	8.25	8.32	8.45	0.12	0.18	0.3	0.02	0.05	0.17	0.03	0.03	0.15
3	0	0.06	0.15	0.1	0.16	0.25	0.11	0.16	0.25	0.03	0.08	0.17	0.09	0.15	0.24
4	0.4	0.34	0.22	0.19	0.12	0.01	0.06	0.13	0.24	0.24	0.3	0.41	0	0.07	0.19
5	0.71	0.64	0.53	0.4	0.33	0.22	0.11	0.04	0.08	0.06	0.01	0.12	0.13	0.2	0.31

Table 2 Relative errors Case a

	Noise Level (%)														
	Mode shape 1			Mode shape 2			Mode shape 3			Mode shape 4			Mode shape 5		
Storey	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10
1	0.66	1.43	3.31	0.33	0.95	2.1	1.94	4.93	10.1	0.75	1.88	3.99	6.78	16.4	32.6
2	0.52	1.99	4.25	6.02	2.89	1.25	2.14	5.4	11.3	0.22	0.48	0.98	1.34	3.38	7.24
3	2.1	5.11	9.89	1.03	2.57	5.43	0.3	0.65	1.26	1.1	2.86	6.11	0.59	1.43	2.92
4	1.16	4.06	9.06	0.31	0.43	0.65	0.38	0.98	2.03	0.25	0.35	0.52	0.07	0.11	0.2
5	1.11	2.03	2.69	0.44	0.44	0.44	0.18	0.23	0.31	0.14	0.2	0.31	0.04	0.02	0.14

Table 3 Relative errors Case a

	Noise Level (%)														
	Mode shape 1			Mode shape 2			Mode shape 3			Mode shape 4			Mode shape 5		
Storey	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10
1	1.12	2.67	5.4	0.23	0.85	2	1.84	4.84	10	0.85	1.99	4.09	6.89	16.6	32.8
2	0.27	0.98	2.35	6.12	3	1.15	2.24	5.5	11.4	0.12	0.38	0.88	1.24	3.28	7.14
3	2.01	5.01	9.78	1.11	2.65	5.52	0.38	0.73	1.34	1.18	2.95	6.2	0.67	1.51	3
4	0.8	2.7	6.44	0.21	0.33	0.55	0.28	0.88	1.93	0.35	0.45	0.62	0.03	0.01	0.09
5	0.28	0.27	1.15	0.34	0.34	0.34	0.08	0.13	0.21	0.04	0.1	0.21	0.14	0.07	0.04

4.2 Data acquisition and processing

The model was clamped to an experiment bench, and submitted to free vibration tests by applying specified initial displacements or velocities to each slab (along the x-axis). Vibration responses were registered with unidirectional accelerometers Kinematrics FBA-11 and processed with an Altus K2 Kinematrics signal processing device. Measurements were taken for a frequency range of 0-50 Hz. and the dynamic response was captured by 3 accelerometers.

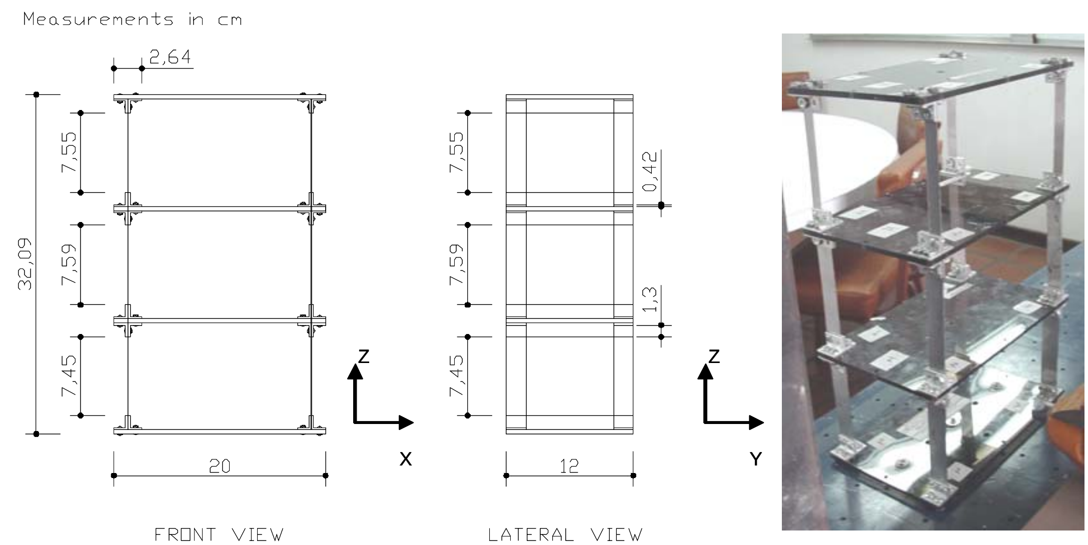

Figure 2 Tested model.

Table 4 Geometric and mechanical properties of the model

Total height		0.321 m
Bay length		0.20 m
Cross section of the columns (w x t)	Storey 1	0.1777 m x 3.86 x 10^{-3}m
	Storey 2	0.1798 m x 3.86 x 10^{-3}m
	Storey 3	0.1772 m x 3.86 x 10^{-3}m
Total mass (model and accelerometers)		2.716 kg
Modulus of elasticity		213.6 GPa

5. STUDY CASES AND IDENTIFICATION RESULTS

After determining the initial stiffness characteristics of the model, its components were modified in a predefined sequence to simulate progressive damage and verify the identification procedures. To simulate structural damage, the widths of the steel columns were decreased. We considered the following case studies:

Case a: the initial, undamaged structure, Case b: 18% stiffness reduction of columns at the first level, Case c: 40% stiffness reduction of columns at the first level, Case d: 19% stiffness reduction of columns at the 2nd level, and 40% at the third level.

The vibration modes are shown in Figure 3. Damage identification was performed using the two methods presented in sections 2.1 and 2.2. The undamaged model was employed as a reference to determine the efficiency of the identification procedures and evaluate the stiffness reduction factors α_i.

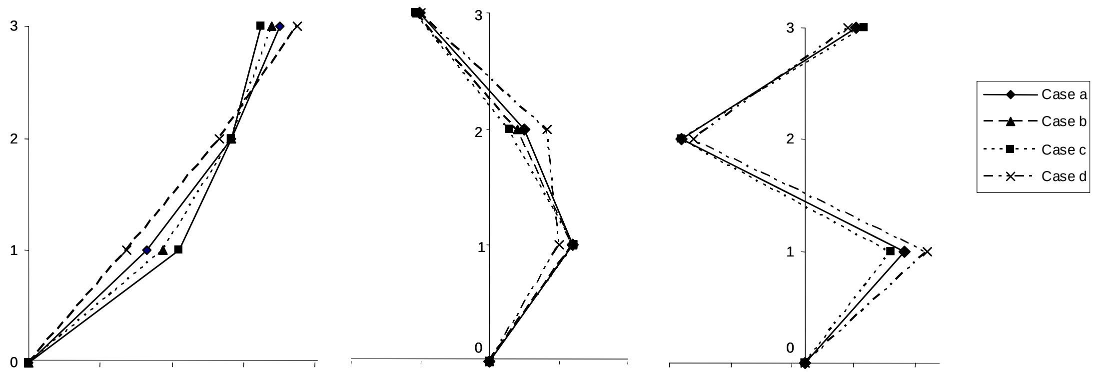

Figure 3 Experimental Mode Shapes for each case.

Table 5 shows the inferred stiffness reduction factors for cases (b), (c) and (d) using the methodology of section 2. The method provides good approximations to the damage values of each floor, and also shows the location of the damage. The quality of damage identification is independent of the mode shape utilized. However, the stiffness reduction factors were less accurate when mode 3 was used.

Table 5 Stiffness changes for each level with known mass values

	Case b				Case c				Case d			
	Real Value	Identified			Real Value	Identified			Real Value	Identified		
		Mode 1	Mode 2	Mode 3		Mode 1	Mode 2	Mode 3		Mode 1	Mode 2	Mode 3
$\alpha 1$	0.82	0.71	0.75	0.61	0.6	0.49	0.51	--	1	0.98	0.96	0.96
$\alpha 2$	1	0.99	1.01	0.98	1	1	0.98	--	0.81	0.76	0.71	0.6
$\alpha 3$	1	0.99	0.98	0.99	1	1.01	0.97	--	0.6	0.56	0.57	0.55

6. CONCLUSIONS

The damage identification procedure was proposed. This procedure may be applied to framed buildings with shear behaviour to evaluate structural damage in terms of stiffness reduction and determine the location of the damage. Although this methodology is limited to framed structures, it has the advantage of using only one modal shape coordinate and a limited number of modes. Other methods may have more extensive applications, but they require more refined finite element models and a complex and expensive series of experimental tests. Typically a great many modal shape coordinates require measurement, including some which are difficult to obtain such as rotations.

Numerical simulations of a finite element model have demonstrated good agreement between the estimated and actual damage. The accuracy of the method is independent of the mode utilised, but the quality of damage identification is sensitive to noise in the mode shapes. The results are more stable under variations of the measured frequency. Since errors in the mode eigenvectors of the damaged structure will affect the damage identification, special attention must be paid to the signal processing.

The method was also applied to a physical scale model of a three-level framed building. Mode shapes were measured in the undamaged structure and three cases representing damaged conditions. The prodecure identified with precision the stiffness change as well as the damage location.

Further research is needed to demonstrate whether damage localization and quantification can be obtained by the present procedure for a real-life structure.

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