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IDENTIFICATION METHOD FOR FLEXURE AND SHEAR BEHAVIOR OF CANTILEVER STRUCTURES

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ABSTRACT :

This article presents an original identification method for the assessment of flexure and shear stiffness of cantilever structures or shear wall buildings. Required data include an initial (theoretical) model, an estimation of lumped mass values (by floor) and an experimental evaluation of two eigenvalues (modal frequencies and their modal shapes). The method estimates coefficients whenever flexural (EI) or shear (GA) values are relevant or irrelevant. A numerical simulation of a real chimney is performed to study the effectiveness of the methodology in identifying damage under noise conditions. A dynamic-test experiment is carried out on a steel cantilever which suffers damage in two sections. The result obtained from the application of the proposed methodology is satisfactory in both numerical and experimental cases, identifying precisely the stiffness changes in the system.

KEYWORDS: experimental testing, structural systems, evaluation and retrofit, structural response, modal analysis

1. INTRODUCTION

There is always a difference between the theoretical model of a building and the real one. Structural stiffness can vary due to time degradation, building modifications, damage, overloads or seismic effects. Structural damage is not always visible, due to the level of damage or the difficulty of accessing the elements of the structure. For this reason, the vibration-based methods are promising because they are nondestructive and the vibration signal of a structure is easily measurable using properly installed sensors.

Several stiffness identification methods have been developed taking as data modal experimental results and structural typology, leading to stiffness changes and damage evaluation (**Baruch**, **Kabe**, **Papadoupulus**). There are algorithms that allow the location and quantification of the damage in certain structural cantilever typologies, shear buildings, and others (**Garcés et al., Ricles, Yuan**). By knowing the stiffness or flexibility matrix topology, we can reduce the number of modal forms and the necessary coordinates for the structural estimation. In general, this reduces the number of measurement points on the structures. This type of method adopts the methodology developed in this paper. The aim of this paper is to present and to demonstrate the efficacy of an identification procedure for flexural stiffness (EI) and shear stiffness evaluation (GA/γ) for cantilever structures with predominant behavior on flexure, shear or both with the use of two modal forms and their frequencies. The methodology uses the knowledge of the matrices forms of a typical cantilever to simplify the identification method.

The following paper presents the mathematical development of the identification methodology. This method is later applied to locate the stiffness changes in a numerically simulated chimney and the stiffness changes in a steel cantilever, which was previously studied in a free-vibration essay. Results illustrate that the proposed methodology successfully determines the location and the type of damage in the structure with accuracy.

2. SHEAR AND FLEXURAL STIFFNESS EVALUATION FOR CANTILEVER STRUCTURES

2.1 General Methodology for flexural and shear stiffness evaluation

Fig. 1 shows the structural model with flexural and shear behavior, rigid floors, non vertical deformations and global consistent masses. Dynamic parameters can be obtained from:

$$(\lambda_i^{-1} - F.M)\phi_i = 0 \tag{2.1}$$

with F=flexibility matrix, M= Mass matrix, $\lambda_i=\omega_i^2$, ω_i= i^{th} modal frequency, i= 1 to N, $\phi_i = \phi i^{th}$ eigenvector

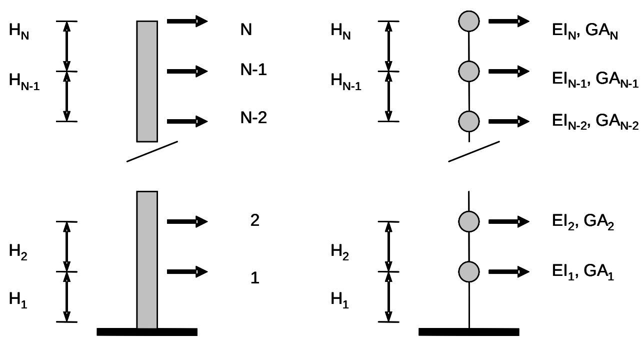


Figure 1. N dof Shear wall structure

Taking into consideration two eigenpairs obtained from experimental procedures, Eq.2.1 can be rewritten as:

$$\begin{bmatrix} a_{11}^a & b_{11}^a & 0 & \cdots & 0 \\ a_{11}^b & b_{11}^b & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ a_{N-1,1}^a & b_{N-1,1}^a & & a_{N-1,N-1}^a & b_{N-1,N-1}^a \\ a_{N-1,1}^b & b_{N-1,1}^b & & a_{N-1,N-1}^b & b_{N-1,N-1}^b \end{bmatrix} \begin{Bmatrix} 1/(EI)_1 \\ 1/(GA/\gamma)_1 \\ \vdots \\ 1/(EI)_{N-1} \\ 1/(GA/\gamma)_{N-1} \end{Bmatrix} = \begin{pmatrix} \phi_a^1/\omega_a^2 \\ \phi_b^1/\omega_b^2 \\ \vdots \\ \phi_a^{N-1}/\omega_a^2 \\ \phi_b^{N-1}/\omega_b^2 \end{pmatrix} \tag{2.2}$$

Stiffness coefficients are represented by each floors unknowns (EI) and (GA/γ). A more detailed explanation is included in ref. **(Garcés et al.)**. Eq. 2.2 establishes a system of 2(N-1) equations and variables with:

$$a_{ik}^a = \sum_{l=k}^{N}\sum_{u=l}^{N} m_{lu}.\phi_a^u.\alpha_{ijk}.1_{k\le Min(i,j)} \qquad b_{ik}^a = \beta_k.\sum_{l=k}^{N}\sum_{u=l}^{N} m_{lu}.\phi_a^u 1_{k\le Min(i,j)}$$

$$a_{ik}^b = \sum_{l=k}^{N}\sum_{u=l}^{N} m_{lu}.\phi_b^u.\alpha_{ijk}.1_{k\le Min(i,j)} \qquad b_{ik}^b = \beta_k.\sum_{l=k}^{N}\sum_{u=l}^{N} m_{lu}.\phi_b^u 1_{k\le Min(i,j)} \qquad \text{for k = 1 to N}$$

$$with : 1_{k\le Min(i,j)} \begin{cases} 1 : if\ k \le Min(i,j) \\ 0 : otherwise \end{cases} \tag{2.3}$$

$$\alpha_{ijk} = H_k\left[\left(\sum_{l=k+1}^{i} H_l\right)\left(\sum_{l=k+1}^{j} H_l\right) + \frac{H_k}{2}\left(\sum_{l=k+1}^{i} H_l + \sum_{l=k+1}^{j} H_l\right) + \frac{H_k^2}{3}\right] \qquad \beta_k = H_k$$

Experimental eigenvectors are:

$$\phi_a = \{\phi_a^1 \cdots \phi_a^i \cdots \phi_a^N\},\ \phi_b = \{\phi_b^1 \cdots \phi_b^i \cdots \phi_b^N\} \qquad (2.4)$$

with H_k = height of story k, E_k = Modulus of Elasticity of story k, G_k = Shear modulus of story k, (A_k/γ)= shear transversal section of story k.

Eigensystems singularities reduce by 1 the number of independent equations, so a simplifying aditional condition is introduced: the damage level is considered to be equal for shear and flexural behaviors in the last floor

$$(EI)_i = (1 - D_i^f).(EI)_i^0 \quad and \quad (GA/\gamma)_i = (1 - D_i^v).(GA/\gamma)_i^0 \qquad (2.5)$$

with: D^f= flexural damage, D^v= shear damage, $(EI)_i^0$ = initial flexural stiffness, $(GA/\gamma)_i^0$ = initial shear stiffness. For the last floor **(Garcés et al.)**:

$$D_N^f = D_N^V = 1 - \frac{a_{NN}^b \dfrac{1}{(EI)_0^N} + b_{NN}^b \dfrac{1}{(GA/\gamma)_0^N}}{\dfrac{\phi_b^N}{\omega_b^2} - \left(\sum_{k=1}^{N-1} a_{Nk}^b \dfrac{1}{(EI)_k} + b_{Nk}^b \dfrac{1}{(GA/\gamma)_k}\right)} \quad (2.6)$$

Once the factor values (*EI* and *GA*) for each storey are known, the stiffness matrix is completely known.

3. NUMERICAL STUDY

Before conducting experimental verification, first the effectiveness of the identification method was evaluated through numerical simulated damage identification. Simulated real steel chimney without damage and with assumed damage sections are considered. Figure 2 shows the geometric values of the steel chimney employed for the numerical study, from **Ambrosini et al**. The chimney is a cylindrical steel structure of 28 m high with 0.914 m diameter, cross section is 12 mm at the base and 3 mm at the top (Fig. 2). The simulated steel chimney is divided into 10 two-dimensional sections (Fig. 2). Three sections (No. 2, 5 and 8) are assumed to be subjected to 10%, 30% and 10% stiffness reduction. The modal data (mode shapes and frequencies) before and after damage have been calculated by using the finite element program **SAP2000**.

3.1 Effects of errors in dynamics measurements

In order to study the effect of noise on the measurement of mode shapes and frequencies for the stiffness identification method, the mode shapes obtained from the numerical simulation were corrupted using the **Sohn and Law** algorithm :

$$\phi_c(n) = \phi\left(1 + \frac{p}{100}R\right) \qquad (3.1)$$

Where $\phi_c(n)$ is the corrupted mode shape, ϕ is the uncorrupted mode shape obtained from numerical simulations, p is a specified percentage of noise level, and R is a random number between 0 and 1. A set of ten vibration tests was carried out.

Three study cases were proposed to study the effect of measurement noise on the damage identification: Case a: frequency is corrupted and modal shape uncorrupted, Case b: frequency is uncorrupted and modal shape corrupted, Case c: frequency is corrupted and modal shape corrupted with same noise level. Two values noise level were considered: 5% and 10%.

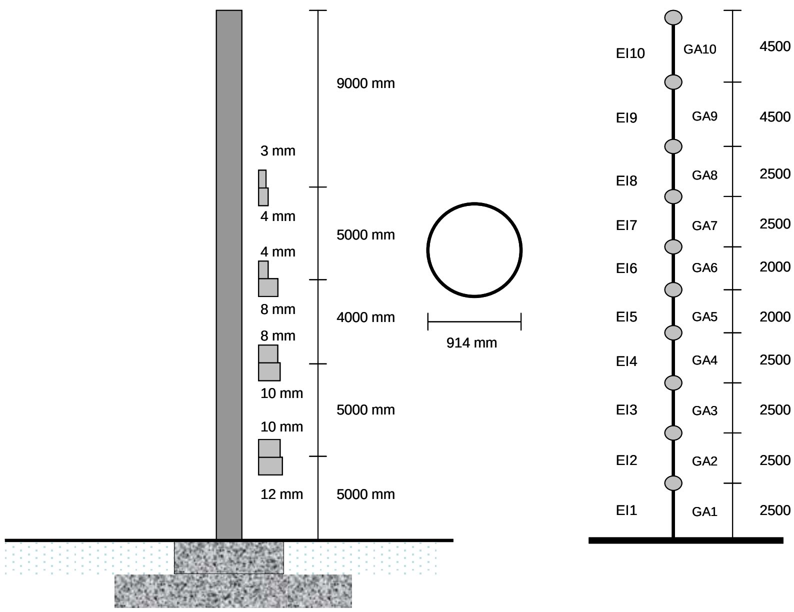


Figure 2 A simulated steel chimney (**Ambrosini et al.**).

3.2 Results

The stiffness identification is carried out using Eqs. (2.2) and (2.6) with noisy mode shapes and mass matrix, the coefficients *EI* and *GA* were obtained for undamaged and damaged states.

The damage identification with different level of noise in frequency or mode shapes was performed to verify the efficiency and accuracy of the proposed method.
Tables 1 to 2 show the relative errors in the damage identification for the defined cases in 3.1. The following observations can be made:

1. The quality of the damage identification is more sensitive to the perturbation of the mode shapes than that of frequencies.
2. The estimation of coefficients GA is made with less precision than coefficients EI. As expected a better adjustment of the flexural coefficient was obtained than the shear coefficient, for the chimney is mainly characterized by a flexural behavior. Numerical simulations for the shear wall buildings shows similar precision in the identification of flexural and shear coefficients (**Garcés et al**).
3. The damage identification with level noise in the modal parameters give a reasonable agreement between the damage estimated and damage assumed. For the EI coefficients, in all cases the methodology identifies with

precision the location of the stiffness changes as well as the variation of stiffness. For the study noise level the relative error of the estimation remain smaller that 9%.
4. The Shear coefficients (GA) are identified with acceptable precision. Only the large error was present and was not corrected, for example in section 8 case c with noise level 5%, the obtained error is closed to 20%.

4. EXPERIMENTAL ASSESSMENT

4.1 Model tested

The model test item is a wide-flanged steel I-beam (IPN 80), consisting of a 80 mm deep web and a 42 mm wide flange. The Beam is 3.13 meters in length, divided in 5 sections. The model was fixed at the reinforced concrete beam (figure 3).

In order to obtain lower values of the modal frequencies and to obtain more mode shapes with the equipment available, the beam was excited in axis X direction (figure 3). Previous numerical simulations established a predominant flexural behavior for the beam. For this reason, the experimental study is limited only for estimate coefficients EI for each section.

Table 1 Relative error Coefficients EI

	Noise Level					
	Case a		Case b		Case c	
Section	5%	10%	5%	10%	5%	10%
1	0.00	-0.01	0.01	0.00	0.00	0.01
2	0.00	-0.01	0.12	0.06	-0.27	-0.02
3	0.00	0.00	0.17	-0.15	-0.09	0.28
4	0.01	-0.04	-0.50	0.08	-0.19	-0.80
5	-0.01	0.13	-3.43	-4.13	-3.69	-8.11
6	0.00	0.03	-0.79	-4.05	-2.12	-2.34
7	0.01	-0.06	0.32	0.85	0.17	1.31
8	-0.01	0.04	0.96	1.44	-0.28	1.22
9	0.00	0.01	-0.38	3.15	0.49	8.00
10	0.00	0.03	3.01	8.41	0.76	5.75

Table 2 Relative error Coefficients GA

	Noise Level					
	Case a		Case b		Case c	
Section	5%	10%	5%	10%	5%	10%
1	0.01	0.00	-0.28	0.01	0.08	-0.29
2	0.12	0.06	-1.90	7.73	-1.74	-3.03
3	0.17	-0.15	-1.44	-0.80	0.52	-1.80
4	-0.50	0.08	0.77	-0.16	1.84	1.49
5	-3.43	-4.13	3.03	1.72	-8.79	-8.43
6	-0.79	-4.05	3.57	3.98	5.79	3.96
7	0.32	0.85	-0.79	3.99	4.25	3.93
8	0.96	1.44	0.77	-3.13	-1.87	-7.14
9	-0.38	3.15	3.73	6.52	18.90	5.93
10	3.01	8.41	3.01	8.41	0.76	5.75

4.2 Experimental analysis and dynamic properties identification

The beam was submitted to a free vibration test applying initial displacement or velocity conditions in each coordinates of measurement. Vibration responses were registered with unidirectional accelerometers Kinemetrics FBA-11 and processed with an Altus K2 Kinemetrics signal processing device. Measurements were taken for a frequency range of 0-50 Hz. and the dynamic response was captured by 5 accelerometers, obtaining three of the five fundamentals frequencies of the model.

5. STUDY CASES AND ESTIMATION RESULTS

Consecutive damage was introduced to the beam, in order to evaluate the proposed methodology. Saws were made in the beam flanges. These saws were localized in the half of the section. Three study cases were established: Case a: Initial undamaged beam (reference for evaluation of coefficient change), Case b: Beam with damage in the 2nd section and Case c: Beam with damage in the 2nd and 4th sections.

For each of the cases described above, free vibration tests were performed with excitation in different coordinates, in order to obtain records and to choose those with more quality and information. Dynamic properties were obtained for every case: Modal frequencies and mode shapes (figure 4). Once the beam mass is estimated and the experimental data analyzed, a linear system is formed (2.2). Solving the system, we obtain changes of coefficient for every case in relation to the initial structure (case a).

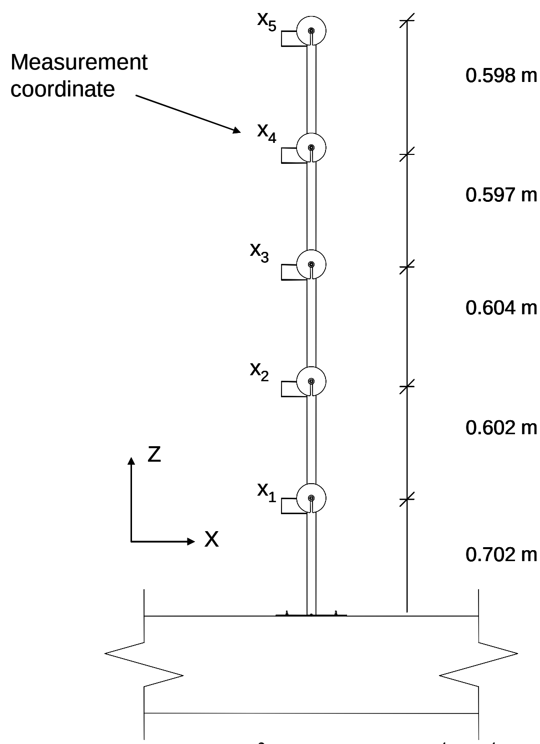
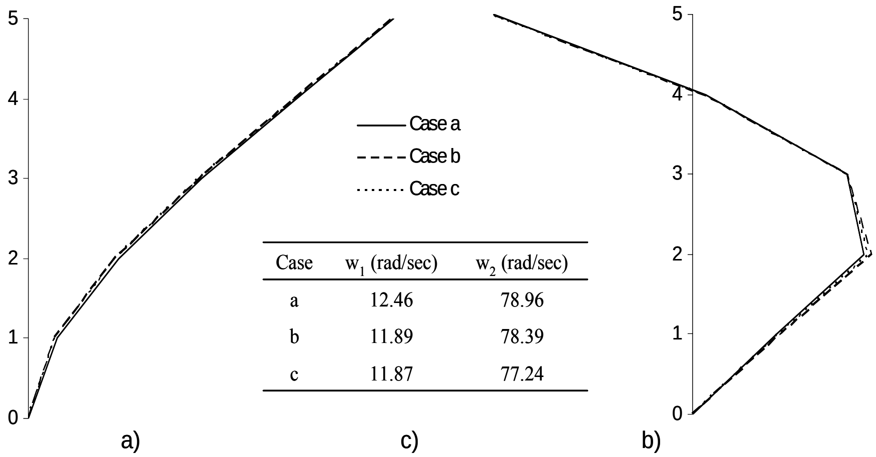


Figure 3 Tested Model. L= 3.13m A= 0.0757 m^2, I_{xx}= 6.29 x 10^{-4} cm^4, E= 205.4 GPa, W (model and accelerometers) = 24.76 kg



Case	w_1 (rad/sec)	w_2 (rad/sec)
a	12.46	78.96
b	11.89	78.39
c	11.87	77.24

Figure 4 Modes and frequencies of free vibration of the cantilever

Table 3 shows the results of damage estimation in case b, and c. In each of the two studied cases, the method shows the location of the damage; in the case of undamaged sections, the maximum error estimation is 3 %, in relation to reference values (case a).

Table 3 Coefficient changes estimation of the cantilever

	Case b	Case c
Section	EI_f/EI_o	EI_f/EI_o
1	1.00	1.00
2	0.72	0.72
3	1.01	1.01
4	0.97	0.88
5	1.02	1.00

6. CONCLUSION

The damage identification procedure for cantilever structures was proposed. This methodology requires a known mass matrix and two natural frequencies with their corresponding mode shapes.

The stiffness identification procedure was illustrated with a numerical example of a real chimney, achieving good precision for stiffness changes in each section under different noise signal conditions. In addition, the stiffness estimation methodology was applied in an experimental study of a steel cantilever beam. Damage was performed in two sections of the beam. The method identified with precision the stiffness change as well as the damage location.

This approach can be applied in cantilever structures (chimneys, control towers, grandstands roofs, etc.). An important advantage is the need of only two mode shapes and simple dynamic tests. This method requires only unidirectional coordinates; does not require rotational coordinates and refined FE models. Therefore, it is convenient to apply this approach in order to identify structural cantilever typologies.

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