

# **SEISMIC RETROFIT OF MICROELECTRONICS FACTORIES USING VISCOUS DAMPERS**

**Jenn-Shin Hwang 1 , Shiang-Jung Wang 2 , Yin-Nan Huang 3 and Jay-Fu Chen 4**

**1**  *Professor, Dept. of Construction Engineering, Taiwan University of Science and Technology, Taipei, Chinese Taiwan. Also, Division Head, Center for Research on Earthquake Engineering(NCREE) of Taiwan* 

**2**  *PHD Student, Dept. of Civil Engineering, Taiwan University, Taipei, Chinese Taiwan. Also, Assistant Researcher, Center for Research on Earthquake Engineering(NCREE) of Taiwan* 

**3**  *Post-Doctoral Research Associate, Dept. of Civil, Structural and Environmental Engineering, State University of New York at Buffalo, Buffalo, New York, USA.* 

**4**  *Graduate Student, Dept. of Construction Engineering, Taiwan University of Science and Technology, Taipei, Chinese Taiwan. Email: JSH@mail.ntust.edu.tw, jshwang@ncree.org.tw* 

#### **ABSTRACT :**

The implementation of viscous dampers to microelectronics factories has been previously proved not to affect the micro-vibration of the factories in operation such that the vibration-sensitive manufacturing process will not be interfered. Therefore, a seismic retrofit strategy which employs the viscous dampers installed in between the exterior and interior structures of the "fab" structure (microchip fabrication facilities are often referred to as "fabs" in micro-electronics industry) is proposed in the study. The design formulas corresponding to the proposed retrofit method is derived using the non-classical damping theory. Based on the study, it is found that the added damping ratio to the fab structure depends greatly on the frequency ratio of the two structures in addition to the damping coefficients of the added dampers. Outside the band width of the frequency ratio in which the added damping ratio is very sensitive to the variation of the frequency ratio, the added damping ratio can be well captured using the classical damping theory.

**KEYWORDS:** seismic retrofit, viscous damper, micro-electronics factory, seismic design, passive control

#### **1. INTRODUCTION**

In the semiconductor industry, microchip fabrication facilities (fab) may be one of the most important and expensive structures. The cost of the civil engineering construction is only about 3% of the total cost of establishing a typical semiconductor factory including the manufacturing facilities. Apparently, the seismic resisting capability of the nonstructural components including the microchip manufacturing facilities is extremely important since these manufacturing facilities housed in the factory are much more valuable compared with the building itself. However, the functionality of these valuable high-tech manufacturing facilities during and after an earthquake event also greatly depends on the seismic performance of the fab structure. During the 1999 Chi-Chi earthquake of Taiwan, the measured maximum ground acceleration at a science-based industrial park was about 0.12g. However, the seismic responses of the fab structures were sufficiently large to shut down the production lines, destroy the work-in-process, and damage some nonstructural components. The interruption on the operation has resulted into a great indirect loss.

SEAOC VISION 2000 [1] has categorized microchip facilities as hazardous facilities to emphasize the importance of the seismic safety of these valuable high-tech industrial facilities. These facilities should remain fully operational after an occasional earthquake (50% exceeding probability in 50 years) and remain operational after a rare earthquake (10% exceeding probability in 50 years). Therefore, the seismic performance criterion for the high-tech industrial factories may be simply stated as: the production lines should be capable of recovering in a very short period after a major quake with some minor calibration of manufacturing facilities of



which some are designed to be automatically shut down when a large vibration is detected [2]. In addition, the retrofit of existing microelectronics factories should address the following issues: (1) enhancing the seismic safety for both the structural and nonstructural components in the fab structure; (2) not increasing the micro-vibration level in the structure after retrofit; and (3) not introducing large vibration and noise that may interfere the manufacturing of microchip during the construction of the retrofit work. In order to conform to the above requirements, the application of viscous dampers that can greatly reduce the acceleration and displacement responses of the fab structure and enhance the micro-vibration control of the structure [2] may be one of the most acceptable retrofit methods. It requires minimum construction work and generates very little interference to the manufacturing lines.

In the paper, the seismic vulnerability of the existing fab structures is first described, and then the design theory is formulated corresponding to the proposed retrofit strategy that employs viscous dampers installed at the separation gap between the interior and exterior structures of the fab structure. The effectiveness of the retrofit method in reducing the seismic responses at the cleanroom has been verified from a numerical example.

#### **2. SEISMIC VULNERABILITY OF MICROELECTRONICS FACTORIES**

The experience from the 1999 Taiwan Chi-Chi earthquake has shown that the microchip factories may be seismically vulnerable and a major earthquake may result in a great economical loss. A typical microelectronics factory structure in Taiwan is illustrated in Figure 1, in which the fab structure is composed of an interior structure and an exterior structure [2]. The working space between the interior structure and shell structure is the cleanroom where microchips are manufactured. The shell structure is basically a one-story structure with a log span roof truss. Separation gaps are provided between the intermediate columns and the interior structure such that the ambient vibration of the exterior structure will not significantly transfer to the cleanroom. The purposes of these separation gaps are (1) to minimize the transmission of the possible micro-vibration induced by the wind load and other service loads to the cleanroom during daily operation; and (2) to provide the path for the circulation and filtering of the air in the cleanroom.





The interior structure is typically a two-story reinforced concrete structure. The roof of the interior structure is denoted as the cleanroom floor and the second floor of the interior structure is denoted as the subfab floor, hereafter. The floor slab of the cleanroom is normally a waffle slab or a flat slab. If a waffle slab is used, the depth of the slab may be up to 120 *cm* while a flat slab may have a thickness of 60 *cm* to 80 *cm* . The reason for using so thick a slab is to minimize the vertical micro-vibration of the clean room induced by walkers and manufacturing facilities in operation. A large mat foundation may be as thick as 2.4 *m* so that the sufficiently large mass can help minimize the micro-vibration, which could be induced by the adjacent traffic loads and possible construction, transmitting to the cleanroom.



In order to identify the dynamic characteristics of fab structures, ambient vibration tests have been conducted for an existing fab structure in Taiwan. The fundamental vibration mode shapes of the interior structure in two lateral directions are identified and shown in Figure 2 from which it is observed that the deformation mostly concentrates on the second story. These irregular shapes are because (1) some partition walls are provided in the first floor for operation safety and storage reasons; and (2) the seismic reactive weight on the cleanroom floor is approximately three times greater than that on the subfab floor. Therefore, the facilities located in the cleanroom may be more vulnerable than those in the subfab. Besides, from the serviceability point of view, the vibration-sensitive equipment in the cleanroom may be automatically shut down in the frequently occurring small earthquakes. The interruption of operation and the damage of work-in-process may be costly for this industry in a high seismicity zone as Taiwan. For larger earthquakes, the large acceleration response occurring at the cleanroom may cause the long term loss of functionality of the manufacturing facilities, even though the appearance of the facilities seems to be structurally undamaged.

### **3. STRATEGY FOR SEISMIC RETROFIT**

The retrofit of a microelectronics factory in operation is extremely difficult due to the following reasons: (1) the subfab is full of manufacturing and supporting facilities, and the locations available for implementing seismic protective systems are very limited; (2) The vibration induced by the retrofit work should be within certain limit without affecting the operation of those vibration-sensitive lithography and metrology facilities which are required to be operated in a vibration environment of *VC* D or *VC* E [2, 3]; (3) The cleanness of the cleanroom should not be affected by the retrofit work so that the "yield rate or chip probe yield" (the percentage of good chips) will not be reduced. The hole drilling for connecting the necessary additional structural members to the structure should generate very limited dust and leave no slurry behind; (4) No welding jobs are allowed inside and outside the factory during the damper installation process.

According to the aforementioned construction difficulties, a convenient and feasible retrofit strategy is to install viscous dampers along the gap existing between the exterior and interior structures, as shown in Figure 3. The concept of connecting the adjacent structures with energy dissipation devices has attracted the attention of many researchers [4-9] due to its ability in improving the wind-resistant or earthquake-resistant capacity of each structure, as well as to reduce the chances of mutual pounding. Corresponding to the proposed retrofit method, design formulas are derived first so that the damping coefficients of the viscous dampers can be determined with respect to a desired added damping ratio in this paper. In addition, the effectiveness of this retrofit strategy has also to be ascertained. The analytical and parametric studies on the coupling system with viscous dampers are conducted to derive the design formulas for achieving a desired added damping ratio to the fab structure. The numerical study is also conducted to validate the proposed preliminary design formulas.



Figure 3 An installation scheme of viscous dampers Figure 4 A simplified two-DOF system





#### **4. RETROFIT THEORY**

In order to derive the design formula, the retrofitted fab structure is simplified as two single degree of freedom systems connected with viscous dampers as shown in Figure 4. The two SDOF systems, denoted as structures *a* and *b* , represent the exterior and interior structures in the fab structure. For design purpose, the equivalent damping ratio of the model should be derived in terms of the damping coefficients of the dampers and other dynamic characteristics of the structures. The corresponding equation of motion to the simplified model connected with the linear viscous damper can be written in a matrix form of

$$
[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -[m]\{I\}\ddot{x}_g
$$
\n(4.1)

where  $[m] = \begin{vmatrix} m_a & b \\ 0 & m \end{vmatrix}$  $\overline{\phantom{a}}$  $\begin{vmatrix} m_a & 0 \\ 0 & m \end{vmatrix}$ ⎣  $=\vert$ *b a m0*  $[m] = \begin{bmatrix} m_a & 0 \\ 0 & m \end{bmatrix}$ ;  $[c] = \begin{bmatrix} c_a + c_d & -c_d \\ 0 & m_c \end{bmatrix}$  $\overline{\phantom{a}}$  $\begin{vmatrix} c_a + c_d & -c_d \end{vmatrix}$ ⎣  $\mathsf I$  $-c_d$   $c_h$  +  $=\begin{bmatrix} c_a + c_d & -b \end{bmatrix}$ *d*  $\mathbf{c}_b$   $\mathbf{c}_d$  $a \perp \theta_d$   $\qquad \theta_d$  $c_d$   $c_h + c$  $c_a + c_d$  – *c*  $c = \begin{pmatrix} c_a & c_d & c_d \\ c_a & c_d & c_d \end{pmatrix}$  ;  $[k] = \begin{pmatrix} a & c_d & c_d \\ c_d & c_d & c_d \end{pmatrix}$  $\overline{\phantom{a}}$  $\begin{vmatrix} k_a & 0 \\ 0 & k \end{vmatrix}$ ⎣  $=\vert$ *b a k0*  $k_a = 0$  $k = \begin{bmatrix} a_a & b \\ 0 & k_b \end{bmatrix}$  ;  $\{I\} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$  $\overline{\phantom{a}}$  $\overline{\mathcal{L}}$  $=\begin{cases} I \\ I \end{cases}$  $I = \{\}$  and  $\{x\}$ ⎭  $\left\{ \right\}$  $\vert$  $\overline{a}$  $=\left\{\right.$ *b a x x*  $x = \{x \in \mathbb{R}^n : x \in \mathbb{R}^n \}$ 

 $x_a$ ,  $x_b$  = the relative displacements of structures *a* and *b* to the ground;  $m_a$ ,  $m_b$  = the masses of structures *a* and *b*;  $c_a$ ,  $c_b$  = the inherent viscous damping constants of structures *a* and *b*;  $k_a$ ,  $k_b$  = the lateral stiffness of structures *a* and *b*;  $c_d$  = the damping constant of the viscous damper;  $\ddot{x}_e$  = ground acceleration; Obviously, it shows a non-proportional damping in Equation (4.1). If ignoring the off-diagonal terms in [*c*] and using the classical damping assumption, one can easily obtain the system damping ratios corresponding to the fundamental vibration modes of the two structures as

$$
\xi_{\text{eff},a} = \frac{c_a + c_d}{2m_a \omega_a} = \xi_a + \frac{c_d}{2m_a \omega_a} = \xi_a (1 + c_d / c_a)
$$
\n(4.2)

$$
\xi_{\text{eff},b} = \frac{c_b + c_d}{2m_b \omega_b} = \xi_b + \frac{c_d}{2m_b \omega_b} = \xi_b (1 + c_d / c_b)
$$
\n(4.3)

where  $\omega_a = \sqrt{k_a/m_a}$ ;  $\omega_b = \sqrt{k_b/m_b}$ ;  $\xi_a = c_a/2m_a\omega_a$ ;  $\xi_b = c_b/2m_b\omega_b$ ; and  $\xi_{eff,a}$ ,  $\xi_{eff,a}$  = the effective damping ratio of structures *a* and *b* , respectively. The effective damping ratios shown in Equations (4.2) and (4.3) are irrational because the simplification of [*c*] uncouples Equation (4.1) and overlooks the interaction between structures *a* and *b*. To have a better insight on the equivalent damping ratio, the nonproportional damping theory is assumed and the equation of motion is solved using state-space method [10-12].

Assuming that  $\omega_b / \omega_a$  is the ratio of the natural frequencies of structure *b* to structure *a*, and  $c_d / c_a$  and  $c_d/c_b$  are the ratios of the damping coefficient of the added viscous damper to the damping coefficients of structure *a* and structure *b* , respectively, a parametric study is conducted for the first and second modal composite damping ratios,  $\zeta_1$  and  $\zeta_2$ , with  $\omega_b / \omega_a = 1$  through 10,  $c_d / c_a = 1$  through 6,  $c_d / c_b = 1$  through 6 corresponding to the assumed damping ratios  $\zeta_a = \zeta_b = 5\%$ . Some results are illustrated in Figure 5, from which it is observed that  $\zeta_1$  and  $\zeta_2$  are essentially skew-symmetric with respect to  $\omega_b / \omega_a = 1$ . The effective damping ratios of the first and second modes,  $\zeta_1$  and  $\zeta_2$ , of the system can be well captured by Equations (4.2) and (4.3) if the two fundamental frequencies  $\omega_a$  and  $\omega_b$  are well separated. It implies that the damping coefficient  $c_d$  of the added viscous dampers should be calculated with respect to structure *a* or structure *b* corresponding to the desired added damping ratio, as depicted by Equations (4.2) and (4.3). In other words, if structure *b* is selected to be added with a target damping ratio,  $\omega_a$  and  $\omega_b$  are well separated and



 $\omega_b/\omega_a < 1$ , then the effective damping ratio of the structure  $\zeta_{\text{eff},b}$  is equal to  $\zeta_1$ . On the other hand, if  $\omega_b / \omega_a > l$ , then  $\xi_{eff, b}$  is equal to  $\xi_2$ .



It is worthy of noting that the effective damping ratio varies significantly within certain frequency ratio bandwidths centered approximately at  $\omega_b / \omega_a = 1$ . These bandwidths of the frequency ratio are functions of  $\omega_b/\omega_a$ ,  $c_d/c_a$  and  $c_d/c_b$ . For a given combination of  $\zeta_a$ ,  $\zeta_b$ ,  $\omega_b/\omega_a$ ,  $c_d/c_a$  and  $c_d/c_b$ , the *n*<sup>th</sup> mode shape vector  $\{\phi_n\}$  can be can be expressed in the form of amplitude and phase angle

$$
\{\phi_n\} = \begin{Bmatrix} \phi_{an} \\ \phi_{bn} \end{Bmatrix} = \begin{Bmatrix} \text{Re}_{an} + i \text{ Im}_{an} \\ \text{Re}_{bn} + i \text{ Im}_{bn} \end{Bmatrix} = \begin{Bmatrix} A_{an} e^{i\theta_{an}} \\ A_{bn} e^{i\theta_{bn}} \end{Bmatrix}, \qquad (n = 1, 2)
$$
 (4.4)

where Re<sub>an</sub>, Im<sub>an</sub> = the real and imaginary parts of  $\phi_{an}$ ; Re<sub>bn</sub>, Im<sub>bn</sub> = the real and imaginary parts of  $\phi_{bn}$ ;  $A_{an}$ ,  $A_{bn}$  = the amplitudes of structures *a* and *b*, respectively, corresponding to the *n*<sup>th</sup> vibration mode; and  $\theta_{an}$ ,  $\theta_{bn}$  = the phase angle of structures *a* and *b*, respectively, corresponding to the *n*<sup>th</sup> vibration mode. The modal damping ratio can then be determined in terms of the phase lag  $\theta_{an} - \theta_{bn}$ 

$$
\xi_n = \frac{(c_a + c_d)A_{an}^2 + (c_b + c_d)A_{bn}^2 - 2c_d A_{an} A_{bn} \cos(\theta_{an} - \theta_{bn})}{2\sqrt{(m_a A_{an}^2 + m_b A_{bn}^2)(k_a A_{an}^2 + k_b A_{bn}^2)}}
$$
(4.5)

Typical results for the phase lag,  $\theta_{a1} - \theta_{b1}$ , of the first mode of vibration structures are shown in Figure 6 which explains why the effective damping ratio is varying at the frequency ratio bands around  $\omega_b / \omega_a = 1$  as depicted in Figure 5. It is valuable to identify the bandwidth of this undesirable frequency ratio region so that the unreliable retrofit design can be avoided. From the parametric study, the undesired frequency ratio bands corresponding to the assumed damping ratios  $\xi_a = \xi_b = 2\%$  and  $\xi_a = \xi_b = 5\%$  are identified and summarized in Figure 7. These undesired frequency ratio bands are determined for those frequency ratios corresponding to the effective damping ratio that has a 2% difference from the damping ratio predicted by Equations (4.2) and (4.3). The procedure for the design of linear viscous dampers for the retrofit of the fab

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structures can be summarized as follows:

- 1. Ascertain that the frequency ratio,  $\omega_b / \omega_a$ , is outside the undesired frequency-ratio band.
- 2. Select the target structure between the interior and exterior structures, and determine the target added damping ratio to the designated structure.
- 3. Calculate the damping coefficient of the viscous dampers,  $c_d$ , using Equations (4.2) and (4.3).
- 4. Check the seismic performance of both structures. If the performance is not acceptable, adjust the design of the dampers.



Figure 6 Relationship of the first modal phase lag  $(\theta_{a_1} - \theta_{b_1})$  and frequency ratio  $\omega_b / \omega_a$  $(\xi_a = \xi_b = 5\%, C_d/C_a = 2, C_d/C_b = 1-6)$ 



Figure 7 Frequency ratio bands with respect to  $C_d/C_a = 1 - 6$  and  $C_d/C_b = 1 - 6$ (a)  $\xi_a = \xi_b = 2\%$ ; (b)  $\xi_a = \xi_b = 5\%$ 

#### **5. NUMERICAL VALIDATION**

A sample microelectronics factory is selected to numerically validate the retrofit method proposed in this study. The exterior and interior structures are identified as "structure *a* " and "structure *b* ", respectively. The estimated seismic reactive weights on the roof and the second floor of the exterior structure are equal to  $W_{a1}$  =3.53 kN /  $m^2$  and  $W_{a2}$  =5.4 kN /  $m^2$ , respectively. The seismic reactive weights per unit area on the cleanroom floor and the subfab floor are correspondingly equal to  $W_{b1} = 14.7 kN/m^2$  and  $W_{b2} = 5.4 kN/m^2$ . The first mode natural frequencies of the exterior and interior structures in the X direction are  $f_a$ =2.43 *Hz* 



and  $f_b = 1.13 Hz$ . The corresponding mode shape vectors are  $\langle 1.0, 0.85 \rangle^T$  and  $\langle 1.0, 0.52 \rangle^T$ . If the two structures are simplified as two single degree of freedom systems in the X direction, the corresponding first modal masses per unit area are calculated as  $m_a = 4641 \, kN - \text{sec}^2/m$  and  $m_b = 9117 \, kN - \text{sec}^2/m$ . Assuming the two structures possess the same inherent viscous damping ratio of  $\xi_a = \xi_b = 2\%$ , the corresponding damping coefficients are calculated to be  $c_a = 4\pi m_a f_a \xi_a = 2834 kN - \text{sec}/m$  and  $c_b = 4\pi m_b f_b \xi_b = 2596 kN - \text{sec}/m$ . If a target damping ratio of 8% is to be added to the interior structure, the viscous dampers should possess a total damping coefficient of  $c_d = 4c_b = 10384 kN - \text{sec}/m$ . For the ratios of damping coefficients,  $c_d/c_a = 3.7$  and  $c_d/c_b = 4.0$ , the undesired frequency ratio band identified in Figure 7 is approximately between 0.8 and 1.2. Therefore, the added damping ratio to the interior structure can be calculated using Equations (4.2) and (4.3).

The elastic seismic responses of the structure subjected to the N-S component of the 1940 El Centro earthquake are calculated using SAP2000 and summarized in Figures 8. The figure shows that the elastic seismic responses at the cleanroom floor for the structure with added dampers are almost identical as those for the structure without dampers but with an inherent viscous damping ratio of 10%. From the result, it is concluded that the damping ratio can be predicted by Equations (4.2) and (4.3) if the frequency ratio of the two structures is outside the undesired frequency ratio band. In addition, the retrofit method proposed in the study is capable of effectively reducing the seismic responses of the interior structure. The responses at the roof of the exterior structure subjected to the El Centro earthquake are shown in Figure 9. It can be seen that the response at the roof is also reduced with the added dampers. However, the response modification is not as significant as that of the interior structure. A reduction of 12% on the peak response is observed while a 30% reduction was expected from the response spectra of the earthquake corresponding to the natural frequency of the exterior structure for an increase of damping ratio from 2% to 10%. The primary reason for this discrepancy is that the dampers are installed in the manner of Figure 3 in which the dampers are not connected directly to the mass of the exterior structure, but to the access floor slab locating around the mid-height of the column. The flexibility the column may reduce the added damping ratio to the exterior structure. A similar phenomenon was observed in the study of damper-connected adjacent buildings with different heights [8] in which the lower building received better response control than the taller building where the floor masses of the taller building above the height of the lower building were not connected with dampers.



Figure 8 Comparison of responses of interior structure; (a)&(b): between the structures with added dampers and with desired effective damping;  $(c)\&(d)$ : between the structures with and without added dampers





Figure 9 Comparison of responses of exterior structure (a)&(b): between the structures with added dampers and with desired effective damping;  $(c)$ &(d): between the structures with and without added dampers

#### **6. CONCLUSIONS**

This paper addresses the economic value and seismic vulnerability of microchip fabrication facilities. The application of viscous dampers to seismically retrofit the fab structures is proposed in the study. The construction corresponding to the proposed retrofit method is believed to induce the least interference to an operating fab. The proposed retrofit method is applicable and effective when the frequency ratio of the interior structure to the exterior structure is outside the undesired frequency-ratio band, where the added damping ratio is extremely sensitive to the frequency ratio. The numerical study has proved that the design guidelines provided in this study can well predict the added damping ratio, and the effectiveness of the seismic retrofit strategy is obvious. However, it should be noted that since the dampers are connected to the access floor slab located around the mid-height of the columns of the exterior structure, the response control of the exterior structure is less significant than the control of the interior structure.

#### **REFERENCES**

- 1. Performance Based Seismic Engineering of Buildings, Structural Engineers Association of California Sacramento, California, 1995.
- 2. Hwang, J.S., Huang, Y.N., Hung, Y.H. and Huang, J.C. (2004). Applicability of seismic protective systems to structures with vibration sensitive equipment. *Journal of Structural Engineering* **130:11**, 1676-1684.
- 3. Gordon, C.G. (1991) Generic criteria for vibration-sensitive equipment. *Proceedings of International Society for Optical Engineering* **1619**, 71-85, San Jose, California.
- 4. Kasai K. and Maison B.F. (1992). Dynamics of pounding when two buildings collide. *Earthquake Engineering and Structural Dynamics* **21**, 771-786.
- 5. Gurley, K.A., Kareem A., Bergman L.A., Johnson, E.A. and Klein, R.E. (1994). Coupling tall buildings for control of response to wind, *Structural Safety & Reliability*, 1553-1560.
- 6. Abe, M. and Igusa, T. (1995). Tuned mass dampers for structures with closely spaced natural frequencies. *Earthquake Engineering and Structural Dynamics* **24**, 247–261.
- 7. Luco, J.E. and De Barros, FCP. (1998). Optimal damping between two adjacent elastic structures. *Earthquake Engineering and Structural Dynamics* **27**, 649–659.
- 8. Xu, Y.L., He, Q. and Ko, J.M. (1999). Dynamic response of damper-connected adjacent buildings under earthquake excitation. *Engineering Structure* **21**, 135-148.
- 9. Bhaskararao, A.V. and Jangid, R.S. (2006) Seismic analysis of structures connected with friction dampers. *Engineering Structures* **28**, 690–703.
- 10. Veletsos, A.S. and Ventura, C.E. (1986). Modal analysis of non-classically damped systems. *Earthquake Engineering and Structural Dynamics* **14**, 217-243.
- 11. Hwang, J.S., Chang, K.C. and Tsai, M.H. (1997). Composite damping ratio of seismically isolated regular bridges. *Engineering Structures* **19:1**, 55-62.
- 12. Hwang, J.S. and Tseng, Y.S. (2006) Design formulations for supplemental viscous dampers to highway bridges. *Earthquake Engineering and Structural Dynamics* **34:13**, 1627-1642.