A Two-Step Method for Structural Parameter Identification with Unknown Ground Motion

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ABSTRACT:

Simultaneous identification of structural parameters and inversion of ground motion in time domain from measured structural response time histories only have received great interests in recent years. We propose a two-step method in this paper for the problem concerned. The method first identifies the structural parameters of a shear building using the LSM (Least Squares Method) from the free vibration part of the measured responses of the structural system. After the structural parameters are identified, we get the unknown ground motion using the SAA (Statistical Average Algorithm) by solving the motion equation. To verify the effectiveness of the method, we simulate the response of a linear 10DOF (ten-degree-of-freedom) shear-type structure excited by ground motions and make estimations using the simulated responses assumed as observed records. The results show that the method has the preferable precision.

KEYWORDS: structural parameters identification, ground motion inversion, two-step method, least-squares method, statistical average algorithm

1. INTRODUCTION

System identification was firstly applied in control field. In the past few decades, along with computing technique and experimental technique development, system identification invoked great interests in the world in many fields of engineering application. In the civil engineering field, system identification was introduced in the 1970's, and was named structural system identification.

Since input excitations such as earthquake loads can not be accurately measured and experimental equipments are expensive, structural parameter identification technique without external excitations brought into focus of many scholars in recent years. Great efforts have been made to find a certain algorithm for structural parameter identification and ground motion inversion. Toki et al. [1] proposed a time-domain method, by which structural parameters were identified and input ground motion was inversed from structural responses only. The coda of the measured structural dynamic responses, which was assumed as free vibration, was first utilized to identify the structural parameters with the extended Kalman filter. Then input ground motion was estimated from the measured full records and the identified parameters. Wang and Haldar [2] developed a finite element-based element-level time domain identification technique to identify structural parameters and estimate ground motion without any input information. The method assumed that the first two time sample points of the ground motion were zero. Structural parameters were identified with least-squares criterion, and ground motion was estimated from the identified parameters. The method was an iterative method, and simultaneous estimated structural parameters and ground motion until the convergence of input forces at the first two time points could be obtained with a predetermined accuracy. Wang and Haldar [3] extended their time domain system identification technique to estimate the structural parameters with limited observations. The new method denoted as ILS-EKF-UI by the authors was a combination of an iterative Least-Squares procedure with unknown input excitations (ILS-UI) and the extended Kalman filter method with a weighted global iteration (EKF-WGI). The method was a two-phase method. The first phase was to obtain the initial values of the unknown system parameters and estimate of the unknown input excitation using the ILS-UI technique, and the second phase was to identify all unknown system parameters using the KF-WGI technique and limited response measurements. Li and Chen [4] introduced compound inversion ideas, and proposed a statistical average algorithm (SAA) according to the mechanical characteristic of earthquake loads acting on a multi-story shear building. The ground motions of all DDOFs were first estimated from the measured structural responses and the assumed initial values of structural parameters. The estimated ground motions were then modified to the same using the SAA. A new estimation of the structural parameters was obtained from the modified ground motions and measured responses with the least-squares method (LSM). The iterative procedure was repeating until the results satisfied the predetermined convergence criterion. Chen and Li [5] proposed total compensation method (TCM), and developed their method to identify structural parameter with incomplete external loads and estimated the unknown part of input information. The external loads were first estimated from the measured structural responses and the assumed initial values of structural parameters. Then the estimated loads were modified using the known part of input information. Finally the new values of structural parameters were identified from the measured records and the modified loads. Repeating the iterative procedure until the convergence of structural parameters was satisfied with a predetermined accuracy, and simultaneous given the estimation of structural parameters and all the external loads. Feng et al [6] developed a two-stage method in the time domain for identification of structural parameters with limited observations. The ground motion was first inversed using the substructure technique and compound inversion method. And structural parameters were then identified using the extended Kalman filter method with a weighted global iteration (EKF-WGI) from the inversed ground motion, the measured responses and initial values of the structural parameters according to the estimated values of several parameters at the first stage. Ling and Haldar [7] proposed a finite element-based system identification technique for nondestructive damage evaluation of structures at the element level with unknown input. The feature of the algorithm was that it can identify a structure without any input information. A nonlinear set of equations were transformed to a linear set of ones using the Taylor series approximation. System parameters were first estimated using the modified least-squares method from the observational responses and the assumed initial values of input excitation. And the unknown input force was estimated from the measured responses and the estimated system parameters. The system parameters were updated from the updated input time history, and the unknown input force was updated using the updated structural parameters. The iteration process continued until the convergence of the input force was reached with a predetermined accuracy. Zhao et al [8] developed a hybrid identification method for identification of structural parameters and inversion of ground motion in the absolute coordinate system. The method first identified the structural parameters above the first floor of a multi-story shear building using the least-squares method. The minimum modal information was then introduced to obtain the structural parameter of the first floor of the structure. Finally the ground motion was constructed by solving a first-order differentiation equation.

In this paper, we propose a two-step method for simultaneous identification of structural parameters and estimation of ground motion in time domain. Structural parameters of a multi-story shear building are first identified using the LSM (Least Squares Method) from the free vibration part of dynamic responses of the building. Then we get the unknown ground motion using the SAA (Statistical Average Algorithm) from the full time histories of dynamic responses and the identified structural parameters by solving the motion equation.

2. IDENTIFICATION OF STRUCTURAL PARAMETERS

The motion equation of a linear multiple degree of freedom system can be written in matrix form as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{f}(t) \tag{2.1}$$

Where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are mass, damping and stiffness matrices of the structure, respectively; $\ddot{\boldsymbol{x}}(t),\dot{\boldsymbol{x}}(t),\boldsymbol{x}(t)$ are structural response vectors of displacement, velocity and acceleration at time t , respectively; and $\boldsymbol{f}(t)$ is excitation force vector.

Using formal expressions derived from the finite element method, Eq. (2.1) can be transformed into a linear parameter estimation equation. Thus the mass matrix can be expressed as

$$\boldsymbol{M}=\sum_{i=1}^{r}\boldsymbol{T}_{mi}^{T}\boldsymbol{m}_{i}\boldsymbol{T}_{mi}=\sum_{i=1}^{r}\theta_{mi}\boldsymbol{T}_{mi}^{T}\overline{\boldsymbol{m}}_{i}\boldsymbol{T}_{mi} \tag{2.2}$$

Where $\boldsymbol{T}_{mi}$ is the product of the position matrix and the coordinate-transformation matrix; θ_{mi} is the unknown mass parameter of the i th element; and r denotes the number of the finite elements of the structure.

With the Eq. (2.2), the 1st item of the right part of Eq. (2.1) can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)=\sum_{i=1}^{r}\theta_{mi}\boldsymbol{T}_{mi}^{T}\overline{\boldsymbol{m}}_{i}\boldsymbol{T}_{mi}\ddot{\boldsymbol{x}}(t)=\sum_{i=1}^{r}\theta_{mi}\boldsymbol{R}_{mi} \tag{2.3}$$

Where

$$\boldsymbol{R}_{mi}=\boldsymbol{T}_{mi}^{T}\overline{\boldsymbol{m}}_{i}\boldsymbol{T}_{mi}\ddot{\boldsymbol{x}} \tag{2.4}$$

On defining

$$\boldsymbol{H}_{m}=\begin{bmatrix}\boldsymbol{R}_{m1} & \boldsymbol{R}_{m2} & \cdots & \boldsymbol{R}_{mr}\end{bmatrix},\quad \boldsymbol{\theta}_{m}=\begin{bmatrix}\theta_{m1} & \theta_{m2} & \cdots & \theta_{mr}\end{bmatrix}^{T} \tag{2.5}$$

Eq. (2.3) can be rearranged as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}=\boldsymbol{H}_{m}\boldsymbol{\theta}_{m} \tag{2.6}$$

Similar process can be performed for the 2nd and 3rd item of the right part of the Eq. (2.1). We can obtain the expressions as

$$\boldsymbol{C}\dot{\boldsymbol{x}}=\boldsymbol{H}_{c}\boldsymbol{\theta}_{c} \tag{2.7}$$

$$\boldsymbol{K}\boldsymbol{x}=\boldsymbol{H}_{k}\boldsymbol{\theta}_{k} \tag{2.8}$$

Where

$$\boldsymbol{H}_{c}=\begin{bmatrix}\boldsymbol{R}_{c1} & \boldsymbol{R}_{c2} & \cdots & \boldsymbol{R}_{cr}\end{bmatrix},\quad \boldsymbol{\theta}_{c}=\begin{bmatrix}\theta_{c1} & \theta_{c2} & \cdots & \theta_{cr}\end{bmatrix}^{T} \tag{2.9}$$

$$\boldsymbol{H}_{k}=\begin{bmatrix}\boldsymbol{R}_{k1} & \boldsymbol{R}_{k2} & \cdots & \boldsymbol{R}_{kr}\end{bmatrix},\quad \boldsymbol{\theta}_{k}=\begin{bmatrix}\theta_{k1} & \theta_{k2} & \cdots & \theta_{kr}\end{bmatrix}^{T} \tag{2.10}$$

$$\boldsymbol{R}_{ci}=\boldsymbol{T}_{ci}^{T}\overline{\boldsymbol{c}}_{i}\boldsymbol{T}_{ci}\dot{\boldsymbol{x}} \tag{2.11}$$

$$\boldsymbol{R}_{ki}=\boldsymbol{T}_{ki}^{T}\overline{\boldsymbol{k}}_{i}\boldsymbol{T}_{ki}\boldsymbol{x} \tag{2.12}$$

Substituting Eq. (2.6), Eq. (2.7) and Eq. (2.8) into Eq. (2.1), and considering the j th sample points ($t=j\cdot\Delta t$), we can get the expression as

$$\boldsymbol{f}_j = \boldsymbol{H}_j \boldsymbol{\theta}_j \tag{2.13}$$

Where

$$\boldsymbol{H}_j = \begin{bmatrix} \boldsymbol{H}_m & \boldsymbol{H}_c & \boldsymbol{H}_k \end{bmatrix}_j, \qquad \boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_m^T & \boldsymbol{\theta}_c^T & \boldsymbol{\theta}_k^T \end{bmatrix}^T \tag{2.14}$$

For all the sample points, Eq. (2.13) can be transformed to

$$\boldsymbol{F} = \boldsymbol{H}\boldsymbol{\theta} \tag{2.15}$$

Where

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{f}_1^T & \boldsymbol{f}_2^T & \cdots & \boldsymbol{f}_s^T \end{bmatrix}^T, \qquad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1^T & \boldsymbol{H}_2^T & \cdots & \boldsymbol{H}_s^T \end{bmatrix}^T \tag{2.16}$$

And s denotes the number of the sample points.

Using the classical least-squares method (LSM), we can get the equation for identifying structural parameters as

$$\boldsymbol{\theta} = \left(\boldsymbol{H}^T \boldsymbol{H}\right)^{-1} \boldsymbol{H}^T \boldsymbol{F} \tag{2.17}$$

Eq. (2.17) is a set of equation for estimating structural parameters with known input information. Considering the input excitation measured difficultly, we propose the following two-step method.

3. PROPOSED ALGORITHM

The proposed method is based on the linear finite element-based time-domain algorithm without input information, and including two steps for estimation of structural parameters and input ground motion of a multi-storey shear-type building. The method first identifies the structural parameters using the least-squares method from the free vibration part of the structural dynamic responses. After the structural parameters are identified, the unknown ground motion is estimated using the statistical average algorithm by solving the motion equation.

Without losing any generality, we can assume the structure system as a time-invariant mass system and a lumped mass system. Considering the free vibration part of the measured structural responses, the value of the input excitation is zero. Eq. (2.15) is still the parameter identification equation, but expressions of some variables are transformed to the following ones as

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{g}_1^T & \boldsymbol{g}_2^T & \cdots & \boldsymbol{g}_s^T \end{bmatrix}^T \tag{3.1}$$

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_c^T & \boldsymbol{\theta}_k^T \end{bmatrix}^T \tag{3.2}$$

$$\boldsymbol{H}_j = \begin{bmatrix} \boldsymbol{H}_c & \boldsymbol{H}_k \end{bmatrix}_j \tag{3.3}$$

$$\boldsymbol{g}_j = -\boldsymbol{M}\ddot{\boldsymbol{x}}_j \tag{3.4}$$

Using Eq. (2.17), and Eq. from (3.1) to (3.4), we can identify the structural parameters such as damping and stiffness coefficients. The estimation values of structural parameters are:

$$\tilde{\boldsymbol{\theta}} = \left(\boldsymbol{H}^T \boldsymbol{H}\right)^{-1} \boldsymbol{H}^T \boldsymbol{F} \tag{3.5}$$

Substituting Eq. (3.5) into motion equation (2.1), we can get the input excitation of all dynamic degree-of-freedoms at every sample points. The input excitation vector at j th sample point is:

$$\boldsymbol{f}_j = \boldsymbol{M}\ddot{\boldsymbol{x}}_j + \boldsymbol{C}\dot{\boldsymbol{x}}_j + \boldsymbol{K}\boldsymbol{x}_j, \quad (j = 0, \cdots s) \tag{3.6}$$

Where

$$\boldsymbol{f}_j = \begin{pmatrix} f_{1,j} & f_{2,j} & \cdots & f_{n,j} \end{pmatrix}^T \tag{3.7}$$

And n is the number of DDOFs; $f_{i,j}$ denotes external load of i th DDOF at j th sample point.

When the external excitation is ground motion, we can get the expression of the excitation of i th DDOF at j th sample point as

$$f_{i,j} = m_i \ddot{x}_{g,j}, \quad (i = 1, \cdots, n \quad j = 0, \cdots, s) \tag{3.8}$$

Since the accelerations of all the dynamic degree-of-freedoms are the same, using the statistical average algorithm, we can obtain the expression of the ground motion at j th sample point as

$$\ddot{x}_{g,j} = \frac{1}{n}\sum_{i=1}^{n}\frac{f_{i,j}}{m_i}, \quad (j = 0, \cdots, s) \tag{3.9}$$

From the measured structural responses at all the sample points, the ground motion time history can be estimated by solving the Eq. (3.9).

4. NUMERICAL EXAMPLE

A ten-storey shear building is considered here [9], as shown in Fig. 1. A stiffness proportional damping matrix is assumed for this example. The actual values of the parameters are shown in Table 1.

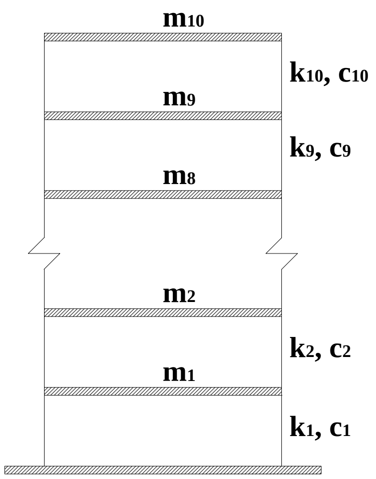

Fig. 1 Shear-type building model

Table 1 Actual values of structural parameters

Floor	Mass	Stiffness	Damping coefficient
1	$1.06*10^5$	$6.8*10^8$	$0.441810*10^{-2}$
2	$1.06*10^5$	$4.5*10^8$	
3	$1.06*10^5$	$4.5*10^8$	
4	$1.02*10^5$	$4.5*10^8$	
5	$1.02*10^5$	$2.2*10^8$	
6	$1.02*10^5$	$2.2*10^8$	
7	$0.90*10^5$	$2.2*10^8$	
8	$0.90*10^5$	$1.5*10^8$	
9	$0.90*10^5$	$1.5*10^8$	
10	$0.80*10^5$	$1.5*10^8$	

The structure is excited by a ground motion shown in Fig. 2. The theoretical responses of all ten DDOFs are calculated in terms of acceleration, velocity and displacement. And the responses of 1st, 5th and 10th floor are shown in Fig. 3, Fig. 4 and Fig. 5, respectively. After the theoretical responses are obtained by dynamically analyzing the structure, the input ground motion is assumed unknown. The task is to identify the structural parameters and to estimate the ground motion using the proposed algorithm with the structural responses only.

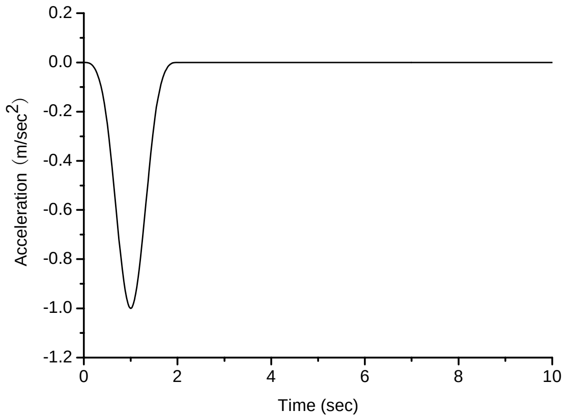

Fig. 2 Input excitation

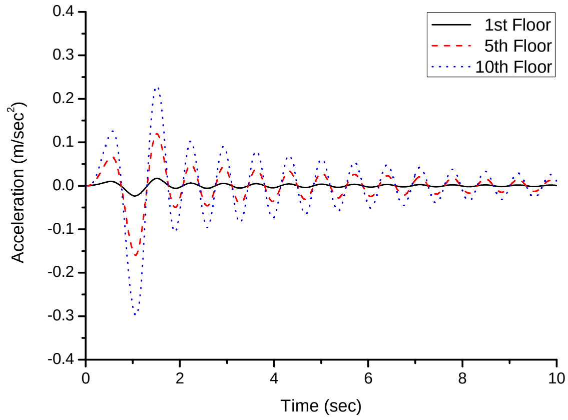

Fig. 3 Acceleration responses of 1st, 5th and 10th floor

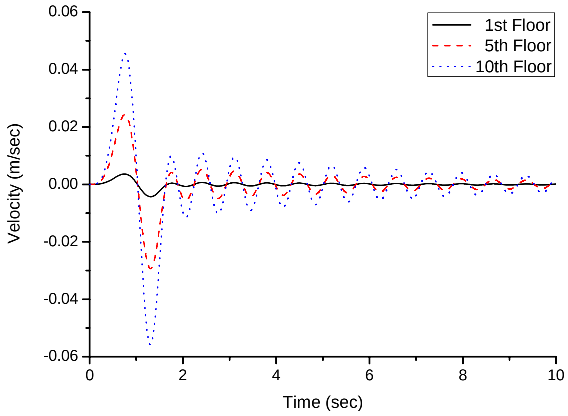

Fig. 4 Velocity responses of 1st, 5th and 10th floor

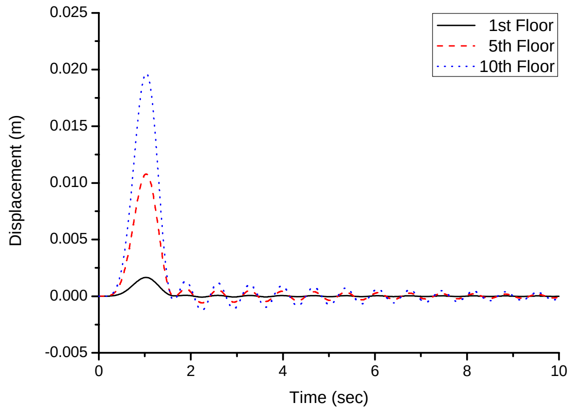

Fig. 5 Displacement responses of 1st, 5th and 10th floor

Although the records are available for a long duration, only the responses from 5.0 to 5.5 second are considered here as the free vibration for identifying the structural parameters. Furthermore, it is assumed that the time interval is 0.01 second. The stiffness proportional damping is assumed to two cases: known and unknown proportional damping coefficient. For the known case, the actual and the identified values of stiffness parameters ($\boldsymbol{K}$) are shown in Table 2. For the unknown case, the actual and the identified values of stiffness parameters ($\boldsymbol{K}$) and damping coefficient (β) are shown in Table 3. The ground motion is inversed from the full time histories of the dynamic responses and the estimated parameters. And the actual and the inversed ground motions of the two cases are shown in Fig. 6 and Fig. 7, respectively.

For the two cases, the maximum error in the stiffness estimation is only 0.01%, and the maximum error for damping coefficient is also 0.01%. And the estimated ground motions have a good agreement with the actual ones for the two cases. Although the ratio of the stiffness to the damping coefficient exceeds 10^{10}, the maximum error is still very small and does not change bigger. The structural parameters are identified in the first step using only 0.5 second of dynamic responses, and in the second step the ground motion is estimated using the full records of output responses and the identified parameters.

Table 2 Identification of stiffness parameters with known damping coefficient
(Time interval = 0.01, sample points = 51)

Floor	***K***		
	Exact	Result	Error (%)
1	$6.8*10^8$	$6.80005*10^8$	0.00
2	$4.5*10^8$	$4.49996*10^8$	0.00
3	$4.5*10^8$	$4.49996*10^8$	0.00
4	$4.5*10^8$	$4.49995*10^8$	0.00
5	$2.2*10^8$	$2.19988*10^8$	0.01
6	$2.2*10^8$	$2.19987*10^8$	0.01
7	$2.2*10^8$	$2.19988*10^8$	0.01
8	$1.5*10^8$	$1.49989*10^8$	0.01
9	$1.5*10^8$	$1.49989*10^8$	0.01
10	$1.5*10^8$	$1.49990*10^8$	0.01

Table 3 Identification of stiffness parameters and damping coefficient with unknown damping coefficient
(Time interval = 0.01, sample points = 51)

	Floor	Exact	Result	Error (%)
K	1	$6.8*10^8$	$6.80005*10^8$	0.00
	2	$4.5*10^8$	$4.49996*10^8$	0.00
	3	$4.5*10^8$	$4.49996*10^8$	0.00
	4	$4.5*10^8$	$4.49995*10^8$	0.00
	5	$2.2*10^8$	$2.19988*10^8$	0.01
	6	$2.2*10^8$	$2.19987*10^8$	0.01
	7	$2.2*10^8$	$2.19988*10^8$	0.01
	8	$1.5*10^8$	$1.49989*10^8$	0.01
	9	$1.5*10^8$	$1.49989*10^8$	0.01
	10	$1.5*10^8$	$1.49991*10^8$	0.01
β	—	$0.441810*10^{-2}$	$0.441849*10^{-2}$	0.01

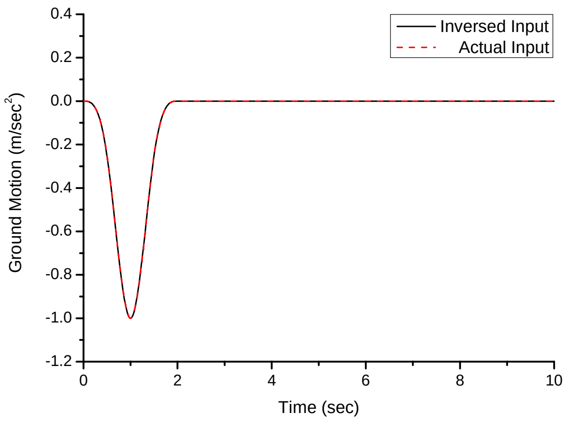

Fig. 6 Inversion of ground motion with known damping coefficient from dynamic responses only

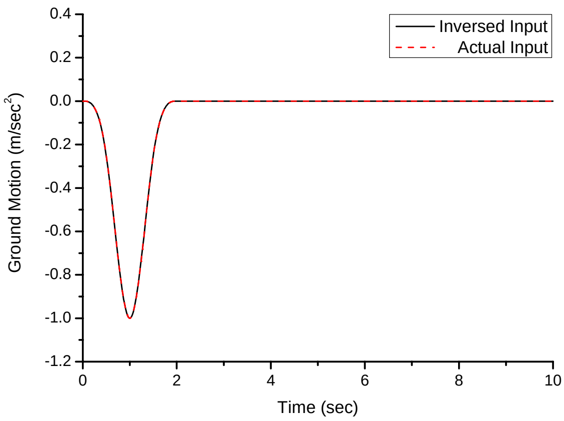

Fig. 7 Inversion of ground motion with unknown damping coefficient from dynamic responses only

5. CONCLUSIONS

A two-step method is proposed in this paper for simultaneous identifying structural parameters and inversing ground motion in time domain without excitation information. The derivation of the proposed method shows that the unknown input can be of any type and can be exerted at any location. This makes the method extremely robust. We give a numerical example of a 10-degree-of-freedom shear-type structure excited by the ground motion to verify the procedure. The results show that the proposed method identify the structural parameters and estimate the ground motion very well, and only a short duration record is required for the successful implementation of the proposed method.

The structural parameters and the excitation time history are estimated with directly solving the linear parameter equation and motion equation. The computing time is very short since the proposed method need not the iterative computing. The method is proposed here to theoretical study for estimation of structural parameters and input excitation. And in the future we will consider some noise in the output responses to estimate the parameters and the excitation with some filtering techniques.

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