# A DIRECT BOUNDARY ELEMENT METHOD APPLIED TO STUDY THE SEISMIC RESPONSE OF TRIANGULAR VALLEYS: 

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#### Abstract

: The direct boundary element method is well studied for the analysis of the seismic response of triangular valleys. The main purpose of this article is conducting the numerical parametric analyses in the time domain to gaining insight into amplification pattern of 2D triangular valleys. Clear perspectives of the amplification patterns of the valley are presented by investigation of the frequency-domain responses. It is shown that wavelength and site geometry are the independent key parameters governing the valley's amplification pattern. Some simple relationships are obtained based on study results which could be used in seismic microzonation and seismic design of structures founded inside the valley.


KEYWORDS: topography effect, triangular valley, boundary element, amplification

## 1. INTRODUCTION

Based on prior observations (Friuli, Italy 1976, Irpinia, Italy 1980, Chile 1985,...) it has well been recognized that by changing the amplitude, frequency content and duration of ground shaking, the local geological and topographical conditions can yield concentrated damages during earthquakes. Complex nature of the seismic wave scattering by topographical structures can be studied accurately and economically by advanced numerical methods under realistic conditions. The BEM is a very effective numerical tool for dynamic analysis of linear elastic bounded and unbounded media. The method is very attractive for wave propagation problems, because the discretization is done only on the boundary, yielding smaller meshes and systems of equations. Another advantage is that this method represents efficiently the outgoing waves through infinite domains, which is very useful when dealing with scattered waves by topographical structures. When this method is applied to problems with semi-infinite domains, there is no need to model the far field. Prompted by observational and instrumented evidence, the problem of scattering and diffraction of seismic waves by topographical irregularities has been studied by several authors using numerous numerical methods, but attempts have seldom been made to express the parametric analysis results in terms of microzonation study requirements. (Boore 1972) modeled the effects of a ridge by using finite differences. Significant interest in ground motion variations across canyon geometries was generated by recordings of unusually large amplitude on a canyon rim near Pacoima Dam during the 1971 San Fernando earthquake. This led to studies of the effects of canyon topography on ground motion by (Trifunac, 1973),(Wong and Trifunac, 1974) among others. These investigators generally assumed linear-elastic medium and simple canyon geometry under SH waves (e.g., semi-cylinder or semi-ellipse). Similar studies for P and SV waves have been performed by (Wong ,1982) and (Cao and Lee,1990), and indicate amplification levels generally smaller than those for SH. Canyon geometry is also significant, with amplification being negligible for shallow canyons (ratio of depth to width < 0.05). For deep canyons, edge amplification is not significantly different than that discussed above, but more base de-amplification occurs (Wong and Trifunac, 1974). (Sánchez-Sesma \& Rosenblueth, 1979) presented a method for calculating the two-dimensional scattering of incident SH waves by canyons of arbitrary shape. They formulated the problem in terms of a Fredholm integral equation of the first kind with the integration path outside the boundary.(Shah et al.1982) studied scattering of antiplane shear waves (SH) in two dimensions by surface and near-surface defects in a homogeneous, isotropic elastic semi-infinite medium .They finally studied a problem of multiple scattering by a triangular canyon and a nearby circular tunnel.(Kawase et al 1988) calculated time-domain response of a semicircular canyon for incident SV , P and Rayleigh waves by the discrete wavenumber boundary element method.
(Vogt et al 1988) calculated the generalized scattered motion in the region of a canyon of arbitrary shape in a horizontally layered half-space using the indirect-boundary-element method in the frequency domain.(Vaziri\&Trifunac1988b) verified the scattering and diffraction of plane P and SV waves in two dimensional valleys.(Eshragi \& dravinski 1989) by using a wave function expansion technique investigated scattering of elastic waves by three-dimensional canyons embedded within an elastic half-space.
(Sánchez-Sesma et al.1993) applied a boundary integral formulation to model the ground motion on alluvial valleys under incident P, S and Rayleigh waves.(Sánchez-Sesma\&Luzon,1995) used a simplified indirect boundary-element method (BEM) to compute the seismic response of three-dimensional alluvial valleys under incident P , S , and Rayleigh waves.(Moczo et al. 1996) investigate an antiplane 2D resonance in a certain class of the sedimentary structures using the finite-difference modeling and they showed the 2D resonance which may develop in the valleys do not satisfy Bard and Bouchon's existence condition. Their results confirmed that the resonance phenomenon is quite robust and that it is to be expected in many configurations of sediment valleys or basins.
(alvarez-Rubioa et al.2005) used the boundary element method for the analysis of the seismic response of valleys of complicated topography and stratigraphy. (Kamalian et al 2006) have presented advanced formulation of the time-domain two-dimensional hybrid finite element-boundary element method (FEM/BEM) and have applied to carry out site response analysis of homogeneous and non-homogeneous topographic structures subjected to incident in-plane motions. (Kamalian et al 2007, 2008) have used direct boundary element method to study the Amplification Pattern of 2D Semi-Sine Shaped Valleys and hills Subjected to Vertically Propagating Incident Waves. In this article, to able to gaining insight into seismic response of triangular valleys a series of numerical parametric analyses have been conducted by using time domain direct boundary element method. To this end we exploited a general purpose two-dimensional nonlinear two-phase BEM/FEM code named as HYBRID. Several examples including site response analysis of half-plane, horizontally layered sites, canyons, alluvial valleys and ridge sections subjected to incident $P$ and $S V$ waves were solved in order to show the accuracy and efficiency of this implemented BE algorithm in carrying out site response analysis of topographic structures.

## 2. METHODOLOGY OF PARAMETRIC ANALYSIS

In all analyses the vertically propagating incident P , SV waves of Ricker type are adopted as a dynamic excitation. The Ricker type wave equation can be expressed as:

$$
\begin{equation*}
f(t)=\left[1-2 \cdot\left(\pi \cdot f_{p} \cdot\left(t-t_{0}\right)\right)^{2}\right] e^{-\left(\pi \cdot f_{p} \cdot\left(t-t_{0}\right)\right)^{2}} \tag{1}
\end{equation*}
$$

in which, $f_{p}$ and $t_{0}$ denote the predominant frequency and an appropriate time shift parameter, respectively. In case of SV waves, $f(t)$ designates the horizontal component of the incident motion while the vertical one is zero, and in case of P waves, vice versa. The boundary conditions consisted of the traction free ground surface and the seismic loading was introduced through the term of incident motion. The geometry of the 2D homogenous valley and Ricker wave time history are demonstrated in Fig. (1), (2) respectively.
In order to apply the results of analyses to frequencies and geometrical conditions different from those of this study models, all results were presented in dimensionless forms. To achieve this goal the well known dimensionless period $\mathrm{T}=\mathrm{tc}_{2} / 2 \mathrm{~b}$ (or its inverse: the dimensionless frequency), which means physically the ratio of the incident's wave length to the width of the valley and the shape ratio of valley $\mathrm{SR}=\mathrm{h} / \mathrm{b}$, are used in which C 2 is wave propagation velocity, b is the half width of the valley, t is time and h is the depth. This study involves a wide range of shape ratios to considering the geometry of the valley: $0.1,0.3,0.5,0.7,1.0,1.2,1.5,2,3,4$. Poisson ratio is chosen 0.33 and seismic wave frequency is adopted 3 Hz . Based on engineering interests, a dimensionless period interval of 0.25 to 8.33 was considered, which corresponds to incident waves with wave lengths of 0.25 to 8.33 times the valley's width. This broad period interval was divided into the five subintervals namely: ( $0.25-0.5$ ), ( $0.50-1.00$ ), (1.00-2.00), (2.00-4.17) and (4.17-8.33), corresponding to incident waves with very short, short, medium, large and very large wave lengths, respectively. Five above-mentioned intervals are shown by P1, P2, P3, P4 and P5 respectively. For the sake of simplicity and following the well known concept of average horizontal spectral amplification (AHSA) defined by (Borcherdt et al 1994) as spectral ratios representing averages over short, intermediate, mid and long period bands, five distinct amplification factors were computed for every point along the valley, by averaging the corresponding amplification curve over each of the above mentioned five period subintervals P1 to P5.

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Figure 1 The geometry of 2D slope


Figure 2 Ricker type wave time history

## 3. GENERAL AMPLIFICATION PATTERN

Figure 3 demonstrate clear perspectives of the time domain amplification pattern of a 2 D valley with the shape ratio of 1.0 subjected to an incident P and SV wave. The receiving points are arranged within an interval of $-5 b$ to 5 b from the valley's center. As can be inferred in the motion components consistent with excitation namely horizontal Component for SV incident wave and vertical component in P incident wave, the ground motion has an amplification of 2.0 but in the motion components opposite to excitation no considerable amplification is produced. The displacement time history of the neighboring points of the edge of the valley are composed of incoming, reflected, refracted and Rayleigh wave effects. As expected the interference of four aforementioned waves produces the largest value of the motion amplitude in the edge and in its neighboring points. Refer to Figure 3 As the incoming waves arrive to the lower points of the valley, the diffracted waves travel through the inclined free surface until they reach to the edge and then move to the points far from the edge. Considering the displacement time histories of the points far from the edge, enable us to observe different diffracted waves. Since diffracted SV and Rayleigh waves propagate with lower velocity compare to diffracted P waves so they arrive in time lag to the points far from the edge. These two kinds of diffracted waves can easily be distinguished in Figure 3.

## 3. WAVE LENGTH, SHAPE ARTIO AND WAVE TYPE EFFECT

Figures 4 demonstrate in a more detailed form, the dependency of the amplification potential of 2D triangular valleys subjected to vertically incident P, SV waves on the wave length and shape ratio. The amplification curves for the points within the valley are categorized according to the wave length of the incident wave, for six shape ratios of $0.1,0.5,1,3$ and 4 . The amplification curves are only shown for consistence components of motion. As can be seen, irrespective of the shape ratio, the wave length plays a key rule in determining the amplification curve of the valley. For both P,SV incident waves two distinct zones along the valley could be distinguished: The first zone is the central part of the valley in which the ground motion is usually de-amplified, irrespective of the length of the incident wave and of the valley's shape ratio. The second zone consists of the edge and its adjacent region in which the ground motion could be considerably amplified, depending on the length of the incident wave and on the valley's shape ratio. If the valley is impinged by a very long to long incident wave, the edge zone would experience amplification factors of one and greater than one, which increase along with the shape ratio. In this case, the maximum amplification factor occurs at the edges and the amplification curve decays toward the center of the valley. If the incident wave possesses a length of medium size, although the same behavior would exist, but as the shape ratio increases, smaller parts of the edge zone would be amplified and more oscillation of the amplification curves could be seen. If the valley is impinged by a short to very short incident wave, the same behavior would still be seen, whereas the oscillations of the amplification the curves would be intensified and valleys with smaller shape ratios would be mostly amplified. Figure 4 shows that the amplification patterns SV and P waves are similar in general. In spite of these similarities, there exist some minor differences. When the valley is subjected to short period excitation (P1, P2) although SV incident waves produce


Figure 3. Amplification patterns of a triangular valley with shape ratio of 1 in case of incident SV (right) and P (left) waves (The symbols 'Hrz' and 'Vrt' present the horizontal and vertical components of amplification, respectively.)
the largest amplifications in large shape ratios but no considerable amplification is observed for smaller shape ratios. Conversely, considerable amplifications can be concluded in the lower shape ratios in the case of P wave excitation. In medium period subinterval (P3) the valleys with smaller shape ratios become more vulnerable to SV wave excitation. In long to very long excitation (P4, P5) only under SV waves the amplification potential of the valleys is more pronounced. Figure 5 depicts the amplification curves for constant shape ratios which are drawn for different periodic subintervals. It can be seen that as the shape ratio increases besides the short period components the effect of long period components of incident waves becomes critical and the relief can significantly amplify the low frequency seismic motions.
The variation of de-amplification and amplification factor in the center and edge of the valley with dimensionless period is demonstrated in Figure 6 for SV wave excitation and some different shape ratios. The natural period of the valley can be obtained from Figure 6 for each shape ratio. It can be inferred that as the shape ratio increase the natural period increases (or characteristic frequency decreases). In other word as the shape ratio increase the larger wavelengths can be amplified by the valley.

## 4.LIMITING PERIOD

As mentioned in previous section the amplification of the valley is mainly governed by shape ratio and wave length. In spite of this fact, there are some useful engineering indexes, noticeable in seismic microzonation
works, which could be evaluated as a function of only the shape ratio. Aiming at this goal the limiting dimensionless period $T_{\text {limit }}$ of incident waves that cause an amplification factor of 1.1 at the edges is used. The incident waves which their period is greater than the $T_{\text {limit }}$ produce amplification at the edges less than 10 percent and the topography effect could be practically ignored in this case. In the following some of the seismic characteristics of semi sine valleys like $T_{\text {limit }}$ and peak amplification, which is studied by (kamalina et al 2007) will be compared with this article results. The approximate linear variation of the limiting dimensionless period with shape ratio for both kinds of valleys is demonstrated in Figure 7.
As Figure 7 indicates, since compare to the triangular valleys the semi sine valleys have greater area so in all shape ratios the semi sine valleys have a larger limiting periods, in other word the minimum frequency that is required to create a considerable amplification in semi sine valleys is smaller than that of triangular valleys. The $T_{\text {limit }}$ for semi sine and triangular valley can be approximated as equations 2 and 3 respectively:

$$
\begin{align*}
& T_{\text {limit }}=0.3+3.4 \text { (S.R) }  \tag{2}\\
& T_{\text {limit }}=0.1+1.7 \text { (S.R) } \tag{3}
\end{align*}
$$

## 5.PEAK AMPLIFICATION

In order to investigate the effect of the shape of the valley on the amplification factor, the relation between the peak amplification and shape ratio is approximated by linear relationship and is compared for two triangular and semi sine valleys in Figure 8-a As it can be seen there is no considerable difference between two amplifications and two kinds of valleys produce roughly the same peak amplifications when they have an equal shape ratios. The maximum amplification factors of the edge estimated by equations (4) and (5) are comparable to the maximum value of 1.4 proposed by the AFPS code for seismic design of structures in topographic areas. As Figure 5 indicates there is notable spatial variation of the motion amplitude along the valley, especially in the case of valleys with shape ratios of more than 1.0 , which can yield considerable relative displacements important for line structures such as bridges, dams and life-lines. To consider this effect, Figure 8-b approximates the maximum relative amplification of the edge with respect to center of the valley, as a linear function of the shape ratio for two triangular and semi sine shapes. The linear equations correspond to each case are also described. Linear peak amplification relationships for semi sine and Triangular valleys can be expressed as equations (4),(5):

$$
\begin{align*}
& A_{\max }=1+0.3 \mathrm{SR}  \tag{4}\\
& A_{\max }=1+0.25 \mathrm{SR} \tag{5}
\end{align*}
$$

As can be seen, in the case of semi sine and triangular valleys with a shape ratio of 1.5 , the amplitude of motion at the edge of the valley could be around 8 and 12 times of that at its center respectively, which could not be ignored in seismic design of line structures.

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Figure 4. Wavelength effect and shape ratio effect on averaged amplification curves of 2D triangular shaped Valleys subjected to vertically propagating incident SV (left), P (right) waves.( $\mathrm{x} / \mathrm{b}$ is normalized distance from center of the valley )

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Figure 5 The amplification curves of 2D semi-sine-shaped valleys subjected to vertically propagating incident SV waves for different periodic subintervals for different shape ratios


Figure 6. The variation of de-amplification and amplification factor in the center and edge of the valley whit dimensionless period


Figure 7 Comparison of limiting dimensionless period in semi sine and triangular valleys


Figure 8 Variation of relative and Max. amplification with shape ratio

## 6.CONCLUSIONS

This paper presents clear perspectives of the amplification patterns of 2D homogenous triangular valleys subjected to vertically propagating SV and P waves, obtained by an extensive numerical parametric analysis using the time domain BEM. It is shown that:

1. The amplification potential of the valley is strongly influenced by the wave length of the incident wave and by the shape ratio.
2. The topography effect could be ignored, only if the valley has a shape ratio of less than 0.1 or is subjected to incident waves with wave lengths of greater than $T_{\text {limit }}$ times its width. The coefficient $T_{\text {limit }}$ could be estimated by equation (3) and is usually less than four times the shape ratio.
3. Although the amplification potential of the valley increases with the shape ratio, but the increasing rate depends on the wave length and varies across the valley.
4. Two distinct seismic zones could be distinguished along the valley: The center zone in which the motion is mostly de-amplified; the edge zone in which the ground motion could be considerably amplified, especially if impinged by incident waves possessing wave lengths of equal or twice the width of the valley.
5. The maximum amplification factor along the valley occurs usually at the edge and has an increasing rate of 0.25 times the increasing rate of the shape ratio.

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