THREE-DIMENSION DYNAMIC SOIL-STRUCTURE INTERACTION ANALYSIS USING THE SUBSTRUCTURE METHOD IN THE TIME DOMAIN

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ABSTRACT :
Based on the wave propagation theory and the lumped-mass explicit finite element procedure with a local transmitting artificial boundary, the substructure method is formulated with step-by-step integration and central difference approach. Since the explicit finite element and step-by-step integration scheme is adopted, this presented method leads to the ability in computational feasibility and the applicability in engineering. In addition, the local transmitting artificial boundary results in the significant reduction in computational effort with little loss of accuracy. To illustrate the efficacy of the procedure, numerical examples are studied for a structure founded on a rigid foundation which is on the surface of semi-infinite soil medium. The soil-structural dynamic predictions under the assumption of rigid foundation on a uniform simi-infinite medium using this substructure method are in good agreement to the reference solutions by direct method. Another numerical example is studied for the influence by the big stiffness of soil medium on the dynamic characters of the soil-structure system. This prediction can also be in agreement the reference by the typical method. These predictions can display the applied feasibility in engineering.

KEYWORDS:
soil-structure interaction, substructure method, transmitting boundary condition

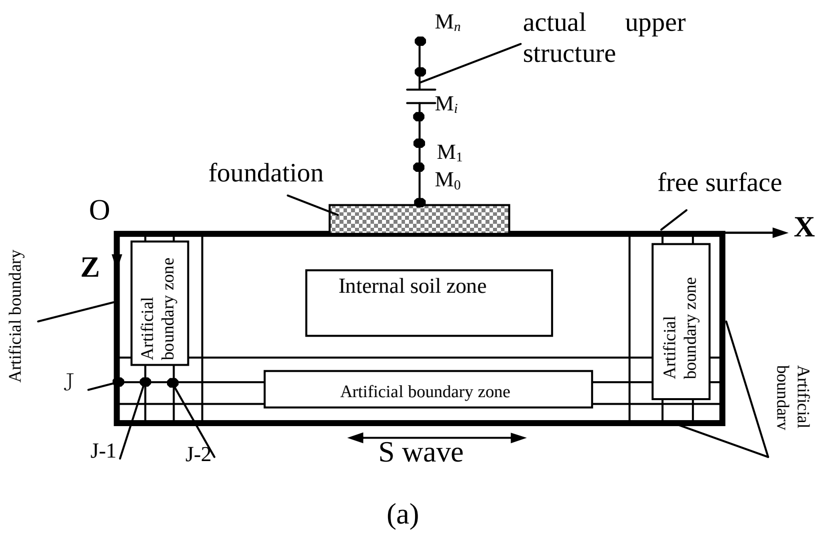
1. INTRODUCTION SECTION HEADING

Since some structures such as nuclear power plants and high building have three-dimensional size, the effect of soil and structure dynamic interaction can be rationally considered only when three-dimension analysis is performed because of structural integrity. There have been many studies on SSI theories and methods which are generally classified into direct method and substructure method either in time domain or in frequency domain. In the direct method, the upper structure, the rigid foundation and the bounded soil medium which is adjacent to the foundation are modeled as a whole with the artificial boundary. Since all the soil-structure system is modeled, the computational cost for the complex system is generally too high even to solve the three-dimension linear elastic system. In the substructure method, the soil-structure system is divided into two or more substructures. Each substructure is modeled separately and is connected to the general structure through the interface of adjacent to other substructures. The earliest substructure developments have been made for several effective analytic solutions considering the simi-infinite soil zone as homogeneous, isotropic and elastic medium as substructure [Luco & Westmann, 1971; Oien, 1973; Wong & Luco, 1978]. After solving the impedance function of the simi-infinite soil zone, the interaction force which the soil applies to the actual structure is replaced by the impedance function [Veletsos & Wei, 1971; Wong & Luco, 1978]. However, it is difficult for the analysis method to solve the soil-structure interaction problems when the semi-infinite soil zone is heterogeneous or anisotropic or nonlinear. Latterly, the researcher's interests have been growing in developing the discrete method to deal with the soil-structure interaction problems [Shah & Wong, 1982; Wolf, 1988; Zhang, Wegner & Haddow, 1999]. Compared with the direct method, the substructure method can simplify the computational model, decrease the computational cost greatly and easily dealt with the complex soil-structure system by dividing the whole system into necessary substructures. However, since most of substructure analysis methods are developed to use implicit equations to solve this kind of problems, the computational freedoms will be relative to the complexity conditions of soil medium and actual upper structure. This treatment often also leads to difficulty in dealing with the engineering problems. Since the numerical procedure using either direct or substructure method can not directly model the unbounded soil medium, the finite soil zone have to be considered. The effect of the truncated soil medium is generally considered approximately by the artificial boundary such as a viscous boundary [Lysmer & Kuhlemeyer 1969], a superposition boundary [Smiyh, 1974], and several others [Lysmer & Waas 1972; Liao et al 1984; E.Kausel 1988].

In this paper, a new three-dimension substructure numerical procedure is presented for analysis of dynamic soil-structure interaction. Utilizing the advantage of local transmitting artificial boundary [Liao et al 1984] for wave propagation explicit numerical procedure, the time-domain explicit finite element method is formulated with step-by-step integration with central difference and lateral difference approach. The key to the computational efficiency of the numerical procedure is the application of the explicit procedure for soil zone and local transmitting artificial boundary as well as substructure treatment. Based on this numerical procedure, the corresponding time-domain explicit finite element program T3DSSI is programmed to analyze the dynamic soil-structure interaction problems. Program T3DSSI has such advantages as simple input data, fast computational speed and extensive engineering applicability. Under the assumption of rigid foundation, the structure vibration frequencies can be obtained using ABAQUS mode-based analysis. The substructure method computed by program T3DSSI is tested by direct method and by a practical example on the vibration frequencies of a structure on rigid medium. It shows that the soil-structural dynamic predictions by this substructure method are in good agreement to the reference solutions.

2. THE THREE-DIMENSION SUBSTRUCTURE METHOD IN TIME DOMAIN

As shown in Fig.1 (a), a soil-structure dynamic interaction model is composed of actual upper structure, surface foundation and finite soil zone with the artificial boundary. Under the rectangular coordinate system (*x*-, *y*-, *z*-), the model is applied to the analysis of S wave propagation through the semi-infinite soil medium to the actual structure.



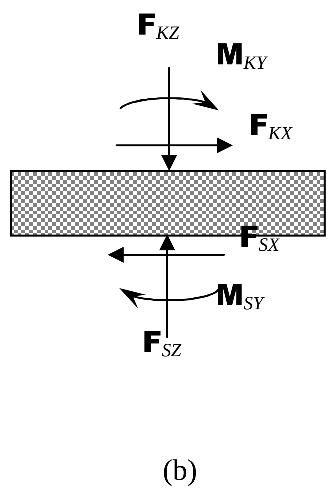


Figure 1 (a) A typical computational model for the soil-structure dynamic interaction analysis; (b) The interaction forces among soil medium, rigid plate and upper structure

2.1. Equation of motion of soil domain

As shown in Fig. 1 (a), the finite soil zone is divided in two parts as the artificial boundary domain and the internal computational domain. Also as shown in Fig. 1 (a), the nodes of the finite element mesh in soil zone are divided into the artificial boundary nodes, the internal nodes, and the nodes connecting with the foundation. To internal nodes of soil domain, when the explicit lumped-mass finite element method in time domain is adopted, the equation of motion can be expressed as:

$$\{U_i\}^{p+1} = 2\{U_i\}^{P} - \{U_i\}^{p-1} - \Delta t^2 [M_i]^{-1} [\sum_l (1+\frac{\mu}{\Delta t})[K_{il}]\{U_l\}^p - \sum_l \frac{\mu}{\Delta t}[K_{il}]\{U_l\}^p - \{F_i\}^p] \quad (2.1)$$

where $[M]$, $[K]$, and $\{F\}$denote the diagonal lumped-mass matrix, the stiff coefficients matrix , and the external loading vector, $\{U\}$ $\{\dot{U}\}$ and $\{\ddot{U}\}$ denote respectively the displacement, the velocity and the acceleration of internal node, μ denotes the viscosity damping coefficient in proportion to the velocity of internal node. The subscript *i* or *l* denotes the internal node and subscript *il* denotes the relationship between the internal node *i* and the node *l* adjoined with node *i*. The superscript *p* refers to the increment number and the relationship between time *t* and no negative increment number *p* is $t=p\triangle t$.

The history of seismic wave is assumed to be inputted from bottom the artificial boundary nodes, and the scattering wave can be transmitted out through them. The motion of the artificial boundary nodes can not be computed by equation (2.1) since the motion of nodes which connect the artificial boundary but outside computational zone is unknown. As shown in Fig. 1 (a), assumed the node J on the artificial boundary, the node J-1, the node J-2, …are denoted to locate the same normal line of the artificial boundary as the node J and arranged near the node J inside the soil computational zone in turn. The displacements of the scattering wave at the node J at moment $(p+1)\triangle t$ on the artificial boundary can be determined by the displacements of the scattering wave at the node J, and the node J-1, the node J-2, …at time $p\triangle t$ and ($p-1)\triangle t$,…, the recurrence formulation is [Liao et al 1984] :

$$\{U_{JR}\}^{p+1} = \sum_{n=1}^{N} (-1)^{n+1} C_n^N \{U_n\} \quad (2.2)$$

where $\{U_{JR}\}^{p+1}$ denotes the displacement vector of scatting wave at the node J on the artificial boundary at the moment $(p+1)\triangle t$, N is the transmitting orders, C_n^N is the binomial coefficient, and the displacement vector$\left\{U_n\right\}$ is denoted as:

$$\left\{U_n\right\} = \left\{\{U_J\}^{p-n+1} \quad \{U_{J-1}\}^{p-n+1} \quad \{U_{J-2}\}^{p-n+1} \quad \cdots \quad \{U_{J-n}\}^{p-n+1}\right\}^T \quad (2.3)$$

where $\{U_J\}^{p-n+1}$ denotes the displacement vector of scatting wave at the node J on the artificial boundary at the moment $(p\text{-}n+1)\triangle t$, and $\{U_{J-i}\}^{p-n+1}$ denotes the displacement of scatting wave at the node J-i near the artificial boundary inside the soil zone at the moment $(p\text{-}n+1)\triangle t$. Therefore, the displacements field of the whole wave field at the nodes $\{U_J\}^{p+1}$on the artificial boundary can be determined as follow:

$$\{U_J\}^{p+1} = \{U_{JR}\}^{p+1} + \{U_{JI}\}^{p+1}$$

where the displacement vector$\{U_{JI}\}^{p+1}$ denotes the free field value when the node J is on the side artificial boundary, or the input wave value when the node J is on the bottom artificial boundary. The motion of the nodes connected with the foundation is decided by the foundation motion which will be presented in section ***2.3***.

2.2. Equation of motion of structure

In this section, the actual upper structure model is simplified as a three-dimension lumped-mass beam element model as shown in Fig. 1 (a). All the nodes of beam elements are classified as the internal nodes and the nodes connected with the foundation. With the Rayleigh damping, the equation of motion to the internal nodes of structure can be expressed as:

$$\{U_i\}^{p+1} = 2\{U_i\}^p - \{U_i\}^{p-1}$$
$$\left\{-\Delta t^2 [M_i]\sum_i\left[(1+\frac{\beta}{\Delta t})[K_{il}]+\frac{\alpha}{\Delta t}[M_{il}]\right]\{U_l\}^p - \sum_l(\frac{\beta}{\Delta t}[K_{il}]+\frac{\alpha}{\Delta t}[M_{il}])\{U_l\}^{p-1} - \{F_i\}^p\right\} \quad (2.4)$$

where $[M]$, $[K]$, $\{F\}$, $\{U\}$, $\{\dot{U}\}$ and $\{\ddot{U}\}$ denote similarly to the equation (2.1), and α, β denote the Rayleigh damping coefficients. The motion of the nodes connected with the foundation is decided by the foundation motion which will be presented in next section.

2.3. Equation of motion foundation

The connection part between the actual upper structure and soil medium structure is the foundation by which the interaction forces from the unbounded soil medium are transferred. Since the foundation is often stiff and thick, it is often assumed to be a rigid plate in computational model. That is to say that the motion of the foundation only includes the six components which are three concentrated force components ($\mathbf{F_X}$,$\mathbf{F_Y}$,$\mathbf{F_Z}$) and three concentrated moment components ($\mathbf{M_x}$, $\mathbf{M_y}$, $\mathbf{M_z}$) applied to the centre of the rigid plate. The total forces applied to the rigid plate which given by the actual upper structure and soil medium make the rigid motion of the foundation. The key problem analyzing the dynamic soil-structure interaction is to decide the total forces applied on the foundation. The model used to describe the interaction forces among soil medium, rigid plate and upper structure is shown in Fig. 1 (b).
The node *k* is assumed the one of the connecting points between the soil and the foundation. Since the deformation of soil domain, the force at node K applied by the soil domain is

$$\{F_k\} = \{F_{kx} \quad F_{ky} \quad F_{kz} \quad M_{kx} \quad M_{ky} \quad M_{kz}\}^T \quad (2.5)$$

where F_{kx}, F_{ky}, F_{kz} respectively denote the concentrated force components at node *k* along the *x*-, *y*-, and *z*- coordinate directions, and M_{kx}, M_{ky}, M_{kz} respectively denote the concentrated moment components at node *k* along the *x*-, *y*-, and *z*- coordinate directions, and then

$$\{F_k\} = -\sum_i(1+\frac{\mu}{\Delta t})[K_{kl}]\{U_l\}^p + \sum_l\frac{\mu}{\Delta t}[K_{kl}]\{U_l\}^{p-1} \quad (2.6)$$

The total force at the foundation applied by the soil domain is computed as:

$$\{F_D\} = \sum_m [A]_k^T\{F_k\} \quad (2.7)$$

where $\{F_D\} = \{F_{DX} \quad F_{DY} \quad F_{DZ} \quad M_{DX} \quad M_{DY} \quad M_{DZ}\}^T$ is the total force vector at the foundation applied by the soil domain, *m* is the total number of the nodes connecting with the foundation, and

$$[A]_k = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta Z_k & -\Delta Y_k \\ 0 & 1 & 0 & -\Delta Z_k & 0 & \Delta X_k \\ 0 & 0 & 1 & \Delta Y_k & -\Delta X_k & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\Delta X_k, \Delta Y_k, \Delta Z_k$ is respectively the relatively coordinate values of node k to the shape center of rigid plate.
The node i is assumed the one of the points of the upper structure, but it is not connected with the foundation. Since the motion of structure, the inertia force at node i will bring about applied forces to the foundation. The inertia force of node i is given:

$$\{F_i\} = \{F_{ix} \quad F_{iy} \quad F_{iz} \quad M_{ix} \quad M_{iy} \quad M_{iz}\}^T \tag{2.8}$$

where F_{ix}, F_{iy}, F_{iz} respectively denote the concentrated force components at node k along the x-, y- and z- coordinate directions, and M_{ix}, M_{iy}, M_{iz} respectively denote the concentrated moment components at node i along the x-, y- and z- coordinate directions, and then from equation (2.4), the inertia force of node i is given:

$$\{F_i\} = -\sum_l [(1+\frac{\beta}{\Delta t})[K_{il}] + \frac{\beta}{\Delta t}[M_{il}]]\{U_l\}^p - \sum_l (\frac{\beta}{\Delta t}[K_{il}] + \frac{\alpha}{\Delta t}[M_{il}])\{U_l\}^{p-1} \tag{2.9}$$

where all the variables have been denoted in equation (2.4). Similarly, the total forces on the foundation applied by the structure can be expressed as:

$$\{F_S\} = \sum_n [A_i]^T \{F_i\} \tag{2.10}$$

where $\{F_S\} = \{F_{SX} \quad F_{SY} \quad F_{SZ} \quad M_{SX} \quad M_{SY} \quad M_{SZ}\}^T$ is the total force vector at the foundation applied by the structure, n is the total number of upper structure nodes except the nodes connected with the foundation, and the definition of matrix $[A_i]$ is the same as the equation (2.7).

From the equation (2.7) and equation (2.10), the total forces applied by both soil domain and upper structure are:

$$\{F\} = \{F_D\} + \{F_S\} = \sum_m [A_k]^T \{F_k\} + \sum_n [A_i]^T \{F_i\} \tag{2.11}$$

where the total force vector $\{F\} = \{F_x, \quad F_y, \quad F_z, \quad M_x, \quad M_y, \quad M_z\}^T$, and the meanings of other components have been denoted in the last two sections.
The equation of motion of rigid foundation is written:

$$[M]\{\ddot{U}\} = \{F\} \tag{2.12}$$

where $[M]$ is the diagonal lumped-mass matrix of rigid plate, the nodes connecting with the foundation of soil medium and actual upper structure, and $\{\ddot{U}\}$ denotes the acceleration vector of rigid plate system, and $\{F\}$ has been denoted in last equation.
Using the explicit lumped-mass finite element method in time domain, the motion equation (2.12) of rigid plate can be expressed as:

$$\{U_F\}^{p+1} = 2\{U_F\}^p - \{U_F\}^{p-1} - \Delta t^2 [M]^{-1}\{F\} \tag{2.13}$$

where $\{U_F\}^p$ denotes the displacement vector of rigid plate at the moment $p\triangle t$ and the other variables have been denoted in this and last two sections. The motion of the nodes connected with foundation either in the structure or in the soil domain is decided by the rigid motion of plate. The expression of the motion of these nodes can be given as follow:

$$\{U_i\}^{p+1} = [A]\{U_F\}^{p+1} \tag{2.14}$$

where $\{U_i\}^{p+1}$, $[A]$, and $\{U_F\}^{p+1}$ have been denoted in this and last two sections.

3. NUMERICAL EXAMPLES

Case one: dynamic response of rectangular rigid foundation on a uniform simi-infinite medium
As shown in Fig.1 (a), a structure with a rigid thick plate foundation is founded on the surface of the semi-infinite soil layer. Moreover, the actual upper structure simplified to a 9 three-dimension beam element system is ignored to test the Program T3DSSI. The dimension and the density of rigid plate are L×W×H=78m×77m×6.5m, and 2.5T/m^3, respectively. The simi-infinite soil medium is simplified uniform medium, and the transmitting artificial boundary is used to truncate the soil medium. The incident S wave is assumed as a unit pulse with the width of 0.2s from the bottom of the transmitting boundary. The three-dimension mesh is used to model the truncated soil medium. The direct method for this kind module has been used in many papers [Liao et al 1984; Yang et al 2007]. The horizontal accelerate history of rigid foundation using the direct method and substructure method in this paper are shown in Fig.2. Form Fig.2, the results are agreement very well when shear wave velocity *V*s in the rigid plate and uniform medium are chosen 2500 m/s and 200 m/s, respectively.

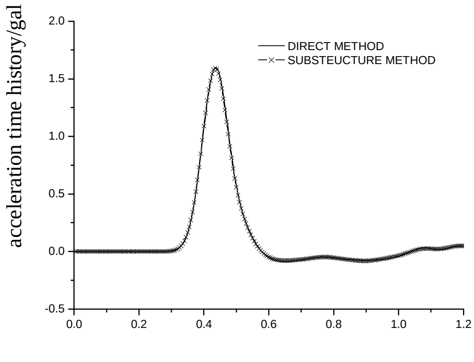


Figure 2 The acceleration time history of the rigid foundation

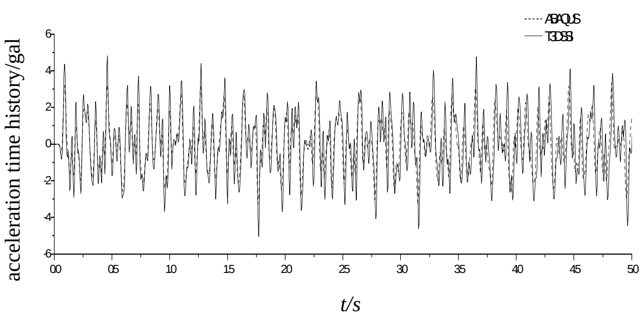


Figure 3 The horizontal acceleration time history of the highest node of the structure

Case two: The dynamic effect of the soil-structure interaction under the big stiffness of soil medium
Also as shown in Fig.1 (a), the actual upper structure simplified to a 9 three-dimension beam element system is

added to the last example. The parameters such as the beam size, area, mass, rotary inertia and moment of inertia are given in Table 1. In addition, the parameters of elastic modulus, shearing modulus of beams is given, respectively as:

$$E=3.0\times10^{10} \text{ and } G=1.2\times10^{10}.$$

The dimension of rigid plate, the transmitting artificial boundary and the incident S wave are assumed the same as last case. The three-dimension mesh is used to model the truncated soil medium. As we known, when the stiffness of soil zone below the upper structure is big enough, the structure applied force by the soil-structure interaction will have a little influence to the structure nature vibration frequencies. This character is often used to prove the effective of the adopted methods. The big stiffness soil medium parameters include shear wave velocity Vs=5000m/s, cut frequency f=25Hz, the densities of soil=2.5T/m^3, Poisson ration ν=0.25, and the damping ratio of beam ξ=0.00.

Table 1 the characters of beam

Number of beam	Length (m)	Area of across section (m^2)	Mass ($\times10^3$kg)	Moment of inertia ($\times10^3$kg·m^2)			Rotary inertia (m^4)		
				x-	*y-*	*z-*	*x-*	*y-*	*z-*
1	6.0	274.7	17040	12.11	12.11	10	167900	167900	10000
2	6.2	274.7	19930	14.17	14.17	10	167900	167900	10000
3	5.8	276.1	19740	13.8	13.8	10	164900	164900	10000
4	6.3	276.8	12040	8.24	8.24	10	154800	154800	10000
5	7.5	175.0	11670	4.05	4.05	10	162500	162500	10000
6	8.0	148.2	14700	4.46	4.46	10	139100	139100	10000
7	10.7	80.6	9300	2.83	2.83	10	76600	76600	10000
8	8.0	70.3	3850	1.94	1.94	10	39200	39200	10000
9	11.5	64.4	35.2	0.79	0.79	10	29600	29600	10000

The structure nature frequencies computing either by transfer function method for T3DSSI or mode-based analysis for ABAQUS can give similar results as shown in Table 2. These results proved the three dimension soil-structure interaction analysis procedure and program T3DSSI to be reasonable under the big stiffness soil medium.

Table 2 The horizontal nature frequencies of structure with damping ratio ξ= 0.0 for the structure

Nature vibration frequencies / Adopted method	1	2	3
ABAQUS（mode-based analysis）	5.031	10.966	18.813
T3DSSI (transfer function method)	5.079	11.038	18.852

The highest node horizontal acceleration time history of substructure module given by T3DSSI is shown in Fig3. To compared, the upper structure with a rigid foundation is modeled using direct method of the commercial software ABAQUAS. With the same unit pulse with the width of 0.05s incident S wave inputted from the rigid plate, the acceleration time histories of the highest node of the structure is also shown in Fig.3. From Fig.3, the dynamic acceleration time history by T3DSSI is in good agreement to the reference solutions using ABAQUS.

4. CLOSURE

A fully numerical procedure is presented to solve the problems of thee-dimension soil-structure interaction analysis. The procedure starts with time-domain substructure idea. The explicit finite element and corresponding transmitting model are taken to model the infinite soil medium. These procedures can solve the high cost in three dimension soil structure interaction computation using direct method. After studying problems on foundation-soft soil medium and big stiff of soil medium-structure systems, the proposed procedures and programs are found to possess reasonable results and high computational cost.

5. ACKNOWLEDGMENT

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