# EFFECT OF PILE SPACING ON PILE BEHAVIOR UNDER DYNAMIC LOADS

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**ABSTRACT :**
This study presents an investigation of the interaction effect of pile spacing effect under transient dynamic loadings. Analyses are carried for non homogeneous soils. The hybrid boundary element formulation is used to represent boundary integral representation of soil domain and pile equations represented by linear structural components. To observe the pile spacing effect on responses of group, a two pile groups is analyzed under triangular axial and lateral pulse loads for homogeneous and Gibson soils, respectively. The analysis is done for pile space diameter ratio of 3,5,7 and 10. Results for axial and lateral modes in non-dimensional forms are presented for homogeneous soils and for Gibson soils.

**KEYWORDS:** Pile-soil interaction, non-homogenous soil, pile spacing, pile dynamics.

## 1. INTRODUCTION

The dynamic analysis of pile foundations has been mostly limited to the frequency domain due to the assumption of linearity in the various formulations that makes problem easier to solve. Because of the analytical complexity involved, time domain analysis had studied relatively less. In this study, a transient analysis of pile behavior is done. The continuum is assumed to be elastic and an efficient step by step time integration scheme is implemented by using an approximate half space integral formulation.

For the elastostatic case of Gibson soil, Poulus (1979) proposed an approximate solution whereby Mindlin's equations (1936) are used in conjunction with the appropriate soil modulus at various points along the pile. Banerjee and Davies (1978) utilized a two layer half space solution to approximately model the soil inhomogeneity. Using a boundary element algorithm they were able to analyze the static behavior of both axially and laterally loaded single piles as well as pile groups.

Banerjee and Mamoon (1990) attempted to synthesize a complete transient equivalent of Mindlin's solutions, but the integrals are in implicit form that preclude analytical integration. For Gibson soil, a similar method of Mamoon and Banerjee (1992) is implemented in the current work to analyze pile and pile groups in time domain.

In this paper, a two pile groups is analyzed under triangular axial and lateral pulse loads for homogeneous and Gibson soils to observe the pile spacing effect on responses of group,. The analysis is done for pile space diameter ratio of 3,5,7 and 10. Results for axial and lateral modes in non-dimensional forms are presented for homogeneous soils and for Gibson soils.

## 2. MODELLING OF PILE-SOIL SYSTEM

The hybrid boundary element formulation is used to represent boundary integral representation of soil domain and pile equations represented by linear structural components.

### *2.1. Soil Continuum Equations*

The elastodynamic, small displacement field in an isotropic homogeneous elastic body is governed by Navier's equation:

$$(\lambda+\mu)\frac{\partial^2 u_p}{\partial x_p \partial x_q}+\mu\frac{\partial^2 u_q}{\partial x_p \partial x_p}-\rho\ddot{u}_q=0 \qquad (2.1)$$

where $\lambda$ and $\mu$ are the Lame's constants

$\rho$ is the mass density of the deformed body

$\ddot{u}_q=\frac{\partial^2 u_q}{\partial t^2}$ are the accelerations and the subscripts $p$ and $q$ ranges from 1 to 3.

Since the solid has semi-infinite extent, the equation is cast in terms of the Green function for the half space, that reduces the domain of integration to the pile-soil interface only. Then, the integral equation can be written as:

$$u_p(\xi,t)=\int_S\int_0^t G_{pq}(x,t;\xi,\tau)t_q(x,\tau)d\tau ds \qquad (2.2)$$

where $u_p$ is the displacements of the soil; $G_{pq}$ is the Green's function for the half space, $t_q$ is the tractions at the pile soil interface, $\xi$ and $x$ are the spatial positions of the receiver and the source point, respectively.

### *2.2. Pile Equations*

Since the pile cross section is much more thinner than the length of it, the piles are modeled as a one dimensional bar.

In the lateral direction, the pile is assumed to act as a thin strip whose behavior is governed by the beam equation. By adding the inertia terms, the governing equations for time harmonic beam subjected to axial and lateral excitations are given by:

$$m\ddot{u}_z^j-E_pA_p\frac{d^2u_z^j}{dz^2}=-\pi Dt_z^j \qquad (2.3)$$

$$m\ddot{u}_x^j+E_pI_p\frac{d^4u_x^j}{dz^4}=-Dt_x^j \qquad (2.4)$$

where, $E_p$ is theYoung's modulus of the pile material; $I_p$ is the second moment of inertia of pile; $A_p$ is the cross-sectional area of the pile; $D$ is the diameter of the pile $u_x^j$ and $u_z^j$ are the lateral and axial displacements at time $t^j$, respectively; $t_x^j$ and $t_z^j$ are the lateral and axial tractions along the pile at time $t^j$, respectively.

The piles are modeled with linear beam column elements. The dynamic equilibrium equations of motion at time $t^j$ are written as follow:

$$\begin{bmatrix} M_{uu} & M_{u\theta} \\ M_{\theta u} & M_{\theta\theta}\end{bmatrix}\begin{Bmatrix}\ddot{u}^j \\ \ddot{\theta}^j\end{Bmatrix}+\begin{bmatrix} C_{uu} & C_{u\theta} \\ C_{\theta u} & C_{\theta\theta}\end{bmatrix}\begin{Bmatrix}\dot{u}^j \\ \dot{\theta}^j\end{Bmatrix}+\begin{bmatrix} K_{uu} & K_{u\theta} \\ K_{\theta u} & K_{\theta\theta}\end{bmatrix}\begin{Bmatrix}u^j \\ \theta^j\end{Bmatrix}=\begin{Bmatrix}f_u{}^j \\ f_\theta{}^j\end{Bmatrix} \qquad (2.5)$$

Though damping matrix is for the structural material included in the equation, in the analysis it is discarded. In equilibrium equation, degrees of freedom corresponding to rotations at the pile nodes need to be eliminated since boundary element formulations contain translation degrees of freedom only. The rotational degree of freedom of the pile top is left in the system of equation, since it will form a part of unknowns to be solved. By using Guyan reduction technique, equations yield to following equation:

$$\overline{M}\ddot{u}^j+\overline{K}u^j=f^j \qquad (2.6a)$$

where,

$$\overline{K} = K_{uu} - K_{u\theta} K_{\theta\theta}^{-1} K_{\theta u} \tag{2.6b}$$

and

$$\overline{M} = M_{uu} - M_{u\theta} K_{\theta\theta}^{-1} K_{\theta u} - (K_{\theta\theta}^{-1} K_{\theta u})^{T} (M_{\theta u} - M_{\theta\theta} K_{\theta\theta}^{-1} K_{\theta u}) \tag{2.6c}$$

It is also can be written in incremental form by:

$$\overline{M}(\ddot{u}^{j} - \ddot{u}^{j-1}) + \overline{K}\Delta u^{j} = \Delta f^{j} \tag{2.7}$$

where $\Delta u^{j}$ and $\Delta f^{j}$ are the current displacement and force increments. The above equation is solved by step by step, using Newmark integration method.

### *2.3. Assembly of Pile and Soil Equations*

It is necessary to introduce a global equation constraint to be able to solve the coupled system of equations. By imposing a constraint on the traction vector $\{t_p\}$. This constraint arises from a consideration of the global equilibrium of the entire system. In an incremental form, this can be represented as:

$$[B_1]\{\Delta t_p^{j}\} - [B_2]\{\ddot{u}_p^{j} - \ddot{u}_p^{j-1}\} - [B_3]\{\ddot{u}_c^{j}\} = \{\Delta F^{j}\} \tag{2.8}$$

Using compatibility relations, final form is presented as:

$$\begin{bmatrix} G+D & b_p \\ B & -E \end{bmatrix} \begin{Bmatrix} \Delta t_p{}^{j} \\ \Delta u_c{}^{j} \end{Bmatrix} = \begin{Bmatrix} R_s^{j} - B_p^{j} \\ \Delta F_c^{j} - H^{j} \end{Bmatrix} \tag{2.9}$$

### *2.4. Formulation for a Pile Group*

For a two pile group, the equilibrium equation is given by using same procedure applied in single pile formulation, one can write the following:

$$\begin{bmatrix} G_{11} + D_{11} + K_{11_{ozd}}^{-1} & G_{12} & b_{p_1} \\ G_{21} & G_{22} + D_{22} + K_{22_{ozd}}^{-1} & b_{p_2} \\ B_1 & B_2 & -E \end{bmatrix} \begin{Bmatrix} \Delta t_{p1}^{j} \\ \Delta t_{p2}^{j} \\ \Delta u_c^{j} \end{Bmatrix} = \begin{Bmatrix} R_{s1}^{j} - B_{p1}^{j} \\ R_{s2}^{j} - B_{p2}^{j} \\ \Delta F_c^{j} - H^{j} \end{Bmatrix} \tag{2.10}$$

In equation (2.10), the indices '1' and '2' resembles to piles in the group.

### *2.5. Extension to multiple groups*

Extension to multiple groups is useful for coupling a superstructure with multiple supports to entire foundation system. After some algebraic substitutions, the final form of assembled equations for two independent groups of piles is presented as follows:

$$\begin{bmatrix} [G + D + K^{-1}]_{11_{ozd}} & G_{12} & b_{p_1} & 0 \\ G_{21} & [G + D + K^{-1}]_{22_{ozd}} & 0 & b_{p_2} \\ B_1 & 0 & -E_1 & 0 \\ 0 & B_2 & 0 & -E_2 \end{bmatrix} \begin{Bmatrix} \Delta t_{p1}^{j} \\ \Delta t_{p2}^{j} \\ \Delta u_{c1}^{j} \\ \Delta u_{c1}^{j} \end{Bmatrix} = \begin{Bmatrix} R_{s1}^{j} - B_{p1}^{j} \\ R_{s2}^{j} - B_{p2}^{j} \\ \Delta F_{c1}^{j} - H_1^{j} \\ \Delta F_{c2}^{j} - H_2^{j} \end{Bmatrix} \tag{2.11}$$

In this equation, the indices '1' and '2' refer to the two independent groups. It shows that between the two groups only a weak coupling exits through the terms $G_{12}$ and $G_{21}$. For groups that are very far to each other, these coupling terms are close to zero and the behavior of the groups are uncoupled.

## 3. NUMERICAL STUDY

It is necessary to prove the validity of presented method by comparing with the available solutions. For this purpose, both direct time domain and Laplace domain solution methods need to be compared. These comparisons were done for homogenous and for Gibson soils. To do that following parameters were used in the analysis:

Pile Length, L = 20. Meter, Pile Diameter , D = 1. Meter
Number of elements, N = 20
Pile to pile spacing for groups, S = 3D
Pile Modulus, $E_p$ = 25.5 GPa
Density of pile, $\rho_p$ = 2400 kg/m$^3$ , Density of soil, $\rho_s$ = 1800 kg/m$^3$, Poisson's ratio of soil, $\upsilon_s$ = 0.4
For homogenous soil, $E_s$ = 51 Mpa
For Gibson soil ($E_s = E_o + mz$), $E_o$ = 12.75 MPa and m = 1.9125 MPa/meter and z is the depth.
The reason to choose these parameters is to attempt to simulate a realistic problem. In other words, L/D = 20, and $E_p/E_s$ = 500 resembles a concrete pile embedded in medium stiff soil.
In all plots, the x-axis is used for non-dimensional time $\tau$ , and given by :

for homogeneous soil , $$\tau = \frac{t}{R}\sqrt{\frac{G_s}{\rho_s}}$$

for Gibson soil , $$\tau = \frac{t}{R}\sqrt{\frac{G_{s,o}}{\rho_s}}$$

where $t$ is total duration of impact loading, $R$ is radius of pile
$G_s$ is shear modulus of homogeneous soil, $G_{s,o}$ is shear modulus of Gibson soil at the top (z = 0)
$\rho_s$ is mass density of soil.

To observe the pile spacing effect on responses of group, a two pile groups is analyzed under triangular axial and lateral pulse loads respectively for homogeneous and Gibson soils. The analysis is done for pile space diameter ratio of 3,5,7 and 10. Results for axial and lateral modes in non-dimensional forms are presented in figures 1 and 2 for homogeneous soils and in figures 3 and 4 for Gibson soils.

## 4. CONCLUSIONS

The time domain analysis of a pile and a pile group was studied for Gibson soil. The hybrid boundary element formulation was used to represent boundary integral representation of soil domain and pile equations represented by linear structural components. A time stepping BEM algorithm together with an implicit time integration FEM scheme was used for modeling the piles.

It can be observed for the axial cases the peak response is almost the same as the single pile for S/D ratio is greater than 5. This means that there is very little effect of neighboring pile when S/D ratio became larger, this is in contrast to the axial static loading behavior where the effect of neighboring piles is still significant at this distance, for the lateral case, the behavior is slightly different, because most of the interactions take place due to the deformation near to ground surface. A higher stiffness more than single pile is observed because of the push pull action developing among the piles.

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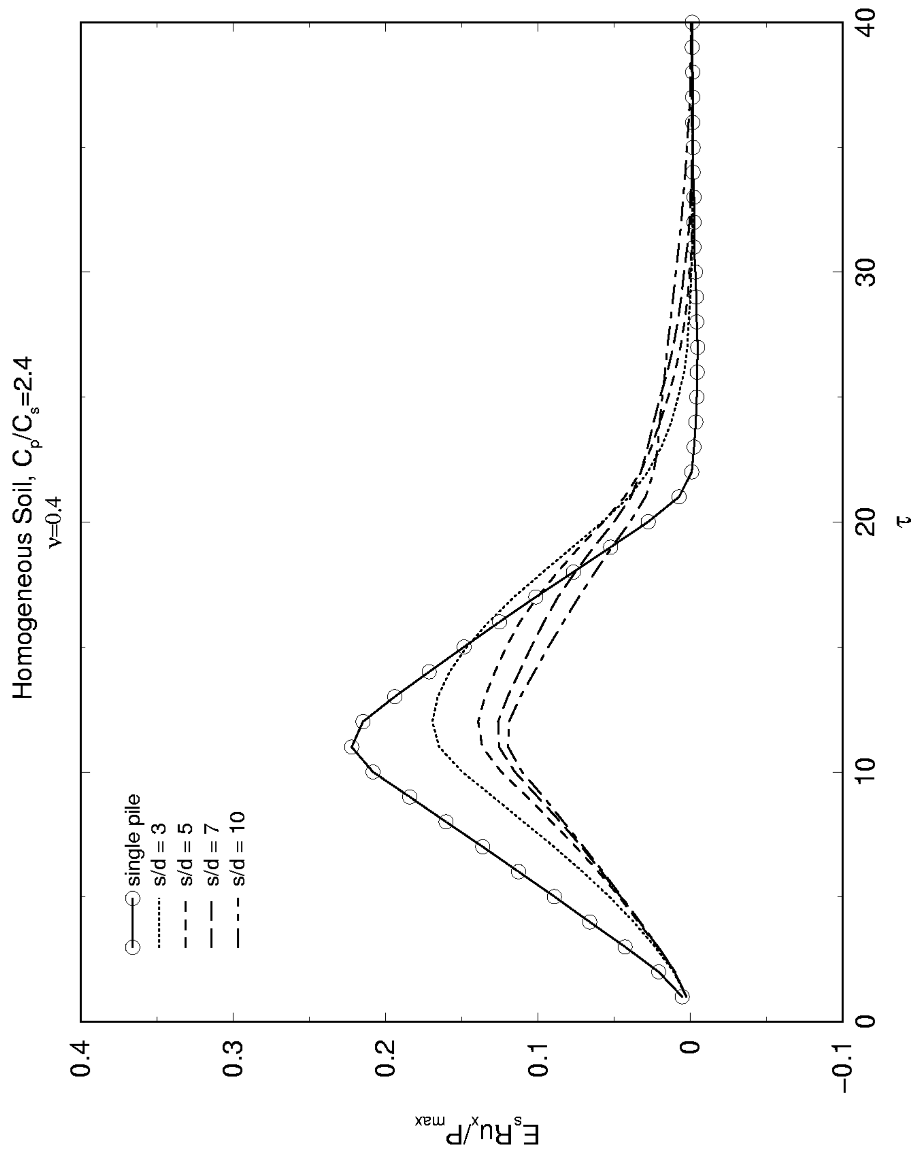


Figure 1. Effect of pile spacing for lateral mode

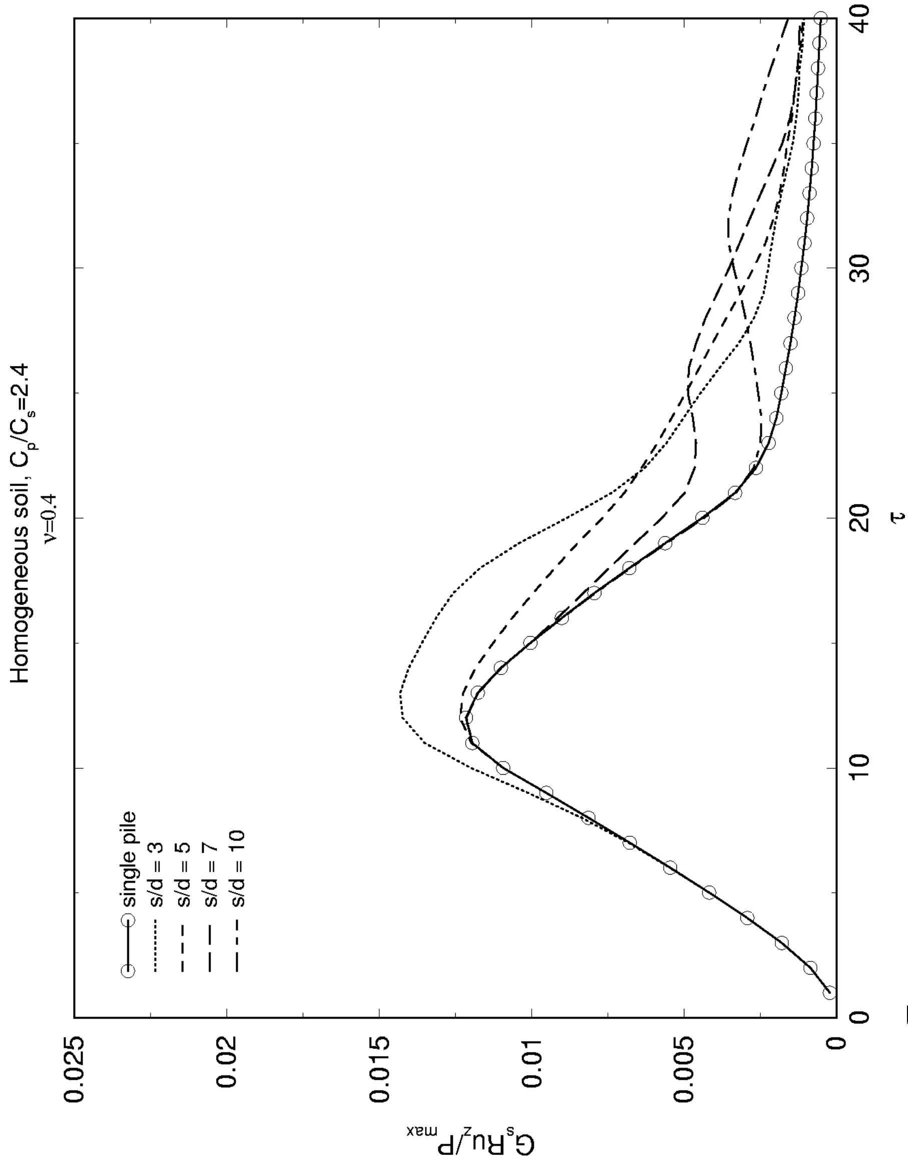


Figure 2.  Effect of pile spacing  for axial mode

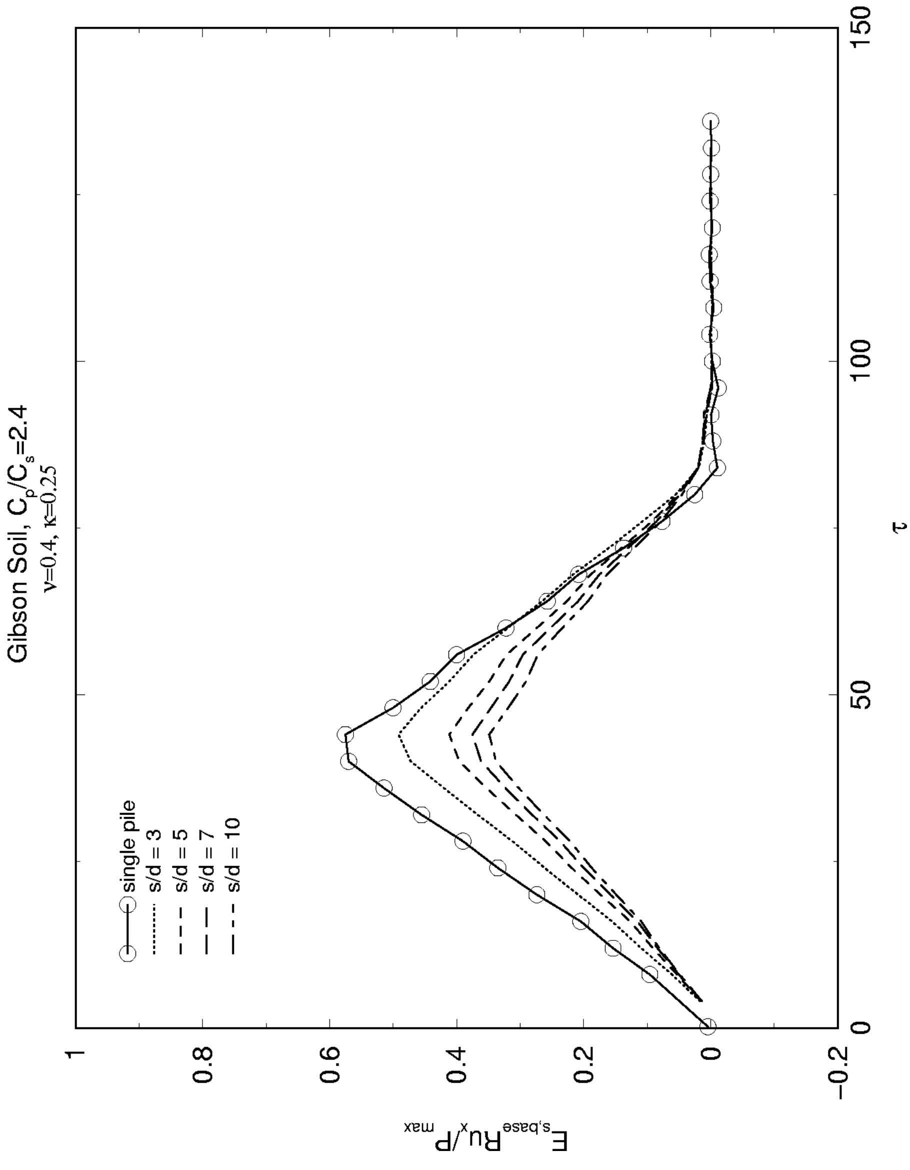


Figure 3. Effect of pile spacing for lateral mode

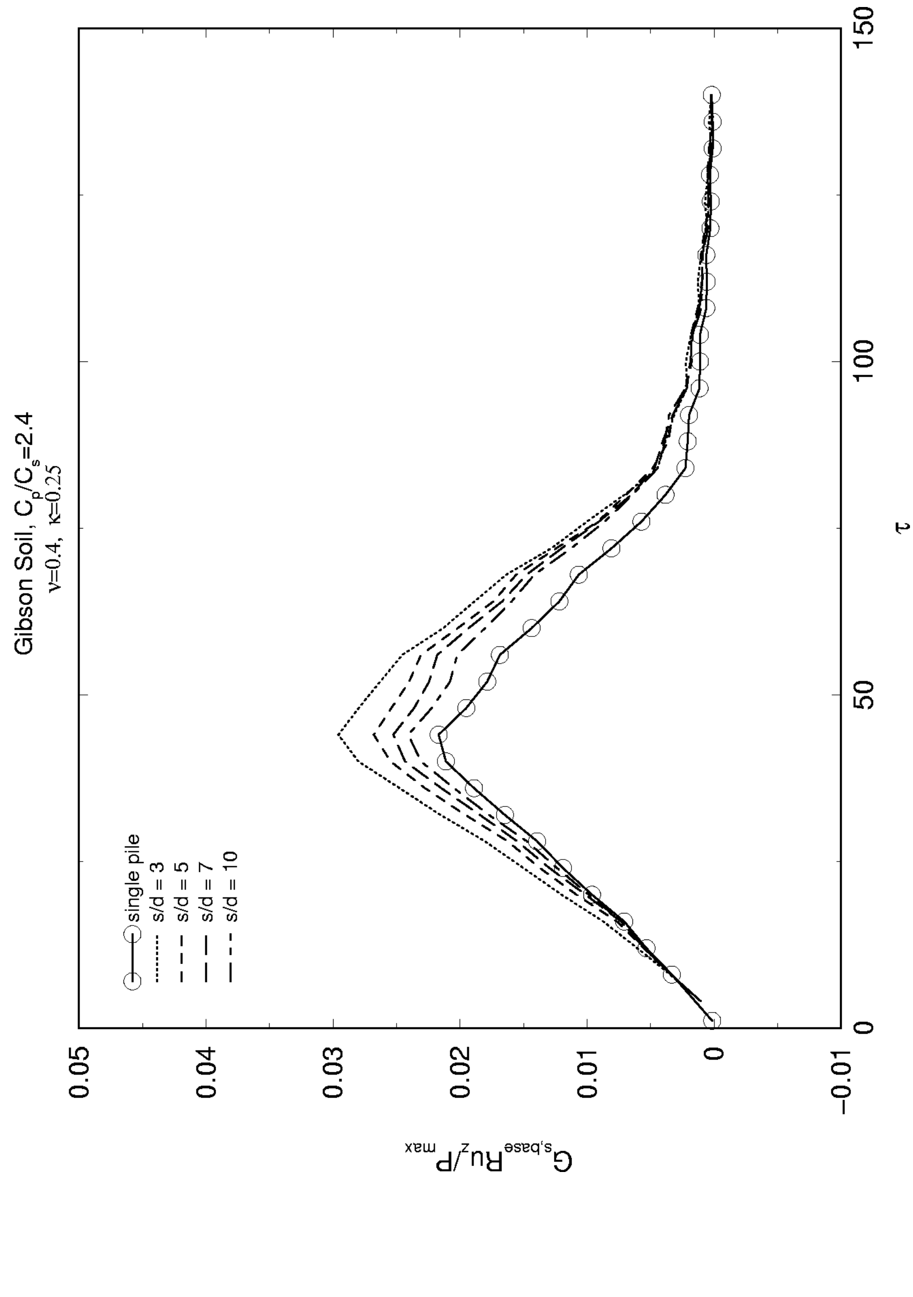


Figure 4. Effect of pile spacing for axial mode