VERTICAL VIBRATION OF PILE IN SATURATED SOIL CONSIDERING SOFTEN REGIONS OF SURROUNDING SOIL

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ABSTRACT :

There has been a considerable interest in dynamic interaction of pile-soil system due to its important applications in machinery foundations, seismic design of structures and pile dynamic testing. Vertical vibration of a pile embedded in saturated soil layer is investigated incorporating the nonlinear behavior of soil in the vicinity of the pile. The pile is dealt with one-dimensional elastic theory. The soil surrounding the pile is divided into two regions: an outer infinite linear saturated poroelastic medium and an inner soften zone in the shape of a hollow cylinder. A set of simplifications is incorporated to soften inner zone and it can be analyzed by cavity expansion theories. Using variable separation method, governing differential equations of the undisturbed region of saturated soil are decoupled by introducing the potential functions. Based on the boundary and continuity conditions of the inner and outer region, pile dynamic responses are presented to an arbitrary vertical load. Numerical results from frequency domain and time domain indicated that the soften region of soil has significant influence on the dynamic response of pile-soil system under vertical vibrating of pile in saturated soil. By comparison of the present solutions with the ones not considering soften region, the results show that there are obvious differences of dynamic responses at pile head. A conclusion can be deduced that the interaction of pile and soil is overestimated under the ideal undisturbed model. And the presented model makes it possible to predict accurately the dynamic responses of pile and soil system.

KEYWORDS: soil-pile interaction, vertical vibration, saturated soil, soften region, dynamic response

1. INTRODUCTION

Vibratory pile driving is a common method for pile sinking. The surrounding soil is disturbed by pile sinking, such disturbance may cause the soil softening, in turn, have an impact on pile vibration. Novak et al. (1978) made a thin-layer-element method, taking into account the disturbance region surrounding the pile, but ignores the soil quality of the disturbed region. Veletsos et al. (1986) corrected this issue. Furthermore, Han et al. (1995) assumed a continuous variation of shear modulus of the disturbed area, and studied the pile vibration in a radial inhomogeneous soil media. The soil is generally regarded as a single phase medium at the soil-pile dynamic interaction in past years, in fact, the soil is a complex three-phase medium, the soil water has more or less effect on the pile vibration. Vibratory pile driving lead to exceed pore water pressure around the pile, so a soften area of the soil around the pile is caused by the corresponding reduction of effective stress (Nogami et al., 1997). Recently, some studies notify the characteristic of the two-phase soil, Zeng et al. (1999) analyzed the dynamic load transfer of a vertical elastic bar through the boundary integral equation in the saturated soil. Li et al. (2004, 2007) studied vertical vibration of pile in saturated soil using the separation of variables under the condition of perfect or imperfect contact. In above solutions, the disturbed area caused by pile sinking is not considered, the modulus of the soil around the pile is presumed as homogeneous along the radial direction, and obviously this is not real. In this paper vibratory pile driving is simplified as expansion of cavity and soil-pile interaction when the surrounding soil is soften due to the expansion.

2. SIMPLIFIED MATHEMATIC MODEL

When a pile is installed in a saturated soil layer using a vibratory pile driving machine, a narrow boundary zone in immediate neighborhood of the pile is aroused, and shear modulus of the region usually decreases with the disturbance of soil expansion and pile vibration. Hereafter, the soil around the pile is divided into two regions: the inner, the soften field and the outer, elastic undisturbed region. A sketch of soil and pile interaction is shown in figure 1, where H and r_0 is the length and radius of pile respectively, $P(t)$ is an arbitrary exciting force applied at the pile head, k_s and k_b denote the subgrade and pile toe reaction coefficients, respectively. μ denotes the initial shear modulus, μ_1 is the shear modulus of disturbed region, c_u is the shear strength of UU test.

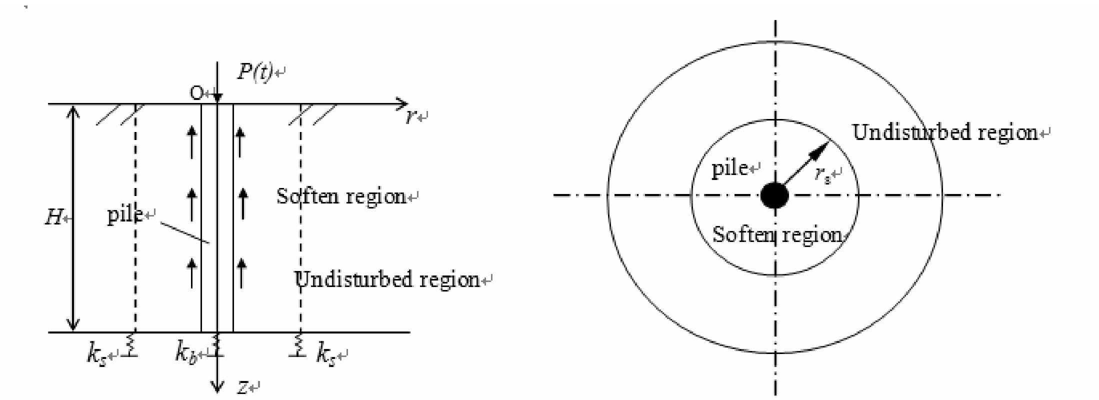

Figure 1 Sketch of soil-pile interaction

2.1. Soften Region

When the pile is strongly excited, nonlinear behavior occurs in the soil region close to the pile. A few rigorous solutions can be used to model the interaction, so, it is common to simply. Assumed that the inner zone is mainly affected by the soil compacting and the effect of pile vibration is minor, the radius of disturbed region can be expressed as $r_s = r_0\sqrt{\mu/C_u}$ by means of cavity expansion theory (Henkel et al., 1966). Assumed that the relationship of stress and strain of inner soil region is linear and neglecting the dynamic interaction, the equation of vertical motion of the inner medium can be written in plane strain as:

$$\frac{\partial(r\tau_{rz1})}{\partial r} = 0, \qquad \tau_{rz1} = \mu_1\frac{\partial u_{z1}}{\partial r} + c_1\frac{\partial \dot{u}_{z1}}{\partial r} \tag{2.1}$$

Where τ_{rz1}, u_{z1} are the shear and vertical displacement of soften region, respectively. c_1 is the coefficient of soil damping. Over-dot denotes the derivative with respect to the time parameter t.
The boundary conditions can be written as:

$$u_{z1}\big|_{r=r_0} = w_b, \quad u_{z1}\big|_{r=r_s} = u_{z2}\big|_{r=r_s}, \quad \tau_{rz1}\big|_{r=r_s} = \tau_{rz2}\big|_{r=r_s}, \quad \tau_{rz1}\big|_{r=r_0} = -f/2\pi r_0 \tag{2.2}$$

Where w_b is the displacement of pile, τ_{rz2}, u_{z2} are the shear and vertical displacement of undisturbed region, respectively. f is the unit skin friction of pile.
In many cases, it is reasonable to simply the soil as a linear visco-elastic material with hysteretic damping, therefore, the shear stress and displacement can be solved in frequency domain. It can be expressed as:

$$\overline{\tau}_{rz1} = \frac{\beta(1+iD)}{\overline{r}}\frac{\overline{u}_{z2}\big|_{\overline{r}=\overline{r}_s} - \overline{w}_b}{\ln\overline{r}_s} \tag{2.3}$$

Where $\overline{\tau}_{rz1} = \tau_{rz1}/\mu$, $\overline{u}_{z2} = u_{z2}/r_0$, $\overline{w}_b = w_b/r_0$, $\beta = \mu_1/\mu$, $\overline{r} = r/r_0$, $\overline{r}_s = r_s/r_0$, D is the coefficient of hysteretic damping.

So we can obtain boundary shear stress are as follows:

$$\bar{\tau}_{rz1}\big|_{\bar{r}=\bar{r}_0=1} = \beta(1+iD)\frac{\bar{u}_{z2}\big|_{\bar{r}=\bar{r}_s} - \bar{w}_b}{\ln \bar{r}_s} \tag{2.4}$$

$$\bar{\tau}_{rz2}\big|_{\bar{r}=\bar{r}_s=r_s/r_0} = \frac{\beta(1+iD)}{\bar{r}_s}\frac{\bar{u}_{z2}\big|_{\bar{r}=\bar{r}_s} - \bar{w}_b}{\ln \bar{r}_s} \tag{2.5}$$

2.2 Undisturbed Region

The undisturbed areas are dealt with two-phase saturated soil theory originated by Biot (1956). For axisymmetric problem, the governing equations of motions of saturated soil can be expressed as:

$$\mu\nabla^2 u + (\lambda_c + \mu)\nabla\nabla\cdot u + \alpha M\nabla\nabla\cdot w = \rho\ddot{u} + \rho_f\ddot{w} \tag{2.6}$$

$$\alpha M\nabla\nabla\cdot u + M\nabla\nabla\cdot w = \rho_f\ddot{u} + m\ddot{w} + b\dot{w} \tag{2.7}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{\partial^2}{\partial z^2}$, u and w denote the average displacement vectors of solid matrix and fluid with respect to solid matrix respectively. ρ and ρ_f denote mass densities of the bulk material and the pore fluid respectively; $\rho = (1-n)\rho_s + n\rho_f$, where n is porosity of soil and ρ_s denotes mass density of grains. $m = \rho_f / n$ denotes a density-like parameter that depends on ρ_f and the geometry of the pores; $b = \rho_f g / k_B$ denotes a parameter accounting for the internal friction due to the relative motion between the solid matrix and the pore fluid, that is, the so-called seepage force, where k_B denotes the coefficient of permeability of the medium and g denotes the gravity acceleration. $\lambda_c = \lambda + \alpha^2 M$, where λ and μ are Lame's constants. α and M denote the Biot's parameters accounting for compressibility in the two-phased material and can be determined by $\alpha = 1 - K_b/K_s$, $M = K_s^2/(K_d - K_b)$, $K_d = K_s[1 + n(K_s/K_f - 1)]$, where K_s, K_f and K_b are the bulk modulus of solid grains, fluid and soil skeleton, respectively.
The boundary conditions of the saturated soil layer are:

(1) Stresses and displacements approach zero at an infinite horizontal distance;

(2) Zero normal stresses on the free surface $\sigma_z(r,0) = 0$

(3) The soil layer bears on an elastic base, $E_s\frac{\partial u_z}{\partial z}(r,H) + k_s u_z(r,H) = 0$

(4) Impervious on pile shaft, $w_r(a,z) = 0$, Zero radial displacement on pile shaft, $u_r(a,z) = 0$

Using variable separation method, the governing differential equations of the undisturbed region of saturated soil are decoupled by introducing the potential functions (Li et al., 2004), by means of the boundary conditions, the shear stress and vertical motion of soil can be obtained as:

$$\bar{\tau}_{zr}\big|_{\bar{r}=\bar{r}_s} = \sum_{n=1}^{\infty}\eta'_{1n}C_{1n}\cosh(h_n z) \tag{2.8}$$

$$\bar{u}_z\big|_{\bar{r}=\bar{r}_s} = \sum_{n=1}^{\infty}\eta'_{2n}C_{1n}\cosh(h_n z) \tag{2.9}$$

Where, C_{1n} is an undefined coefficient, $\eta'_{1n} = 2(1+\frac{h_{1n}^2}{h_n^2})\frac{\lambda_1-\lambda_2}{\lambda_2-\lambda_5}g_{1n}h_nK_1(g_{1n}\overline{r}_s)$,

$$\eta'_{2n} = 2h_nK_0(g_{1n}\overline{r}_s) - \frac{2(\lambda_1-\lambda_5)g_{1n}h_nK_1(g_{1n}\overline{r}_s)K_0(g_{2n}\overline{r}_s)}{(\lambda_2-\lambda_5)g_{2n}K_1(g_{2n}\overline{r}_s)} - \frac{2(\lambda_1-\lambda_2)g_{1n}h_{1n}K_1(g_{1n}\overline{r}_s)K_0(h_{1n}\overline{r}_s)}{(\lambda_2-\lambda_5)h_nK_1(h_{1n}\overline{r}_s)},$$

The coefficient h_n satisfies the transcendental equation: $h_n\sinh(h_n\theta)+k_s^*\cosh(h_n\theta)=0$, where $\theta = H/r_0$ stands for the pile slenderness ratio, $k_s^* = k_s r_0/E_s$ denotes the dimensionless coefficient of subgrade reaction.
$g_{1n}^2+h_n^2=\beta_1^2$, $g_{2n}^2+h_n^2=\beta_2^2$, $h_{1n}^2+h_n^2=\gamma^2$. $I_0(gr)$, $K_0(gr)$ are zero-order and first-order modified Bessel functions of the second kind, respectively.

$\beta_{1,2}^2 = \frac{d_1 \pm \sqrt{d_1^2-4d_2}}{2}$, $\gamma^2 = \frac{-\rho^{*2}\delta^4+(m^*\delta^2+b^*\delta)\delta^2}{m^*\delta^2+b^*\delta}$, $d_1 = \frac{(\lambda_c^*+2)(m^*\delta^2+b^*\delta)+M^*\delta^2-2\alpha\rho^*M^*\delta^2}{(\lambda^*+2)M^*}$,
$d_2 = \frac{(m^*-\rho^{*2})\delta^4+b^*\delta^3}{(\lambda^*+2)M^*}$, $\lambda_i = \frac{-\alpha M^*\beta_i^2+\rho^*\delta^2}{M^*\beta_i^2-(m^*\delta^2+b^*\delta)}$, $i=1,2$, $\lambda_5 = -\frac{\rho^*\delta^2}{m^*\delta^2+b^*\delta}$.
$\lambda_c^* = \lambda^*+\alpha^2M^* = \lambda/\mu+\alpha^2 M/\mu$, $\rho^* = \rho_f/\rho$, $\delta = \sqrt{\rho/\mu}sr_0$, $m^* = m/\rho$, $b^* = br_0/\sqrt{\rho\mu}$.

2.3 Dynamic Response of Pile

The motion of pile can be expressed as:

$$E_b\pi r_0^2\frac{\partial^2 w_b}{\partial z^2} - f(z) = \rho_b\pi r_0^2\frac{\partial^2 w_b}{\partial t^2} \tag{2.10}$$

$$\left.(E_b\pi r_0^2\frac{\mathrm{d}w_b}{\mathrm{d}z}+k_bw_b)\right|_{z=H}=0,\quad \left.\frac{\mathrm{d}w_b}{\mathrm{d}z}\right|_{z=0} = \frac{P(t)}{E_b\pi r_0^2} \tag{2.11}$$

$$\left.w_b\right|_{t=0}=0,\quad \left.\frac{\partial w_b}{\partial t}\right|_{t=0}=0 \tag{2.12}$$

where E_b and ρ_b denote the elastic modulus and density of pile, respectively.

The perfect contact condition is satisfied on the contact surface of pile and soil.

$$u_z(r_0,z)=w_b(z) \tag{2.13}$$

Based on the continuous conditions of the contact surface at the pile and soil, also at the disturbed and the undisturbed regions, the dynamic response of the coupled system can be solved according to the orthogonal property of hyperbolic cosine function. Assumed that $s=i\omega$, the complex stiffness of pile-soil system at pile head can be expressed as:

$$k_d = \frac{E_b^*\kappa(A_1-B_1)}{A_1[1+\sum_{n=1}^{\infty}\frac{-2\beta(1+iD)\eta'_{2n}E_n}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}]+B_1[1+\sum_{n=1}^{\infty}\frac{-2\beta(1+iD)\eta'_{2n}F_n}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}]} \tag{2.14}$$

Where, $$A_1=\frac{\frac{\overline{P}^*}{E_b^*\kappa}[(k_b^*-\kappa)e^{-\kappa\theta}+\sum_{n=1}^{\infty}\frac{-2\beta(1+iD)\eta'_{2n}F_nG_n}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}]}{(k_b^*+\kappa)e^{\kappa\theta}+(k_b^*-\kappa)e^{-\kappa\theta}+\sum_{n=1}^{\infty}\frac{-2\beta(1+iD)\eta'_{2n}(E_n+F_n)G_n}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}},$$

$$B_1=-\frac{\frac{\overline{P}^*}{E_b^*\kappa}[(k_b^*+\kappa)e^{\kappa\theta}+\sum_{n=1}^{\infty}\frac{-2\beta(1+iD)\eta'_{2n}E_nG_n}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}]}{(k_b^*+\kappa)e^{\kappa\theta}+(k_b^*-\kappa)e^{-\kappa\theta}+\sum_{n=1}^{\infty}\frac{-2\beta(1+iD)\eta'_{2n}(E_n+F_n)G_n}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}},$$

$$E_n=\frac{\frac{e^{(\kappa+h_n)\theta}-1}{2(\kappa+h_n)}+\frac{e^{(\kappa-h_n)\theta}-1}{2(\kappa-h_n)}}{[\frac{-\overline{r}_s\ln(\overline{r}_s)\eta'_{1n}}{\beta(1+iD)}+\frac{2\beta(1+iD)\eta'_{2n}}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}+\eta'_{2n}][\frac{\theta}{2}+\frac{\sinh(2h_n\theta)}{4h_n}]},$$

$$F_n=-\frac{\frac{e^{-(\kappa-h_n)\theta}-1}{2(\kappa-h_n)}+\frac{e^{-(\kappa+h_n)\theta}-1}{2(\kappa+h_n)}}{[\frac{-\overline{r}_s\ln(\overline{r}_s)\eta'_{1n}}{\beta(1+iD)}+\frac{2\beta(1+iD)\eta'_{2n}}{E_b^*(h_n^2-\kappa^2)\ln(\overline{r}_s)}+\eta'_{2n}][\frac{\theta}{2}+\frac{\sinh(2h_n\theta)}{4h_n}]}.$$

$G_n=h_n\sinh(h_n\theta)+k_b^*\cosh(h_n\theta)$, $k_b^*=k_b/(E_b\pi r_0)$, $\overline{P}^*=\overline{P}/(\mu\pi r_0^2)$, $E_b^*=E_b/\mu$, $\rho_b^*=\rho_b/\rho$, $i=\sqrt{-1}$, $E_\mathrm{b}^*=E_\mathrm{b}/\mu$, $\rho_\mathrm{b}^*=\rho_\mathrm{b}/\rho$, $\kappa^2=\frac{\rho_b^*\delta^2}{E_b^*}+\frac{2\beta(1+iD)}{E_b^*\ln(\overline{r}_s)}$.

The admittance of pile at the pile head is,

$$|H_v(i\omega)|=|\mathrm{i}\omega/k_d| \tag{2.15}$$

When a transient force such as a half-sine pulse imposed on the pile head, the response of velocities at the pile head can be developed by inverse Fourier transform and convolution.

$$V(t)=IFT\left[H_v(i\omega)\cdot q_{\max}\omega\frac{1+e^{-\pi s/\omega}}{\omega^2+s^2}\right] \tag{2.16}$$

Where $q_{\max}$ is the amplitude of excited forces, s is the Laplace transform of time parameter t.

3. NUMERICAL ANALYSIS

Table 3.1 Parameters of pile and soil system

Parameters		values	parameters		values
Saturated soil	K_s (GPa)	36	Bearing coefficient	k_b^*	2.0
	K_f (GPa)	2.0		k_s^*	1.0
	ρ_s (kg/m^3)	2700	Disturbed region	D	0.02
	ρ_f (kg/m^3)	1000		$\overline{r}_s$	$\sqrt{2}$
	n	0.45		β	0.5
	k_B (m^2)	10^{-6}-10^{-1}	Pile	ρ_p (kg/m^3)	2500
	η (N.s/m^2)	10^{-3}-10^{-1}		C (m/s)	3800

In this section, numerical results are calculated based on the above solutions where parameters are given in table 3.1. Authors may choose how they wish to format the rows and columns. Please be consistent throughout your manuscript. Leave one blank line before the table heading and one blank line after the table.

3.1. Comparison of Perfect Contact Model and Soften Model

The comparisons of pile complex stiffness and admittance amplitude in frequency domain and velocity of reflect wave in time domain between the two pile vibration models are shown in figure 2, where X-axis is the dimensionless frequency $\overline{f} = 2\pi r_0 f / \omega_g$, where ω_g is the natural frequency of pile to satisfy $\omega_g / c = k_b^* \cot(\omega_g \theta / c)$, c is the compressive wave velocity of pile. For the sake of convenience, the dynamic stiffness has been normalized by the static stiffness at $\omega = 0$ as shown in figure 2. $\overline{t} = tc/H$ denotes the dimensionless time.
From figure 2, we can see that the amplitude of complex stiffness, the admittance and velocity of the reflect wave in the present solutions are larger than those in the perfect contact model. Such a situation shows that the perfect bonding condition overestimates the constraint of soil on the pile. When pile is strongly excited, the perfect bonding condition is unsafe for analysis of dynamic response of pile.

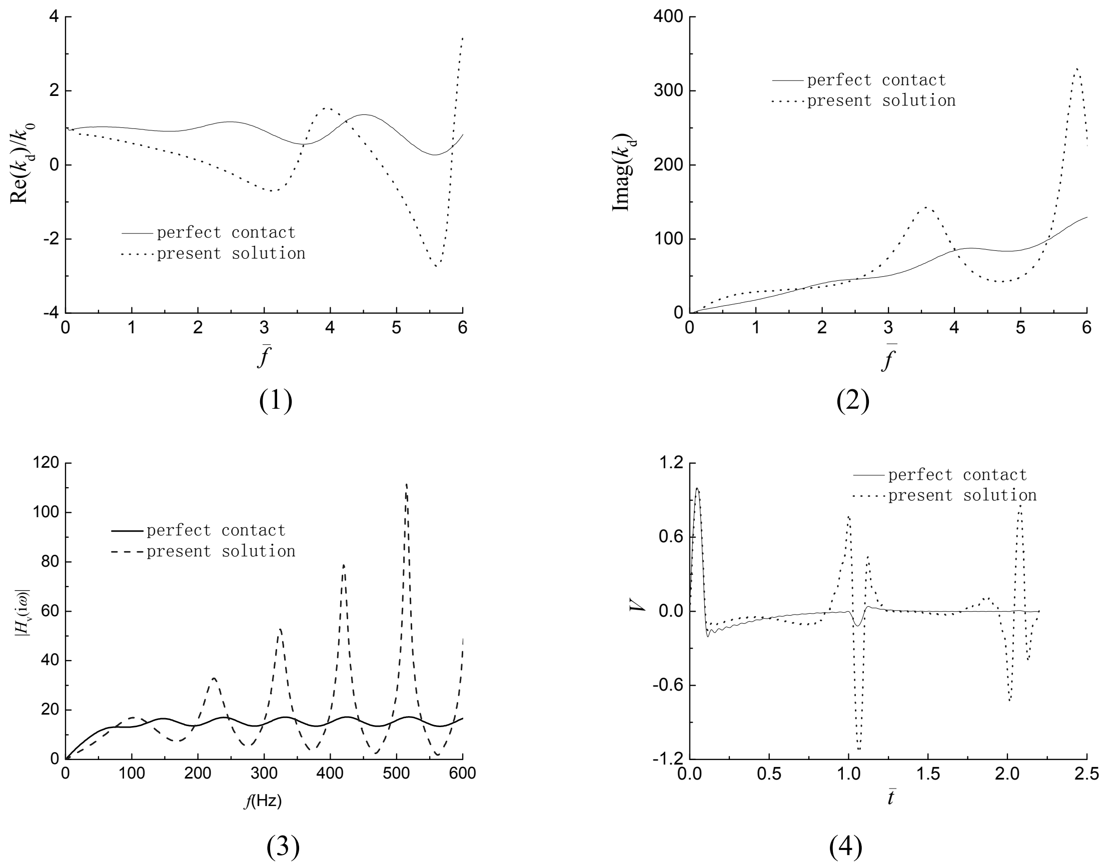

Figure 2 Comparison of pile dynamic response for different model

3.2. Effects of the Soften Region on the Pile Dynamic Response

The effects of soil modulus ratio on the admittance in the frequency domain are obvious in figure 3. Admittance of the peak gradually decreases with the increasing soil modulus ratio. Figure 4 shows that the admittance amplitude increases with the increasing radius of soften zone. At a lager radius, the admittance amplitude of pile amplified generally with the increase of frequency, however, at a smaller one, the admittance remains almost the same.

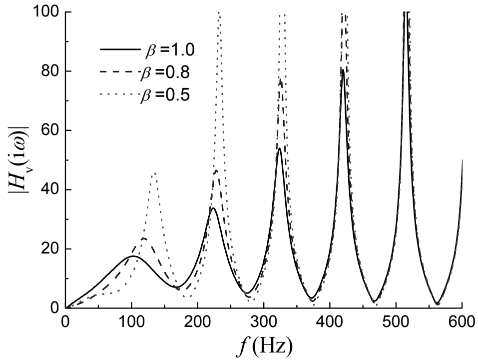

Figure 3 Effects of equivalent modulus on pile admittance

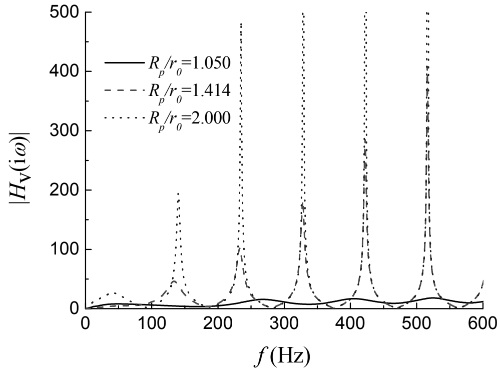

Figure 4 Effects of radius ratio on pile admittance

4. CONCLUSION

In present paper, a simplified model is established to consider the soften region caused by pile sinking, an analytical solution has been derived by means of variable separation method. Numerical results show that the equivalent modulus and radius of soften region have obvious effects on the characteristics of vertical vibration of pile. Under strongly excited, a disturbed field adjacent to the pile occurs and the actual pile-soil interaction is weakened. The perfect contact model overestimates the constraints of soil on the pile and the present model can simulate the soil-pile interaction more reasonable.

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