ESTIMATION OF SHEAR WAVE VELOCITY BY MEANS OF ARRAY MEASUREMENT OF MICROTREMORS USING GENETIC ALGORITHM METHOD

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ABSTRACT:

Shear wave velocity is an important parameter in any dynamic site response analysis. Evaluation of surface geology and site effect in a seismic microzonation study requires a considerable effort and cost for obtaining shear wave velocity (V_s) profile of surface soil layers, if geotechnical and even common geophysical surveys are employed. As a very applicable and cost consumer method, array measurement of microtremores is introduced recently. Although a considerable complexity lies behind theoretical inversion methods, effectiveness and low cost of this method in places such as urban areas makes it very favorable. This method is based on the dispersion phenomenon of Rayleigh surface waves in layered media. Surface wave dispersion curve inversion is a challenging problem for linear inversion procedures due to its highly non-linear nature and to the large numbers of local minima and maxima of the objective function (multi-modality). As the important outcome of the current study, Genetic algorithm (GA) method is utilized in optimizing the inversion of measured dispersion curve of microtremors to obtain shear wave velocity (V_s) profile. This method successfully utilizes recently developed genetic algorithms as a global optimization method. Result of this method is compared to the site profile which obtained by another shear-wave estimation methods.

KEYWORDS: Shear-wave Velocity, Array Measurement, Microtremor, Genetic Algorithm, F-K Method

1. INTRODUCTION

The shear wave profile is the key parameter controlling dynamic response characteristic of soils. Most of the field tests currently used for determining this property require boreholes, and thus may not be performed conventionally at all cases. Few exceptions are surface wave methods that enable one to determine shear wave structures without drilling any borehole. The spectral analysis of surface waves (SASW) method originally introduced by Stokoe and his colleagues into the geotechnical engineering field has been revised and automated, and are now used in various places. Recently, the use of surface waves in microtremors has been also introduced to the geotechnical engineering community for estimating shear structures of sites.

The surface wave methods are based on the dispersion phenomenon of Rayleigh surface waves in layered media. Dispersion refers to the variation of wave velocity as a function of wavelength or frequency of the propagating wave. In simple terms, dispersion occurs because the penetration depth of Rayleigh surface waves increase with increasing wavelength. Rayleigh waves with short wavelength, i.e. high frequencies, propagate only in a shallow zone near the surface, and hence their propagation velocity depends on the properties of the top layers. On the other hand, waves with long wavelengths penetrate deeper into the medium, and therefore their propagation velocity is affected by the properties of bottom layers. In general, the propagation velocity of a Rayleigh surface wave reflects the properties of the layers in which the major part of the wave propagates. In the Rayleigh wave method, surface waves are generated in the test medium over a wide range of frequencies to determine the dispersion curve, i.e. velocity versus wavelength or frequency. The shear wave velocity profile of sediments is then back calculated from the measured dispersion curve that this process known as inversion. The inversion process is based on fitting the experimental dispersion curve to a theoretical model of the shear wave profile that depends on a set of adjustable parameters, e.g. shear wave velocities and thickness of layers. A fitness function is defined as the sum of differences between experimental and theoretical dispersion curves. The best fit between experimental data and the theoretical model is found by using search or optimization technique to locate the minimum of the fitness function. Best-fit parameters in the theoretical model are then accepted as a representation of the actual physical system.

When the considered problem is multi-modal (i.e. the objective function exhibits several local minima and maxima), the traditional approaches can fail because the starting model can be close to some local minimum that will attract it. Therefore, the dependency on an initial model is an important problem when inverting surface wave data. When an appropriate initial model can be generated using a priori information about subsurface structures, linearized inversions can find an optimal solution that is the global minimum of a fitness function. However, if a priori information is either scant or unavailable, the inversion may find a local optimal solution.

To reduce these difficulties, we examined a recently developed nonlinear optimization method that uses a genetic algorithm (GA), which has been applied in several other fields, such as engineering design (e.g., Goldberg, 1989). This algorithm can simultaneously search both globally and locally for an optimal solution by using several models. This method has demonstrated significant potential for solving problems involving multi-modal search spaces.

In this study we were applied frequency-wavenumber (F-K) spectrum analysis of array measurement of microtremores to investigate phase velocity dispersion characteristics (experimental dispersion curve) of microtremors.

The inversion process of the dispersion curve has been the focus of many studies during the last two decades. Other than the traditional gradient-based inversion methods, there are alternative inversion procedures such as simulated annealing and genetic algorithms (GAs).

GAs have recently been used for inversion procedures by several investigators. Hunaidai (1998) and Pezeshk et al. (2005) used GA to invert experimental dispersion curves which obtained by spectral analysis of surface waves (SASW) method. They discussed this method is fast, stable and accurate, with several advantages compared to the traditional methods. Yamanaka et al. (1996) used synthetic and observed earthquake data and examined the applicability of this genetic surface wave inversion method in deducing an S-wave profile for sedimentary layers from short and intermediate period surface wave dispersion data. They demonstrated that the method is robust and can be used to interpret surface-wave dispersion data.

2. F-K SPECTRUM ANALYSIS OF MICROTREMORS

2.1. Array Measurement of Microtremors

A typical setup of the array observation of microtremors is shown in Figure 1. In addition to vertical or three-component velocity sensors, the test equipment consists of amplifiers, lowpass filters, 16 bit A/D converter and a computer, all built in a portable case. The computer equipped with the A/D converter can digitize and analyze microtremor data in the field. The array configuration may not necessarily be circular; however, it is maintained to be circular as much as possible.

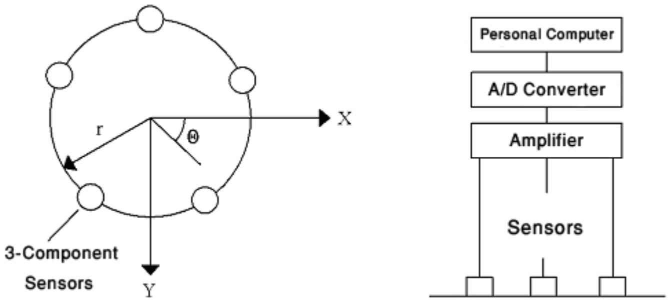

Figure 1. Schematic diagram for Array measurement of microtremors

The wavelength range which produces reliable phase velocities is obtained by Eqn. 2.1.

$$\begin{aligned} D_{\max} &\rangle \lambda_{\max}/3 \\ D_{\min} &\langle \lambda_{\min}/2 \end{aligned} \tag{2.1}$$

in which $D_{\max}$ and $D_{\min}$ are the maximum and minimum sensor spacing and $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum effective wavelengths. Hence, by changing array size, microtremor measurements are repeated, until the phase velocities within the wavelength range of interest are obtained. If more sensors are deployed at different sensor distances, the number of repetition may be reduced.

2.2. Experimental Dispersion Curve

The phase velocity of Rayleigh waves may be determined through the frequency-wavenumber (F-K) spectrum analysis developed by Capon (1969). Assuming that M sensors are available and that the i_{th} sensor is located at the vector position x_i, the F-K spectrum, $P(f,k)$, is defined by

$$P(f,k) = \sum_{i=1}^{M} \sum_{j=1}^{M} A_i^*(f,k) A_j(f,k) G_{ij}(f) \exp[ik.(x_i - x_j)] \tag{2.2}$$

where * denotes complex conjugate, f is the frequency; k is the vector wavenumber in cycles per meter defined as

$$k = |k| \exp(i\theta) \tag{2.3}$$

in which θ is the azimuth of the vector wavenumber, measured clockwise from the x-axis as shown in Figure 1. Based on a direct segment method, the cross power spectrum between the i_{th} and j_{th} sensors at a frequency f, $G_{ij}(f)$, is determined by

$$G_{ij}(f) = \frac{1}{N} \sum_{n=1}^{N} S_{in}(f) S_{jn}^*(f) \tag{2.4}$$

in which N is the total number of the nonoverlapping data segments, and S_{in} is the Fourier transform of the data in the i_{th} sensor and in the n_{th} segment; and for conventional method $A_i = 1$, and for the high resolution method

$$A_i(f,k) = \sum_{j=1}^{M} q_{ij}(f,k) / \sum_{i=1}^{M} \sum_{j=1}^{M} q_{ij}(f,k) \tag{2.5}$$

in which $q_{ij}(f,k)$ is the inverse of the matrix $\exp[ik.(x_i - x_j)] G_{ij}(f)$.

The F-K spectrum is drawn on a two-dimensional wavenumber (k_x-k_y) space for each frequency, as shown in Figure 2 in which the positive k_y-axis points to the north. The spectra are drawn as contours of $-10\log[P(f,k)/P_{\max}(f)]$ in which $P_{\max}(f)$ is the maximum value of $P(f,k)$. The maximum of the spectrum power is indicated by an asterisk and the contours of the spectral power are drawn from 0 to 12 dB in steps of 2 dB. The peak of this F-K spectrum provides the information concerning the phase velocity and the azimuth of the source. If a peak occurs at a distance of $|k_p|$ from the origin at a frequency f, the corresponding phase velocity, c, and the wavelength, λ, can be given by

$$c = 2\pi f / |k_p| \quad , \quad \lambda = 2\pi / |k_p| \tag{2.6}$$

In Figure 2, for example, k_p=0.187 rad/m, $c = 202$ m/s, $\lambda = 33.6$ m, and the azimuth of the source is the northeast. By repeating the above computation over the frequency range for all data from different array diameters, a dispersion curve can be obtained.

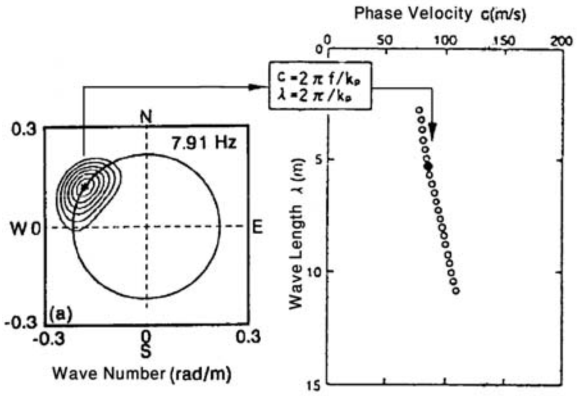

Figure 2. A typical example of F-K spectra and corresponding dispersion data

2.3. Inversion

The last and most important part of utilizing microtremors in estimation of shear-wave velocity profile of sediments is inversion. This part consists of two major steps. The first is to use forward theory, as opposed to inverse theory, to find a theoretical dispersion curve. The second part in this study is to use a Genetic algorithm approach to adjust the theoretical dispersion curve to make it fit to the experimental dispersion curve and finally obtain the shear-wave velocity profile. The forward problem in this study is to solve the Rayleigh dispersion equation for dispersion data which presented blow (Haskell, 1953)

$$F_R[\lambda_m, \mu_m, \rho_m, K, d_m, C] = 0 \tag{2.7}$$

where λ_m and μ_m denote Lame's elastic module of each layer, ρ_m denotes mass density of each layer, K denotes the wavenumber, d_m denotes thickness of each layer, C denotes phase velocity and m denotes number of layers. If pressure wave-velocity (V_p), shear-wave velocity (V_s), thickness of layers (d_m) and mass density of layers (ρ_m) be assumed, Eqn. 2.7 will result in a theoretical dispersion curve.

In the process of the inversion, the values of V_p and ρ_m are rarely changed from their initial values, since the influence of these parameters on the calculated phase velocities is of secondary importance for reasonable initial estimates (Rix et al., 1991). Therefore we can rewrite Eqn. 2.7 for the purposes of this study as:

$$F_R[V_s, d, C, f] = 0 \tag{2.8}$$

Therefore by estimating V_s and *d* the dispersive behavior of the Rayleigh waves will demonstrate. In this equation V_s is a $m \times 1$ vector of shear-wave velocities and d is a $(m-1) \times 1$ vector of thicknesses excluding the half-space.

A computer program developed in MATLAB environment, which has capabilities of solving Eqn. 2.8 and estimating theoretical dispersion curve. The output of program is the fundamental mode's frequencies generated for a typical soil under a set of specific phase velocities.

3. FUNDAMENTALS ON GENETIC ALGORITHMS

The main objectives of this study is to use a genetic algorithm (GA) to estimate shear-wave velocity profiles using data obtained from experimental dispersion curves which obtained by F-K analysis of microtremor measurements. Genetic Algorithms (GAs) have been originally introduced by John Holland and his group at the University of Michigan in the 1970s (Holland, 1975). The fundamental aspect characterizing a genetically based evolutional scheme is the Darwinist paradigm that the fittest survive and reproduce, the others disappear. The results obtained in an inversion process using GA methodology are considered more dependable (Goldberg, 1989; Pezeshk and Camp, 2002) because:

- The GA does not require an initial model to start the optimization.
- GA method use objective function information and a probabilistic transition scheme with no use of gradient information.
- GAs work on a population of possible solution instead of a single solution.
- GAs utilize a coding set of variables instead of variables themselves.

In GAs problem variables are usually created randomly. Variables are grouped in sets of variables which is called strings (chromosome) and composed of a series of characters (gen) that defines a possible solution for the problem. Characters in each string are typically binary numbers, which are evaluated after decoding to real numbers to represent the values of the discrete problem variables for a particular solution. Performance of the problem variables, as described by the objective function and the constraints, is represented by the fitness of each string. A mathematical expression, called a fitness function which calculates a value for a solution of the objective function. The fitter solution gets the higher value and the ones that violate the objective function and constraints are penalized. The fittest and best solutions will survive and get the chance to be a parent of the next generation, like what happens in nature.
The initial population is used to breed new individuals. This is done by means of four genetic operators: (1) Selection, (2) Crossover, (3) Mutation, and (4) Elitism. Individuals are selected for mating on the basis of their fitness, the better the fitness of the individual, the more often it mates. Consequently, fit individuals persist and weak ones die off. In a crossover operator, two selected chromosomes reproduce the next generation. The operator first divides the selected parent chromosomes into segments, and then some of the segments of a parent chromosome are exchanged with the corresponding segment of another parent chromosome. Mutation operators allow good genes that have never appeared before, to be selected and should also ensure that a potentially good component is not lost during reproduction and crossover operations (Goldberg, 1989; Man et al., 2001). Elitism is a strategy often implemented to pass the best individuals of each generation unchanged to the next generation in order to avoid possible loss of good individuals. The process can stop after a fixed number of generations or when the fitness of an individual reaches a certain previously-fixed value.

In this study, we used Genetic Algorithm TOOLBOX of MATLAB, developed by researchers at the University of Sheffield, Department of Automatic Control and Systems Engineering.

4. INVERSION PROCESS OF EXPERIMENTAL DISPERSION CURVE USING GENETIC ALGORITHM

The purpose of this study is to present a new inversion method of experimental dispersion curve of microtremore measurements using genetic algorithm. This method search to find the best combination of the soil layer’s thickness and their corresponding shear-wave velocities to minimize the difference between the experimental and theoretical dispersion curves. The theoretical dispersion curve is obtained by solving the Rayleigh dispersion equation (Eqn. 2.8) for each generation. The deviation of each generation from the experimental dispersion curve is measured by

$$Error=\sum_{i=1}^{I}\left(f_e-f_t\right)^2 \tag{4.1}$$

The optimization problem is minimization of the Error function. Where f_e is a $I\times 1$ vector of frequencies which obtained by experimental dispersion curve for each phase velocities, f_t is $I\times 1$ vector of frequencies which obtained by theoretical dispersion curve for corresponding phase velocities, I is the number of points (phase velocities) which constitute experimental dispersion curve. The optimization problem is subjected to a search space for each unknown parameters and parameters range are normally chosen based on expected variation in parameter values in order to limit the search space. Here they were deliberately chosen to be fairly wide.

The forward problem solution may result in dispersion curves that correspond to several modes of propagation (Nazarian and Stokoe, 1986; Gucunski and Woods, 1991; Tokimatsu et al., 1992a). In general, surface waves consist of the summation of many modes of propagation. However, the fundamental mode usually dominates in a normally dispersive characteristic for the vertical motion in which the phase velocity increases with decreasing frequency (Gucunski and Woods, 1991). In this study, we used the fundamental mode of propagation in the inversion process.

5. EXAMPLES

In according to illustrate the effectiveness of the presented procedure, two examples is presented in this section. The examples are the application of the proposed procedure to determine a set of thickness and shear-wave velocities corresponding to the best match of the theoretical and experimental dispersion curves. The stability of the GA in conforming itself to match a given objective, which in this study is the experimental dispersion curve, is demonstrated. In both examples, the experimental dispersion curve, which are determined from field measurements, are known and given.

5.1. Example 1

In this example, a comparison of the shear-wave velocity profiles obtained from a site in Bam, southeast of Iran, using the proposed GA-based inversion procedure, a downhole seismic survey and geophysical method is presented. The shear-wave velocity profiles used for comparison was obtained from a downhole seismic survey method and geophysical method performed by Building & Housing Research Center of Iran. The experimental dispersion curve for this site was determined using F-K analysis of an array measurement of microtremors. The shear-wave velocity profile is then calculated using the proposed procedure. The design variables for this problem consist of 7 unknowns, 3 thicknesses and 4 shear-wave velocities. The target input data consist of the abscissas of experimental dispersion curve at 21 phase velocities. A population size of 50 was run for 200 generations with an 80% probability of crossover but by proceeding of process it decreased. At each generations, 4 individuals produced by Elitism strategy and remainder individuals produced by Mutation. Figure 3 shows the experimental dispersion curve and the best obtained theoretical dispersion curve at these 21 phase velocities. The convergence history is shown in Figure 4.

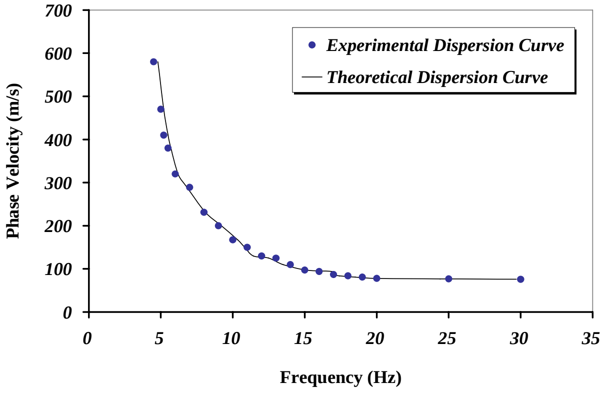

Figure 3. Experimental dispersion curve and the best obtained theoretical dispersion curve for example 1

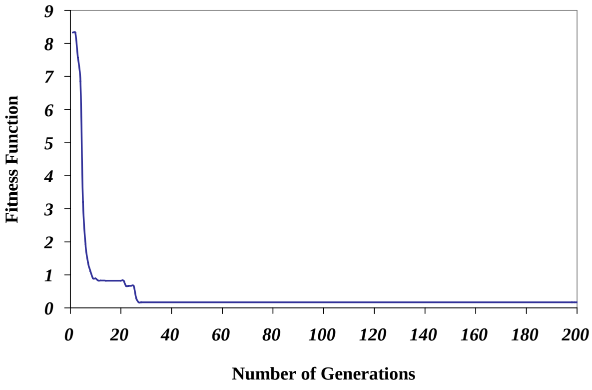

Figure 4. Convergence history of example 1

The algorithm selected 3 thicknesses and 4 shear-wave velocities, as presented in Figure 5. It can be observed in this figure that the proposed method results in a good estimation of shear-wave velocities at different depths in comparison with other methods.

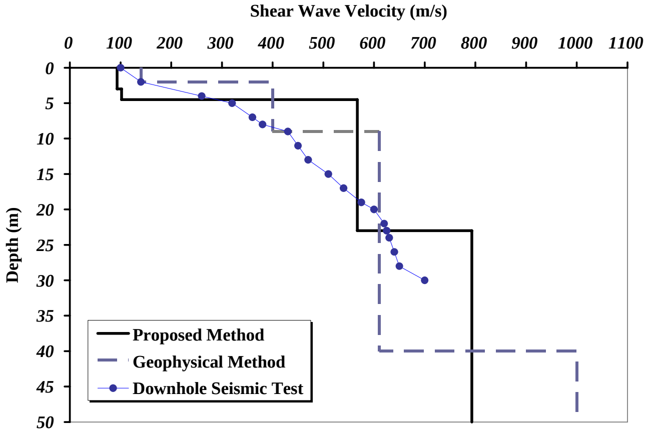

Figure 5. Comparison of the shear-wave velocity profiles which obtained by proposed method, Downhole seismic test and Geophysical method for example 1

5.2. Example 2

The design variables for this problem consist of 9 unknowns, 4 thicknesses and 5 shear-wave velocities. The site is located in Orumieh, northwest of Iran. The procedure is similar to that of example 1. In this example, experimental dispersion curve consists of 21 phase velocities and the number of generations is 450. Figure 6 shows the experimental dispersion curve and the best obtained theoretical dispersion curve at these 21 phase velocities. The convergence history is shown in Figure 7.

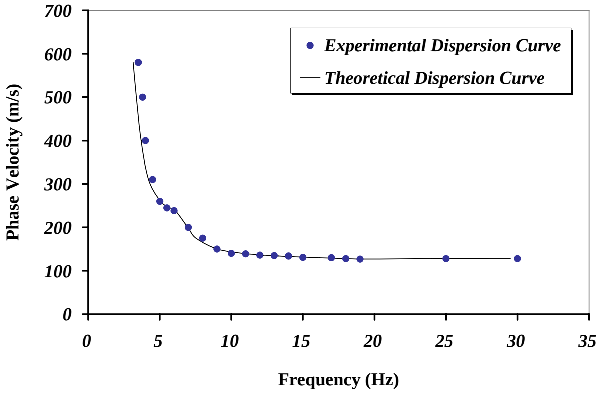

Figure 6. Experimental dispersion curve and the best obtained theoretical dispersion curve for example 2

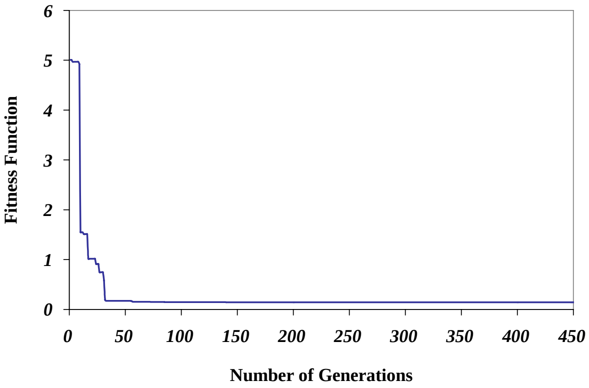

Figure 7. Convergence history of example 2

The algorithm selected shear-wave velocity profile, as presented in Figure 8. This profile is compared to those which obtained by geotechnical and geophysical methods in this Figure. From this Figure, it can be observed that the presented method results in a good estimation of shear-wave velocities at different depths in comparison with the geotechnical and geophysical methods. Also the presented method could estimate a soft layer underlying a stiff layer but the geophysical method couldn't detect it. Nonetheless the shear-wave velocities which presented by new method are smaller than those presented by geotechnical method that it can be because of weak estimation between N_{spt} and V_s by experimental relation which is used.

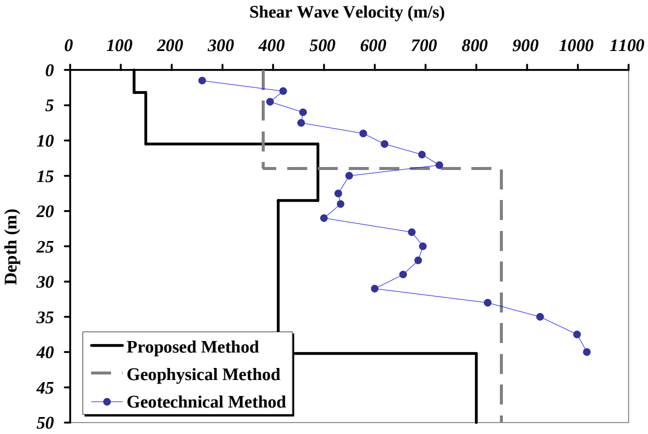

Figure 8. Comparison of the shear-wave velocity profiles which obtained by proposed method, Geotechnical and Geophysical method for example 2

6. CONCLUSIONS

A new inversion method for array measurement of microtremors is presented. This method uses genetic algorithm (GA) to obtain the theoretical dispersion curve that matches the experimental dispersion curve. The input to the GA is either a union of soil thickness and shear-wave velocities. Each generation of individuals is modified through the processes that mimic nature's mating, natural selection and mutation. The process continues until an optimum individual set is obtained which represent a soil profile with a dispersion curve that best matches the experimental dispersion curve. The method is stable and accurate with several advantages compared with the traditional optimization methods. The results of provided examples show acceptable coincidence between the theoretical and experimental dispersion curves. The shear-wave velocity profile of examples obtained by the new method agrees well with the profiles which obtained by another methods such as geophysical method, downhole seismic survey and geotechnical method.

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