A NOTE ON THE STATIONARY MODEL OF EARTHQUAKE INDUCED GROUND MOTION WITH A HU SPECTRUM *

LI Hongjing[1], SUN Guangjun[2] and REN Yongliang[3]

[1] *Professor, College of Civil Engineering, Nanjing University of Technology, Nanjing, China*
[2] *Ph.D. Candidate, School of Civil Engineering, Southeast University, Nanjing, China*
[3] *Master, Jiangsu Mingcheng Architectural Design Co., LTD., Yancheng, China*
Email: harbiner@163.com, gjsun2004@163.com, renyongliang123@163.com

ABSTRACT :
The Hu spectral model, a modified Kanai-Tajimi spectral model for the stationary stochastic process of earthquake ground motion, is analyzed. According to the Hu spectral model, the spectral density function of the Kanai-Tajimi spectrum is modified only during the low frequency range and in good accordance with the Kanai-Tajimi spectrum during the moderate and high frequencies. It is proved that the earthquake-induced ground acceleration process with the Hu spectrum is essentially the result of which a filtered Gaussian white noise process on the rock is filtered by the overlaying soil represented by a linear single-freedom-degree system, namely it is a twice filtered white noise process. This shows that the Hu spectral model is not only concise in the mathematical expression but also distinct in physical meaning and reasonable in practice. Furthermore, the low frequency control factor which determines the low frequency contents in Hu spectral model is investigated and evaluated from the observation data of earthquake ground motion.

KEYWORDS:
earthquake ground motions, stochastic process, Kanai-Tajimi spectrum, Hu spectrum, low frequency control factor

1. INTRODUCTION

The heavy economic loss and human injury are usually caused in a severe earthquake, the mean reason is that the civil engineering facilities are not able to resist the earthquake induced ground motions and failure during the earthquake. In order to reduce the earthquake damages, the most effective measure is to enhance the earthquake-resistant capability of structures and optimize the structural performance due to allowable costs. Therefore, it is necessary to properly analyze and evaluate the seismic responses of structures. At first, the earthquake-induced ground motions ought to be estimated and described.

It is known that the randomness exists in the occurrence of earthquakes in time and space, and the uncertainty is vast in predicting intensities of resulting ground motions, so the earthquake-induced ground motions are usually considered as stochastic processes. Obviously it is reasonable to consistently account for the underlying uncertainties and randomness involved in the earthquake ground motions and evaluate quantitatively the structural vulnerability and safety subject to earthquake excitation on the basis of probabilistic methods. Because the strong motions are rare events, it is practically impossible to obtain a large number of the stochastic process samples of ground motions and determine their probabilistic characteristics by statistic methods. In practice, the random models are usually utilized for the description of earthquake ground motions and the models' parameters may be identified from the actual earthquake records. The random ground motion model is distinct in physical conceptions and it is commonly described by the power spectral density functions in frequency domain.

The stochastic process model to simulate the earthquake ground accelerations was first proposed by Housner (1947), and this model is equivalent to a stationary Gaussian white noise process, by which the ground motion is

* Supported by National Natural Science Foundation of China Under Grant No. 50678084.

assumed as a sequence of consecutive velocity pulses. Subsequently, some filtered white noise models are extensively proposed to represent the earthquake ground motion. The stationary filtered white noise model presented by Kanai and Tajimi (Kanai, 1957; Tajimi, 1960) is a well-known and favorite for many researchers and engineers and widely used in the field of earthquake engineering. In the Kanai-Tajimi spectral model, the rock acceleration is assumed a white noise process and the overlying soil deposits are simulated by a linear single-degree-of-freedom system. However, the lower frequency contents of ground motions are magnified unpractically due to Kanai-Tajimi model, may lead to unreasonable response analysis results for structures with lower natural vibration frequencies. Furthermore, the Kanai-Tajimi spectral model is singular in the point of zero frequency and cannot be integrated twice, so the displacement and velocity variances of the ground motion will be infinite. In order to improve the Kanai-Tajimi spectral model, some modified Kanai–Tajimi models are presented (Hu and Zhou, 1962; Ou, Niu and Du, 1991; Clough and Penzien, 1993; Hong, 1995). Based on these models, further modifications are made by researchers (Du and Chen, 1994; Lai, Ye and Li, 1995; Li and Zhai, 2003). In all the modified Kanai–Tajimi models, the Hu spectral model eliminate successfully the problems of the Kanai-Tajimi spectral model in frequency zero, meanwhile the advantages of this model are retained. The Hu model is consistent properly with the statistic results of the strong motion records and concise in mathematical expression, so it is more reasonable for describing the statistic characteristics of the earthquake-induced ground motions.

In this paper, the physical meaning of the Hu spectral model is interpreted, and the frequency parameter to restrain the low frequency content of earthquake ground motions is discussed. The autocorrelation function of the Hu spectrum is deduced by state space method. These results will provide a basis for random response analysis of the seismic structures in time domain.

2. MODELING OF EARTHQUAKE-INDUCED GROUND MOTION

The power spectral density function of stationary Gaussian process with the power spectrum of Kanai-Tajimi is expressed as:

$$S_a^K(\omega) = \frac{\omega_g^4 + (2\zeta_g\omega_g\omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\zeta_g\omega_g\omega)^2} \cdot S_0 \tag{2.1}$$

where ω_g and ζ_g are the parameters of the overlaying soil deposits; S_0 is the constant spectral intensity of the rock motions.

The Kanai-Tajimi spectrum is the solution that the white noise process is filtered, and the following equations are used for this purpose:

$$a(t) = -2\zeta_g\omega_g\dot{x} - \omega_g^2 x \tag{2.2}$$

$$\ddot{x} + 2\zeta_g\omega_g\dot{x} + \omega_g^2 x = -\ddot{U}(t) \tag{2.3}$$

in which $\ddot{U}(t)$ is the rock acceleration, it is the Gaussian white noise process.

In Hu model, the power spectral density of random ground motion process is modified as:

$$S_a^H(\omega) = \frac{\omega^6}{\omega^6 + \omega_c^6} \cdot S_a^K(\omega) \tag{2.4}$$

where ω_c is the factor of low frequency control.

Comparing with the Kanai-Tajimi model, the Hu spectral model modifies only over the low frequency range of the Kanai-Tajimi spectrum and is in good accordance over the range of high frequency. Obviously the velocity and displacement variance of the ground motion are convergent due to Hu spectral model. Therefore, the Hu spectral model can not only remain the advantages of the Kanai–Tajimi model but also eliminate the drawbacks of the Kanai–Tajimi model.

The rock motion is assumed as the white noise process due to the Kanai-Tajimi model, obviously this does not accord with the realities in physics. In fact, the acceleration of the rock motion induced by an earthquake must be the color noise process with certain characteristics. Assuming it can be expressed by the following equation:

$$S_{\ddot{U}}(\omega)=\frac{\omega^6}{\omega^6+\omega_c^6}\cdot S_0 \tag{2.5}$$

it can be proved that the spectral density function of the ground acceleration $a(t)$ obtained from the filtered rock motion $\ddot{U}(t)$ by equations (2.2) and (2.3) has the same form with the expression of the Hu spectral model.

Thus the Hu spectral model may be considered the improvement of the Kanai-Tajimi spectrum, and can be interpreted physically that the rock acceleration process with the spectral density function defined by Eq. (2.5) is filtered by a linear single-degree-of-freedom system with natural frequency ω_g and damping ratio ζ_g, as a result, it will lead to a stochastic process with the Hu spectrum.

3. INTERPRETATION OF THE MODIFIED KANAI-TAJIMI SPECTRUM MODEL

Since the Hu spectrum is the result of filtered color noise process, what are the properties of the rock acceleration $\ddot{U}(t)$? There are two spectral parameters, S_0 and ω_c, in equation (2.5). Figure 1 shows the relationships between the two parameters and the spectral amplitude.

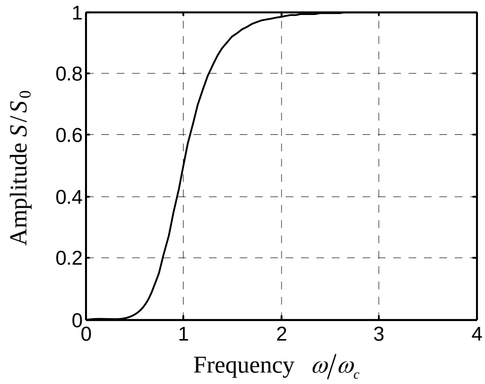

Figure 1 Filtered white noise in rock

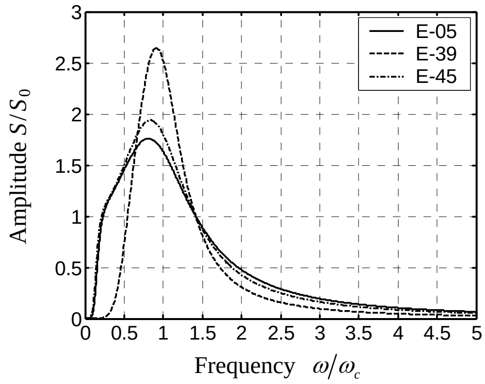

Figure 2 Hu spectral model

Observing the figure 1, the spectral density of the rock acceleration only lies differences with white noise in the lower range of frequencies, and they are well compatible each other over the medium and high frequency range. So the model of the stochastic rock motion with spectrum given by Eq. (2.5) is the modification to white noise model by reducing only the lower frequency contents of the motion. The modified limits are controlled by the factor ω_c, i.e. the frequency contents of the white noise are modified during the approximate range from zero to $2\omega_c$.

Considering the following filter equations:

$$\ddot{U}(t)=\dddot{y}(t) \tag{3.1}$$

$$\dddot{y}+\omega_c^3 y=p(t) \tag{3.2}$$

in which $p(t)$ is the white noise process with spectral intensity S_0.

Let $p(t)=e^{i\omega t}$ and $y=H_{yp}(i\omega)e^{i\omega t}$. Substituting $p(t)$ and $y(t)$ into Eq. (3.2) and considering the condition $e^{i\omega t}\neq 0$ gives the transfer function:

$$H_{yp}(i\omega)=\frac{1}{-i\omega^3+\omega_c^3} \tag{3.3}$$

Then the spectral density function of $y(t)$ is given by

$$S_y(\omega) = \left|H_{yp}(i\omega)\right|^2 S_p(\omega) = \frac{1}{\omega^6 + \omega_c^6} \cdot S_0 \tag{3.4}$$

Considering the relationship:

$$S_{\dddot{y}}(\omega) = \omega^6 S_y(\omega) = \frac{\omega^6}{\omega^6 + \omega_c^6} \cdot S_0 \tag{3.5}$$

The spectral density function of $\ddot{U}(t)$ is deduced as

$$S_{\ddot{U}}(\omega) = S_{\dddot{y}}(\omega) = \frac{\omega^6}{\omega^6 + \omega_c^6} \cdot S_0 \tag{3.6}$$

It is clear that Eq. (3.6) is same with Eq. (2.5). Therefore the rock motion process is a filtered white noise process and the stochastic process with the Hu spectrum is the result that the filtered white noise process is filtered again, i.e. it is the twice filtered white noise process.

In Eq. (2.5), S_0 represents the intensity of the rock acceleration, which depends on the energy released during the earthquake and can be determined by the mean value of the peak ground accelerations. The parameter ω_c limits the range of low frequency reduction, and the more this frequency parameter, the less the low frequency content of earthquake ground motion, so it may be related to the fault mechanisms. In generally, the high frequency contents of the rock motion are abundant when the earthquake occurs, and they are usually reduced by soil filters and some contents with long periods will be amplified in the process of propagation. In this word, it is not only concise in the mathematic expression that the rock acceleration model given by Eq. (2.5) only modifies the frequency contents during the lower range and holds basically the frequency characteristics of the white noise spectrum during the medium and high frequency range, but also physically reasonable, because the influences of the high frequency contents of the rock motion have not been very strong when the motion are propagated at the site.

Total four parameters, ω_c, ω_g, ζ_g and S_0, are involved in the Hu spectral model, and they have been studied by some researchers. The values of the four parameters at two site conditions are suggested by a least squares regression to the power spectral density function obtained from the instrumented data of strong ground motions (Hong, Jiang and Li, 1994), the results are listed in Table 1. Table 2 also provides the values of the four parameters based on three seismic accelerograms received by SMART-1 array (Wang and Jiang, 1997), and the results are shown in Figure 2 as well.

Table 1 The parameters of Hu spectral model (by Hong Feng)

Site Condition	S_0	ω_g (rad/s)	ω_c (rad/s)	ζ_g
Soft soil	1.574	10.25	1.742	1.006
Medium soil	1.076	17.07	2.108	0.7845

Table 2 The parameters of Hu spectral model (by Wang Jun-jie)

No.	S_0 ($\times 10^{-3}$)	ω_g (rad/s)	ω_c (rad/s)	ζ_g
E-05	3.30	16.77	2.65	0.63
E-39	55.00	6.97	3.60	0.40
E-45	14.05	10.59	1.51	0.56

4. CHARACTERISTICS OF ROCK MOTIONS IN TIME DOMIAN

The statistic characteristics of the random ground motion process with Hu spectrum are described by the spectral density function given by Eq. (2.4) in frequency domain, and the statistic characteristics in time domain can be described by the correlation function. Because the Hu model is a twice filtered Gaussian white noise process, the time properties of the rock acceleration can be obtained by using the filter equations (3.1) and (3.2). Introducing the state space vectors, Eq. (3.2) is rewritten as

$$[A]\{\dot{z}\}+[B]\{z\}=\{F_r\}p(t) \tag{4.1}$$

in which

$$\{z\}=\begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix}=\begin{Bmatrix} y \\ \dot{y} \\ \ddot{y} \end{Bmatrix},\ [A]=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},\ [B]=\begin{bmatrix} \omega_c^3 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix},\ \{F_r\}=\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

Because $[A]$ and $[B]$ are symmetric, the characteristic equation of the Eq. (4.1) is

$$([A]\lambda_j+[B])\{\varphi_j\}=\{0\} \qquad j=1,2,3 \tag{4.2}$$

The complex eigenvalues may be solved from Eq. (4.2) as

$$\lambda_1=-\omega_c\,,\quad \lambda_2=\frac{1+\sqrt{3}i}{2}\omega_c\,,\quad \lambda_3=\frac{1-\sqrt{3}i}{2}\omega_c \tag{4.3}$$

Substituting Eq. (4.3) into Eq. (4.2) leads to the complex modes of the system:

$$\{\varphi_1\}=\begin{Bmatrix} 1 \\ \lambda_1 \\ \lambda_1^2 \end{Bmatrix},\quad \{\varphi_2\}=\begin{Bmatrix} 1 \\ \lambda_2 \\ \lambda_2^2 \end{Bmatrix},\quad \{\varphi_3\}=\begin{Bmatrix} 1 \\ \lambda_3 \\ \lambda_3^2 \end{Bmatrix} \tag{4.4}$$

It can be proved that the complex modes are weighted orthogonal with respect to the matrix $[A]$ and $[B]$. The orthogonality may be expressed as

$$\{\varphi_j\}^T[A]\{\varphi_k\}=\{\varphi_j\}^T[B]\{\varphi_k\}=0 \qquad j\neq k$$

$$\{\varphi_j\}^T[B]\{\varphi_j\}=-\lambda_j\{\varphi_j\}^T[A]\{\varphi_j\}$$

The response $\{z\}$ of the system can be expressed as the superposition of the modal contributions:

$$\{z\}=\sum_{j=1}^{3}\{\varphi_j\}h_j \tag{4.5}$$

Substituting Eq. (4.5) in Eq. (4.1) and premultiplying each term in this equation by $\{\varphi_j\}^T$. Because of the orthogonality conditions of the complex modes, the uncoupled equation for each mode can be obtained:

$$\dot{h}_j-\lambda_j h_j=\eta_j p \qquad j=1,2,3 \tag{4.6}$$

Where

$$\eta_j=\frac{\{\varphi_j\}^T\{F_r\}}{\{\varphi_j\}^T[A]\{\varphi_j\}}=\frac{1}{3\lambda_j^2}.$$

The solution to the Eq. (4.6) may be solved as

$$h_j(t)=\int_0^{\infty}\eta_j e^{\lambda_j\tau}p(t-\tau)d\tau \tag{4.7}$$

The correlation function of the complex modal contributions h_j and h_k $(j,k=1,2,3)$ is defined as

$$R_{h_jh_k}(\tau)=E[h_j(t)h_k^*(t+\tau)] \tag{4.8}$$

where the asterisk * denotes the complex conjugate of vector. Substituting Eq. (4.7) in Eq. (4.8) and changing the orders of expected value and integral calculations gives

$$R_{h_jh_k}(\tau)=\eta_j\eta_k^*\int_0^{\infty}\int_0^{\infty}e^{\lambda_j u+\lambda_k^* v}R_p(\tau+u-v)dvdu \qquad j,k=1,2,3 \tag{4.9}$$

where $R_p(\tau)=2\pi S_0\delta(\tau)$, which is the correlation function of the white noise process.

According to the Eq. (4.5) and Eq. (4.6), the responses of the system are given by

$$\ddot{y}(t)=z_3(t)=\sum_{j=1}^{3}\lambda_j^2 h_j(t) \tag{4.10}$$

The correlation function of the response is

$$R_{\ddot{y}}(\tau)=\sum_{j=1}^{3}\sum_{k=1}^{3}\lambda_j^2(\lambda_k^*)^2R_{h_jh_k}(\tau) \tag{4.11}$$

Substituting Eq. (4.9) in Eq. (4.11) gives

$$R_{\ddot{y}}(\tau)=\frac{1}{9}\sum_{j=1}^{3}\sum_{k=1}^{3}\int_0^\infty\int_0^\infty e^{\lambda_j u+\lambda_k^* v}R_p(\tau+u-v)dvdu \tag{4.12}$$

Considering the following relationship:

$$R_{\ddot{y}}(\tau)=-\frac{d^2}{d\tau^2}R_{\dot{y}}(\tau)$$

the correlation function of the filtered white noise process in rock is solved as

$$R_{\ddot{U}}=R_{\ddot{y}}=\frac{\pi\omega_c S_0}{3}[-e^{-\omega_c|\tau|}+e^{\frac{\omega_c}{2}|\tau|}(\cos\frac{\sqrt{3}}{2}\omega_c\tau-\sqrt{3}\sin\frac{\sqrt{3}}{2}\omega_c|\tau|)] \tag{4.13}$$

5. CORRELATION FUNCTION OF THE MODIFIED KANAI-TAJIMI SPECTRUM MODEL

The correlation function of the Hu spectrum can be obtained by random vibration analysis to the single-degree-of-freedom system subject to the seismic excitation with the spectral density function of Eq. (2.5) or correlation function of Eq. (4.13).

The filter equations can be rewritten as

$$[M]\{\dot{u}\}+[K]\{u\}=-\{F_g\}\ddot{U}(t) \tag{5.1}$$

where $\ddot{U}(t)$ is the filtered white noise process with the correlation function given by Eq. (4.13); $\{u\}=\{x,\dot{x}\}^T$ a state vector; $[M]$、$[K]$ and $\{F_g\}$ the mass matrix, stiffness matrix and direction vector, respectively.

$$[M]=\begin{bmatrix}2\zeta_g\omega_g & 1\\ 1 & 0\end{bmatrix},\quad [K]=\begin{bmatrix}\omega_g^2 & 0\\ 0 & -1\end{bmatrix},\quad \{F_g\}=\begin{Bmatrix}1\\0\end{Bmatrix}$$

The modal expansion of displacement vector $\{u\}$ can be expressed as

$$\{u\}=\sum_{j=1}^{2}\{\gamma_j\}q_j \tag{5.2}$$

where $\{\gamma_j\}$ and $q_j(t)$ are the jth complex mode and modal coordinate, respectively.

The correlation function of ground acceleration is expressed as

$$R_a(\tau)=\omega_g^4R_x(\tau)+4\zeta_g^2\omega_g^2R_{\dot{x}}(\tau) \tag{5.3}$$

in which

$$R_x(\tau)=\sum_{j=1}^{2}\sum_{k=1}^{2}R_{q_jq_k}(\tau) \tag{5.4}$$

$$R_{\dot{x}}(\tau)=\sum_{j=1}^{2}\sum_{k=1}^{2}r_jr_k^*R_{q_jq_k}(\tau) \tag{5.5}$$

where r_j is the jth complex frequency, $r_{1,2}=-\zeta_g\omega_g\pm i\omega_D$, $\omega_D=\omega_g\sqrt{1-\zeta_g^2}$.

The correlation function of complex modal response is

$$R_{q_jq_k}(\tau)=\frac{(-1)^{j+k}}{4\omega_D^2}\int_0^\infty\int_0^\infty e^{r_ju+r_k^*v}R_{\ddot{U}}(\tau+u-v)dvdu\quad j,k=1,2\quad \tau\ge 0 \tag{5.6}$$

Substituting Eq. (4.13) in Eq. (5.6), and carrying out the integral calculation leads to

$$R_{q_jq_k}(\tau)=\frac{\pi S_0\omega_c(-1)^{j+k}}{24\omega_D^2}\cdot\frac{1}{r_j+r_k^*}\cdot(\alpha_{jk}e^{-\omega_c\tau}+\beta_{jk}e^{\mu\tau}+\kappa_{jk}e^{\mu^*\tau}+\rho_{jk}e^{p_k^*\tau})\qquad j,k=1,2\quad \tau\ge 0 \tag{5.7)}$$

The coefficients in Eq. (5.7) are

$$\alpha_{jk}=\frac{-2(r_k^*+r_j)}{(r_k^*+\omega_c)(r_j-\omega_c)},\quad \beta_{jk}=s\cdot(\frac{1}{r_k^*-\mu}+\frac{1}{r_j+\mu}),\quad \kappa_{jk}=s^*\cdot(\frac{1}{r_k^*-\mu^*}+\frac{1}{r_j+\mu^*}),$$

$$\rho_{jk}=s\cdot(\frac{1}{r_k^*+\mu}-\frac{1}{r_k^*-\mu})+s^*\cdot(\frac{1}{r_k^*+\mu^*}-\frac{1}{r_k^*-\mu^*})+\frac{-4\omega_c}{(r_k^*)^2+\omega_c^2},\quad s=1+\sqrt{3}i,\quad \mu=(\frac{1}{2}+\frac{\sqrt{3}}{2}i)\omega_c$$

Substituting Eq. (5.7) in Eq. (5.4) and Eq. (5.5), using Euler transform gives

$$R_x(\tau)=\frac{\pi S_0\omega_c}{12\omega_D^2}[b_1e^{-\omega_c|\tau|}+e^{\frac{\omega_c}{2}|\tau|}(b_2\cos\frac{\sqrt{3}}{2}\omega_c\tau-b_3\sin\frac{\sqrt{3}}{2}\omega_c|\tau|)+e^{-\zeta_g\omega_g|\tau|}(b_4\cos\omega_{D_g}\tau-b_5\sin\omega_{D_g}|\tau|)] \tag{5.8}$$

$$R_{\dot{x}}(\tau)=\frac{\pi S_0\omega_c}{12\omega_D^2}[c_1e^{-\omega_c|\tau|}+e^{\frac{\omega_c}{2}|\tau|}(c_2\cos\frac{\sqrt{3}}{2}\omega_c\tau-c_3\sin\frac{\sqrt{3}}{2}\omega_c|\tau|)+e^{-\zeta_g\omega_g|\tau|}(c_4\cos\omega_{D_g}\tau-c_5\sin\omega_{D_g}|\tau|)] \tag{5.9}$$

in which the coefficients are respectively given as

$$b_1=\frac{-4(1-\zeta_g^2)\omega_g^2}{(\omega_g^2+\omega_c^2)^2-4\zeta_g^2\omega_g^2\omega_c^2}$$

$$b_2=\frac{4\omega_g^2(1-\zeta_g^2)[\omega_g^4-2\omega_g^2\omega_c^2(2\zeta_g^2-1)-2\omega_c^4]}{(\omega_g^8-\omega_g^4\omega_c^4+\omega_c^8)+2\omega_g^2\omega_c^2(2\zeta_g^2-1)[\omega_g^4+2\omega_g^2\omega_c^2(2\zeta_g^2-1)+\omega_c^4]}$$

$$b_3=\frac{4\sqrt{3}\omega_g^4(1-\zeta_g^2)[\omega_g^2+2\omega_c^2(2\zeta_g^2-1)]}{(\omega_g^8-\omega_g^4\omega_c^4+\omega_c^8)+2\omega_g^2\omega_c^2(2\zeta_g^2-1)[\omega_g^4+2\omega_g^2\omega_c^2(2\zeta_g^2-1)+\omega_c^4]}$$

$$b_4=\frac{-2\omega_c(1-\zeta_g^2)}{\zeta_g\omega_g}\{\frac{\omega_g^6(4\zeta_g^2-1)+\omega_g^4\omega_c^2(32\zeta_g^4-24\zeta_g^2+3)+\omega_g^2\omega_c^4(8\zeta_g^2-3)+2\omega_c^6}{(\omega_g^8-\omega_g^4\omega_c^4+\omega_c^8)+2\omega_g^2\omega_c^2(2\zeta_g^2-1)[\omega_g^4+2\omega_g^2\omega_c^2(2\zeta_g^2-1)+\omega_c^4]}-\frac{\omega_g^2(4\zeta_g^2-1)+\omega_c^2}{\omega_g^4+2\omega_g^2\omega_c^2(2\zeta_g^2-1)+\omega_c^4}\}$$

$$b_5=\frac{2\omega_c\sqrt{(1-\zeta_g^2)}}{\omega_g}\{\frac{\omega_g^6(4\zeta_g^2-3)+\omega_g^4\omega_c^2(32\zeta_g^4-40\zeta_g^2+11)+\omega_g^2\omega_c^4(8\zeta_g^2-5)+2\omega_c^6}{(\omega_g^8-\omega_g^4\omega_c^4+\omega_c^8)+2\omega_g^2\omega_c^2(2\zeta_g^2-1)[\omega_g^4+2\omega_g^2\omega_c^2(2\zeta_g^2-1)+\omega_c^4]}-\frac{\omega_g^2(4\zeta_g^2-3)+\omega_c^2}{\omega_g^4+2\omega_g^2\omega_c^2(2\zeta_g^2-1)+\omega_c^4}\}$$

$$c_1=b_1\omega_c^2,\quad c_2=\frac{b_2+\sqrt{3}b_3}{2}\omega_c^2,\quad c_3=\frac{b_3-\sqrt{3}b_2}{2}\omega_c^2$$

$$c_4=(1-2\zeta_g^2)\omega_g^2b_4-2\zeta_g\omega_g^2\sqrt{1-\zeta_g^2}b_5,\quad c_5=(1-2\zeta_g^2)\omega_g^2b_5+2\zeta_g\omega_g^2\sqrt{1-\zeta_g^2}b_4$$

Substituting Eq. (5.8) and Eq. (5.9) in Eq.(5.3) and simplifying the expressions as

$$R_a(\tau)=\frac{\pi S_0\omega_c}{12(1-\zeta_g^2)}[A_1e^{-\omega_c|\tau|}+e^{\frac{\omega_c}{2}|\tau|}(A_2\cos\frac{\sqrt{3}}{2}\omega_c\tau+A_3\sin\frac{\sqrt{3}}{2}\omega_c|\tau|)+e^{-\zeta_g\omega_g|\tau|}(A_4\cos\omega_D\tau+A_5\sin\omega_D|\tau|)] \tag{5.10}$$

in which

$$A_1=(\omega_g^2+4\zeta_g^2\omega_c^2)b_1,\quad A_2=(\omega_g^2+2\zeta_g^2\omega_c^2)b_2+2\sqrt{3}\zeta_g^2\omega_c^2b_3,\quad A_3=2\sqrt{3}\zeta_g^2\omega_c^2b_2-(\omega_g^2+2\zeta_g^2\omega_c^2)b_3$$

$$A_4=\omega_g^2(1+4\zeta_g^2-8\zeta_g^4)b_4-8\zeta_g^3\omega_g^2\sqrt{1-\zeta_g^2}b_5,\quad A_5=-8\zeta_g^3\omega_g^2\sqrt{1-\zeta_g^2}b_4-\omega_g^2(1+4\zeta_g^2-8\zeta_g^4)b_5$$

Eq. (5.10) is the expression of the correlation function of the Hu spectral model, which is the inverse Fourier's transform of Eq. (2.4).

6. CONCLUDING REMARKS

(1) The Hu spectral model is an improved scheme to the Kanai-Tajimi model, and essentially the filtered color noise process, thus it is definite in physical conception. The singular point in zero frequency is eliminated due to the Hu model so that the variances of the ground velocity and displacement are finite. The Hu spectral model is consistent well with actual earthquake-induced ground motion.

(2) The low frequency contents of the earthquake ground motion are modified by the low frequency control factor ω_c in the Hu spectral model. The low frequency contents decreases with the increase of ω_c, and the Hu spectral model can be used for the stochastic seismic response analysis of the structures with low frequency as well as medium and high frequency.

(3) The correlation function is the important characteristic of the stationary stochastic process in time domain, by which other statistical properties can be obtained conveniently. The Hu spectral model is a twice filtered white noise process, so the correlation function can be deduced through the filter equations in time domain. These results provide a basis for random response analysis of the seismic structures in time domain.

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