EMPIRICAL RELATIONSHIPS FOR MAGNITUDE AND SOURCE-TO-SITE DISTANCE CONVERSIONS USING RECENTLY COMPILED TURKISH STRONG-GROUND MOTION DATABASE

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ABSTRACT:

We derived empirical relationships for magnitude conversion using the recently compiled Turkish strong-motion database. The database also provides useful information about the relations between different source-to-site distance metrics (epicentral distance, R_{epi}, hypocentral distance, R_{hyp}, Joyner-Boore distance, R_{jb} etc) that are effectively used in the seismic hazard analysis. The empirical magnitude equations re-scale body-wave (m_b), surface-wave (M_s), local (M_L) and duration (M_d) magnitudes to moment magnitude (M_w). We employed ordinary and total least squares regression procedures separately to derive these relationships. The residual analyses conducted for the evaluation of these relationships showed that the expressions obtained from ordinary least squares regression procedure perform better for the conversion of other magnitude scales to M_w. The proposed equations are also compared with the other similar relationships presented in the literature. The preliminary evaluations reveal a fairly good agreement between the proposed relationships and the magnitude conversions described in the other studies. The observations made in this study suggest the reliability of the recently compiled Turkish strong-motion database for more advanced earthquake related studies.

KEYWORDS: magnitude, source-to-site distance, Turkish strong-motion database, ordinary / orthogonal regression.

1. INTRODUCTION

The recently compiled Turkish strong-motion database has indicated that the database mainly consists of five different magnitude scales: M_w, m_b, M_s, M_L and M_d (Erdoğan, 2008). Figure 1.1 shows the distribution of the earthquakes according to the aforementioned magnitude scales. The most common magnitude scale published by the agencies is M_d and this is followed by m_b. Although the most reliable magnitude scale is moment magnitude (it does not suffer from saturation as in the case of other scales), it is the least existing magnitude scale among the others.

An earthquake catalog containing homogeneous size estimations for all events is highly desirable for many earthquake related studies such as seismic hazard assessment, derivation of ground-motion prediction equations, determination of long-term seismic strain rates and nuclear activity verification. The main objective of this study is to derive earthquake magnitude conversion relationships to homogenize the Turkish strong-motion database in terms of moment magnitude (M_w). The proposed equations estimate moment magnitude (M_w) as a function of body-wave (m_b), surface-wave (M_s), local (M_L) and duration (M_d) magnitudes. Both ordinary and total least squares regression procedures are employed separately to compute these empirical relationships. The residual analysis is performed to evaluate the proposed conversion models. The examination of residual trends suggests that the models obtained from the ordinary least squares regression method yield unbiased estimations. The derived empirical relationships are compared with the other relevant studies presented in the literature. Our preliminary analyses indicate a good agreement between the proposed relationships and the results obtained from other studies. Within the context of this study, the relationships between different source-to-site distance metrics are also examined to further ascertain the reliability of Turkish strong-motion database for future engineering seismology and earthquake engineering related studies. Our observations in terms of R_{epi} (distance between the epicenter and the recording station), R_{hyp} (distance between the hypocenter and the station), R_{jb} (closest horizontal distance between the vertical projection of the rupture plane and recording station) and R_{rup} (closest distance from the recording station to the rupture plane) are consistent with the theoretically expected variations in these distance metrics.

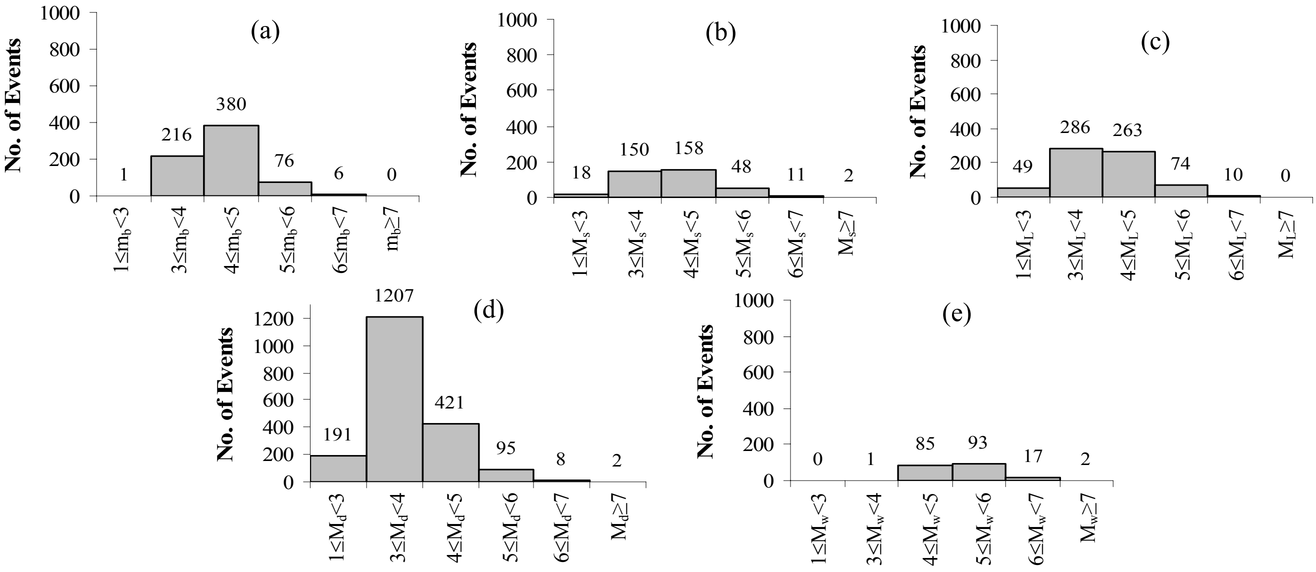

Figure 1.1 Histograms of the events in terms of different magnitude scales: (a) m_b, (b) M_s, (c) M_L, (d) M_d and (e) M_w. The earthquake magnitudes presented in the database are the reliable estimations compiled from different international or national seismic agencies that are evaluated according to a pre-determined priority level (Erdoğan, 2008).

2. MAGNITUDE CONVERSION MODELS

The non-homogeneous distribution between different magnitude scales requires empirical relationships for converting various reported magnitude scales to M_w. In this study, ordinary least squares and total least squares techniques are used to compute the relationships between M_w and the other magnitude scales. Table 2.1 presents the number of data used in the regression analysis for different magnitude pairs.

Table 2.1 Number of events used for establishing the empirical relationships for magnitude conversion

Dependent Variable	*Independent Variable*	*Number of Data*
M_w	m_b	196
M_w	M_s	177
M_w	M_L	156
M_w	M_d	182

Linear regression analyses are performed in this study. It computes the best-fitting line to a given set of points by minimizing the sum of the squares of the residuals or offsets of the points from the line (Draper and Smith, 1980). The ordinary least squares assumes that only the dependent (response) variable is random (Bormann et al., 2007). In other words, the measurement errors are introduced only to the dependent variable and its variance is different than zero (i.e. $\sigma_y^2>0$) while the variance of the independent (predictor) variable is considered as zero (i.e. $\sigma_x^2 \rightarrow 0$).

The total least squares method considers the measurement errors on both dependent and independent variables (Bormann et al., 2007; Carroll and Ruppert, 1996; Castellaro and Bormann, 2007; Castellaro et al., 2006). Unlike the ordinary least squares regression, in the orthogonal regression method, the line equation to be optimized for the best interpolation of the observed points is the weighted orthogonal distance. According to the Castellaro and Bormann (2007), the main obstacle in the application of orthogonal regression is that it requires the knowledge of the variance ratio ($\eta=\sigma_y^2/\sigma_x^2$) between the two variables. This ratio is usually not known because the global standard deviation for a given magnitude scale is meaningful when the corresponding magnitude is reported by at least three station estimates. In this study, the value of η is unknown and to overcome this problem η is set equal to 1 which formally coincides with the assumption that error ratios of different magnitudes are approximately equal. As stated by Castellaro et al. (2006), this is the conventional approach for unknown η.

The regression analyses are performed using the model described below:

$$y = \alpha + \beta x + \varepsilon \tag{2.1}$$

where α and β are the variables to be computed from the ordinary and total least squares methods. The term ε represents the unpredicted or unexplained variation in the response variable and it is conventionally called as “measurement error”. Tables 2.2 and 2.3 lists the computed α and β values through the ordinary and total least squares approaches, respectively. Note that M_w vs. M_s relationship is defined as a bilinear expression since the distribution of M_w vs. M_s scatters requires a bilinear fit to the data for $M_s<5.5$ and $M_s\geq5.5$. Similarly, some other studies in the literature (e.g. Ekström and Dziewonski, 1988; Bungum et al., 2003) consider a bilinear relationship for M_w vs. M_s regressions. The square of multiple correlation coefficients (R^2) are also presented in the tables to quantify how well the linear model assumption describes the overall variation of the data.

Table 2.2 Parameters computed in Eq. (1) using ordinary least squares approach

Parameter	M_w - m_b	M_w - M_s		M_w - M_L	M_w - M_d
		$M_s<5.5$	$M_s\geq5.5$		
α	-0.194	2.484	1.176	0.422	1.379
β	1.104	0.571	0.817	0.953	0.764
R^2	0.851	0.799	0.959	0.776	0.651

Table 2.3 Parameters computed in Eq. (1) using total least squares method

Parameter	M_w - m_b	M_w - M_s		M_w - M_L	M_w - M_d
		$M_s<5.5$	$M_s\geq5.5$		
α	-0.736	2.330	1.117	-0.283	0.561
β	1.216	0.607	0.826	1.094	0.934
R^2	0.842	0.796	0.959	0.758	0.619

The residual analysis is performed to evaluate the empirical equations obtained from the regression analyses. Figure 2.1 presents the residuals scatters for each model. Linear trend lines are also fitted to determine whether the estimations are biased towards conservative or non-conservative values. A significant slope in these linear trends will indicate biased estimations for the concerned functional model. The significance of the slopes in the linear trends is measured by the p-value statistics. A p-value less than 0.05 indicates that the null-hypothesis (slopes of the linear trends are not significant) can be rejected at the 5% significance level. As it is depicted from these plots, the variation in the ordinary least squares residuals is quite random as a function of independent magnitude parameter. The trends do not show any significant tendency towards either conservative or non-conservative estimations for this case (p-values are greater than 0.05). In terms of total least squares regression, slopes of the linear trends are different than zero (p-values are mostly less than 0.05) suggesting the existence of tendency towards either conservative or non-conservative estimations. The examination of residual trends and p-statistics (Figure 2.1) suggest that the ordinary least squares procedure results a more appropriate functional model than the total least squares approach. The poor performance of total least squares regression may stem from the η=1 assumption due to the lack of knowledge of the dependent and independent variable variances.

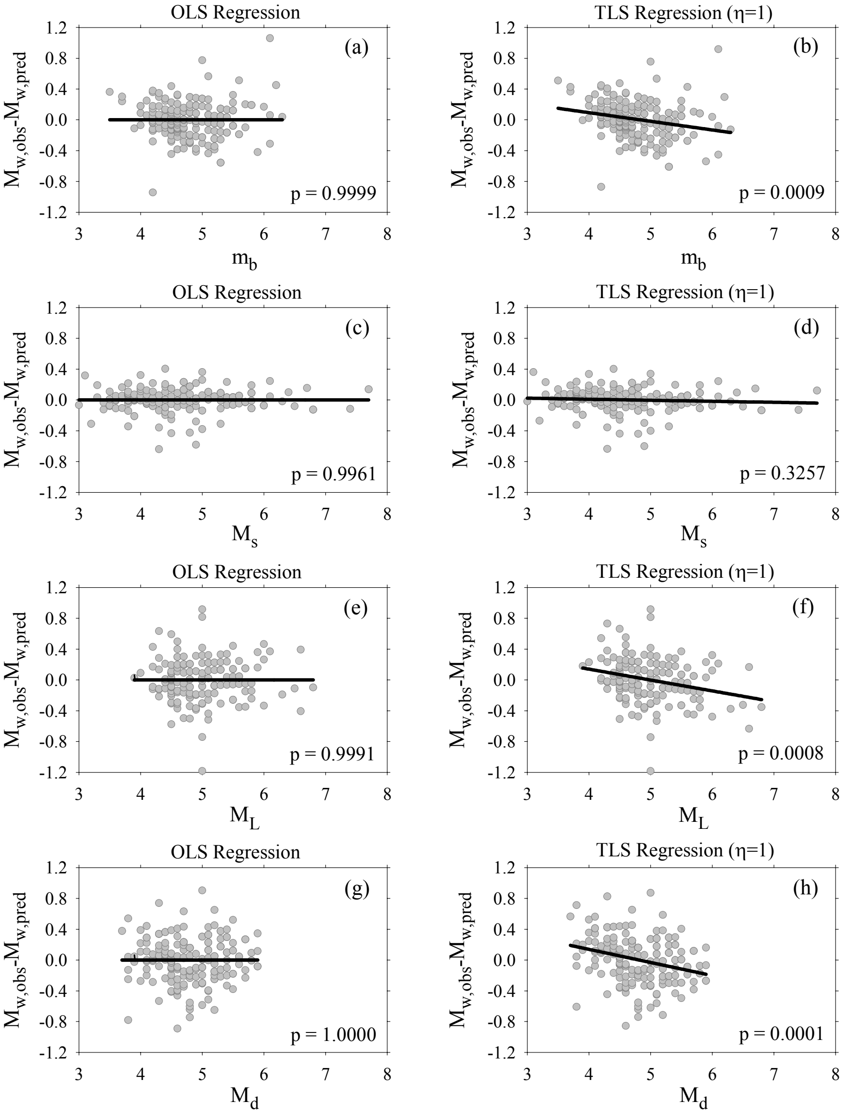

Figure 2.1 Residual scatters of each magnitude couple computed from ordinary least squares (OLS) and total least squares (TLS) regressions

Final empirical predictive equations for the moment magnitude conversion are given in Eqns. 2.2 to 2.5. Note that the equations also present the upper and lower magnitude bounds for each magnitude scale where the conversion equations are valid. M_w vs. M_s conversion relationship has the widest magnitude range. The empirical relationship between M_w vs. M_d is not applicable for $M_d>6$ due to the saturation of duration magnitude. The events with $M_d>6$ are not taken into consideration for the regression analysis to avoid the underestimation of M_w for large magnitude events.

$$M_w = 1.104m_b - 0.194, \quad 3.5 \le m_b \le 6.3 \tag{2.2}$$

$$M_w = 0.571M_s + 2.484, \quad 3.0 \le M_s < 5.5 \tag{2.3.a}$$

$$M_w = 0.817M_s + 1.176, \quad 5.5 \le M_s \le 7.7 \tag{2.3.b}$$

$$M_w = 0.953M_L + 0.422, \quad 3.9 \le M_L \le 6.8 \tag{2.4}$$

$$M_w = 0.764M_d + 1.379, \quad 3.7 \le M_d \le 6.0 \tag{2.5}$$

3. COMPARISON OF THE MODELS

The comparisons of magnitude conversion models are illustrated in Figure 3.1. Note that the models proposed in this study, Deniz (2006), Ulusay et al. (2004) and Kalafat et al. (2007) use national strong-motion datasets whereas the other relationships are derived from different earthquake databases. In the case of M_w vs. m_b relationship (Figure 3.1.a), Deniz (2006) introduces a remarkable difference with this study and the other proposed relationships. Castellaro et al. (2006), Ulusay et al. (2004) and Kalafat et al. (2007) estimate closer results to this study. Braunmiller et al. (2005) underestimates M_w values for $m_b<5$ when compared to the results of this study whereas its estimations start converging to the results of this study for $m_b>5$.

In general, our linear regression models result in fairly similar estimations with the other studies, especially for M_w vs. M_s conversion. The close examination of Figure 3.1.b shows that the relationship proposed by Deniz (2006) describes relatively different estimations with respect to this study. However, other models presented in Figure 3.1.b provide very similar results to this study.

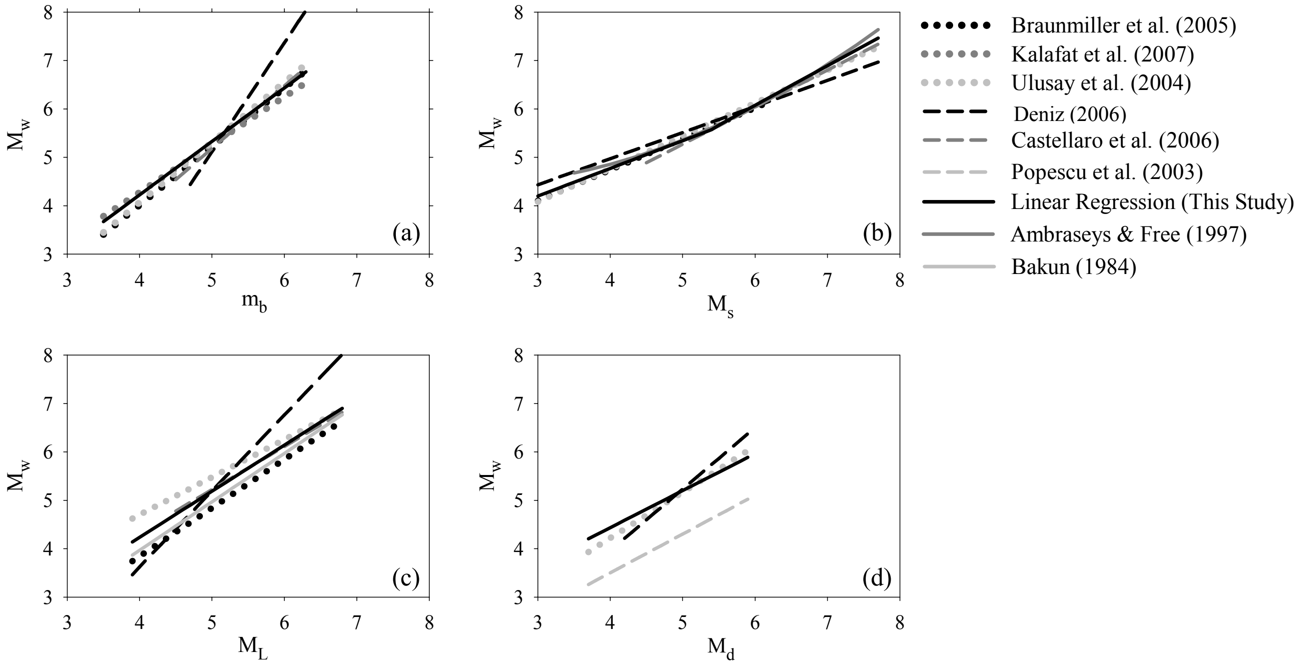

Figure 3.1 Comparisons of the magnitude conversion models with different studies in the literature

When M_w vs. M_L relationship is of concern, it is observed that there are considerable differences between the M_w estimations of this study and the other studies. As it is shown in Figure 3.1.c, except for Deniz (2006) the slopes of the M_w-M_L curves show similarities but the corresponding intercepts take quite different values. Braunmiller et al. (2005) and Bakun (1984) yield similar estimations of M_w from M_L. Our results define relatively conservative M_w estimations with respect to these two studies. The relationship proposed by Deniz (2006) yields significantly different results from all the other studies for this case as well. Note that M_L generally depends on the information disseminated by local seismic agencies. Therefore, discrepancies observed between this study and other international studies are expected due to the differences stemming from databases. However, the observed differences between this study and Deniz (2006) are unexpected since both studies have made use of the Turkish ground-motion database. Note that the relationships derived by Deniz (2006) generally calculate significantly different estimations with respect to other studies investigated here.

For M_w vs. M_d relationships (Figure 3.1.d), Ulusay et al. (2004) and this study establishes almost the same estimations whereas Deniz (2006) calculates larger M_w estimates for $M_d>5$ and underestimates M_w for $M_d<5$. The relationship proposed by Popescu et al. (2003) results in significantly lower M_w values with respect to this study despite having almost the same slope. If all plots are examined carefully, it is seen that Ulusay et al. (2004) gives closer results to this study particularly for $M_w>5$. This could be grossly attributed to the similar databases used by

Ulusay et al. (2004) and this study. The close trends between Kalafat et al. (2007) and this study for M_w vs. m_b relationship can also be explained in a similar manner. The overall picture suggests that the proposed magnitude conversion relationships are in a good agreement with the models proposed by other studies in the literature.

4. SOURCE-TO-SITE DISTANCE RELATIONSHIPS

Description of a consistent distance metric that defines the variation of ground-motion intensity measures (e.g. peak ground-motion values, spectral quantities etc.) is very important because these parameters constitute the primary information in the seismic hazard related studies. The reliability of source-to-site distance information in the recently compiled Turkish strong-motion database is investigated from the relationships between various distance metrics. We considered four different distance metrics that are widely used in seismic hazard studies: epicentral distance (R_{epi}), hypocentral distance (R_{hyp}), Joyner-Boore distance (R_{jb}) and rupture distance (R_{rup}). Figure 4.1 illustrates the comparisons for R_{jb} vs. R_{epi}, R_{rup} vs. R_{epi}, R_{rup} vs. R_{jb} and R_{rup} vs. R_{hyp} in terms of different magnitude intervals. Figure 4.1.a shows that R_{jb} attains smaller values than R_{epi}, especially for large magnitude events ($M_w \geq 6$). The discrepancies between R_{jb} and R_{epi} diminish for large distances except for a few events with $M_w > 7$. For large magnitude events ($M_w > 6$) and epicentral distances less than 40 km, the discrepancy between R_{jb} and R_{epi} becomes noticeable depending on the location of the hypocenter or the dimensions of the fault plane. The scatters in Figure 4.1.b reveal that R_{epi} generally tends to be larger than R_{rup} for increasing magnitude and decreasing distance but this trend is not as clear as in R_{jb} vs. R_{epi} scatters. This might be due to the event-dependent variation in depth as well as the dipping angle that play important roles in the calculation of R_{rup}.

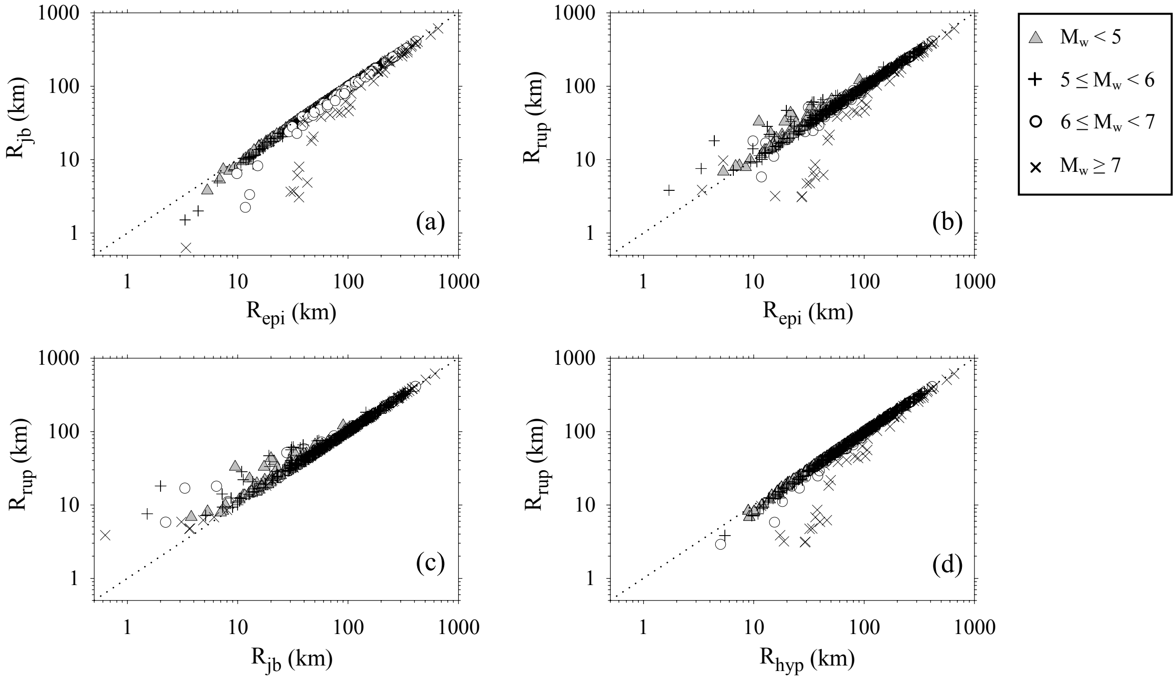

Figure 4.1 Comparison of source-to-site distance metrics in terms of different magnitude intervals

For R_{rup} vs. R_{jb} comparisons (Figure 4.1.c), it is observed that regardless of the variations in magnitude, R_{jb} is generally smaller than R_{rup} for R_{jb}<20 km. For larger distances, R_{jb} is approximately equal to R_{rup}, which means that after approximately 20-30 km, the differences in the definitions of these distance metrics become immaterial. This can be attributed to the importance of earthquake depth that marks the major differences between R_{jb} and R_{rup} for sites close to the fault rupture. As the recording station is located away from the source, depth reduces its significance and consequently $R_{jb} \approx R_{rup}$. Note that the database mainly constitutes of shallow earthquakes and this feature facilitates our observations on the similarity of R_{rup} and R_{jb} values at intermediate and large distances. It is depicted from Figure 4.1.d that R_{hyp} is always equal to or greater than R_{rup}. For $M_w > 7$ events and close to intermediate distances (10 km<R_{hyp}<50 km), the discrepancy between these two distance metrics becomes larger. The increase in discrepancy may stem from the increased dimensions of ruptured fault plane at large magnitude events. Note that for events with M_w<7, regardless of the distance value, R_{hyp} is approximately equal to R_{rup}.

5. LIMITS OF THE TURKISH STRONG-MOTION DATABASE

The recently compiled Turkish strong-motion dataset currently contains 488 records that are "usable" for conducting detailed earthquake engineering and engineering seismology related studies. The term "usable" describes the high quality records having reliable M_w, site class, faulting style and source-to-site distance information. The site class information of these records is obtained from the shear-wave velocity (V_s) profiles of the recording stations that are calculated via MASW method (Yilmaz et al., 2008). Figure 5.1.a presents M_w-R_{jb} scatter of these records. The scatter data is classified according to NEHRP site class information (BSSC, 2003). Note that there is very few ground motions recorded at NEHPR B sites. The dataset can be extended further using the findings of this paper for future studies. Figure 5.1.b shows the extended M_w-R_{jb} scatters when the empirical magnitude conversions and source-to-site distance observations of this study are implemented. While realizing the magnitude conversions, the highest priority is given to the M_w vs. M_s relationship. In the absence of M_s information, the order of preference among the conversion relationships is: M_w vs. m_b, M_w vs. M_L and M_w vs. M_d. Lesser reliability of local and duration magnitudes with respect to the surface- and body-wave magnitudes as well as the smallest dispersion in the M_w vs. M_s relationship (Figure 2.1) constitute the major reasons for the presented priority. A total of 849 good quality waveforms can be added to the dataset when the manipulations discussed above are performed. This database exhibits a good resolution between $3.5 \leq M_w \leq 6.5$ and $1\ \text{km} \leq R_{jb} \leq 200$ km. The scatters presented reveal that there is a certain magnitude gap between 6.5 and 7.0 in our database. The database contains a fairly good amount of records for $M_w \geq 7.0$. Note that similar discussions can also be made for M_w-R_{rup} scatters. We do not present the extent of our database in terms of M_w-R_{rup} information due to the space limitations. The reader is referred to Erdoğan (2008) for a full discussion on the general features of Turkish strong-motion database.

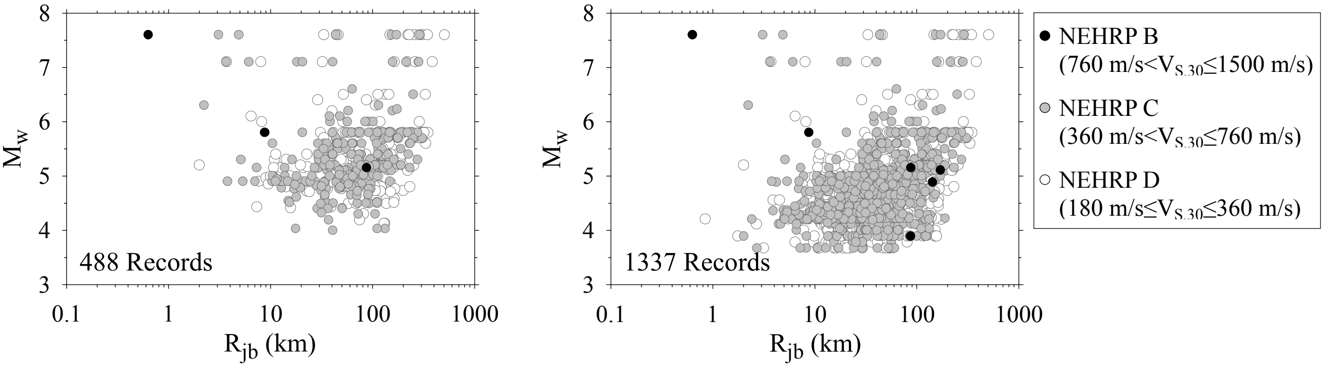

Figure 5.1 M_w vs. R_{jb} scatter plots of the "usable" records for (a) the current and (b) the extended database. For the extended database we used magnitude conversion relationships of this study. Moreover for small magnitude events ($M_w \leq 6$) R_{jb} was approximate as R_{epi} based on the observations highlighted in Section 4.

6. CONCLUSIONS

Empirical relationships between M_w and other classical magnitude scales (m_b, M_s, M_L and M_d) are derived to homogenize the recently compiled Turkish strong-motion database in terms of M_w. The proposed empirical equations are compared with the other studies in the literature. The comparisons indicate that our models result in fairly similar estimations with the other studies, especially for M_w vs. M_s and M_w vs. m_b conversions. The relationships between various source-to-site distance metrics (R_{epi}, R_{hyp}, R_{jb} and R_{rup}) are also investigated within the context of Turkish strong-motion database. The observations presented are consistent with the theoretically expected behavior of the distance metrics investigated. Based on the discussions presented throughout the text, we showed that one can obtain more than 1300 homogenous records in terms of magnitude, distance and site class information from the Turkish strong-motion database. Such a reliable database will certainly enhance the seismic risk and seismic hazard studies in Turkey. It is also believed that this database will constitute valuable information for the worldwide global strong-motion databanks.

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