

# Estimation of Local Spectral Density of Seismogram by Orthogonal Hilbert-Huang Transform and Application of the Spectral Density

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## **ABSTRACT :**

It is necessary to describe earthquake ground motion with local spectral density, which is time-dependent and can reflect the non-stationary characteristics. Firstly, the paper introduces the approach using HHT to estimate local spectral density, which makes the local spectral density estimation very convenient by HHT transform. Secondly, the approach using the orthogonal HHT to estimate local spectral density is provided in the paper as a complementarity of HHT. Comparisons of estimating precision and speed are carried out among orthogonal HHT, HHT, STFT and Multifilter. The study shows that orthogonal HHT is a universal method avoiding leakage of energy compared with conventional HHT. So it is an efficient method to estimate local spectral density. Thirdly, the study illustrates that the local power spectra estimated by orthogonal HHT are able to reveal the temporal-frequency energy distribution for earthquake recordings precisely and clearly. Finally, integrated with the orthogonal HHT estimating local spectral density of earthquake ground motion, time-dependent power spectra is used as target power spectrum and the trigonometric series is used to simulate record of non-stationary earthquake ground motion.

**KEYWORDS:** non-stationary random process, local spectral density estimation, Orthogonal HHT, simulation

## **1. INTRODUCE**

Dr. Norden. E. Huang et al<sup>[1]</sup> developed a kind of theory and a calculating approach in 1998. This approach is particularly suitable for the analyzing and handling the non-stationary data, which called Hilbert-Huang Transform(HHT) including empirical mode decomposition (EMD) and Hilbert spectral analysis. The creation of this approach is introducing Intrinsic Mode Functions (IMF). The approach can decompose the complicated original signals into several simple components called IMF. It has a very good Hilbert transform feature and enables instantaneous temporal-frequency spectrum to be calculated, and it has a clearly physical significance. Based on the local feature of the signals, it is suitable for handling non-stationary signals.

However, HHT cannot keep energy conservation when EMD is carried out. So, this paper provides orthogonal HHT(OHHT) approach to avoid energy leakage. This study seeks to use OHHT approach to estimate the local spectral density of the non-stationary earthquake ground motion and analyze earthquake motion recordings. Time-dependent power spectrum is used as target power spectrum and the trigonometric series is used to simulate record of non-stationary earthquake ground motion. The objective of the study is to reveal useful information from ground motion recordings that might be either hidden or distorted by conventional data-analysis approaches, and to simulate a lot of earthquake samples having same statistical characteristic as well as the real record benefiting to history analysis of structural random response.

# 2. ESTIMATION OF THE LOCAL SPECTRAL DENSITY BY HHT



Huang et al proposed EMD decomposition approach. For given data, the Hilbert transform, c(t), is defined as:

$$\hat{c}\left(t\right) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{c(\tau)}{t-\tau} d\tau$$
(2.1)

Where, P denotes the Cauchy principal value and c(t) is IMF component. With this definition, c(t) and c(t) can be combined to form the analytical signal z(t), given by:

$$z(t) = c(t) + i c(t) = a(t) e^{i\theta(t)}$$
(2.2)

Where, time-dependent amplitude a(t) and phase  $\theta(t)$  are defined as:

$$a(t) = \sqrt{c^{2}(t) + c^{2}(t)}$$
(2.3)

$$\theta(t) = \arctan\left(\hat{c}(t)/c(t)\right)$$
(2.4)

Hilbert amplitude spectrum  $H(\omega, t)$  can be shown in the frequency-time plane, which called Hilbert spectrum, as:

$$H(\omega,t) = \operatorname{Re}\sum_{j=1}^{n} a_{j}(t) e^{i\int \omega_{t}(t)dt}$$
(2.5)

Based on the expandedness of intrinsic modal function, signal describing approach is given. Its amplitude and frequency are changeable unlike fixed amplitude and fixed frequency in Fourier transform.  $H(\omega, t)$  accurately describes the varying rule of signals amplitude according time and frequency. So when energy is taken into account, the local power spectral density of original random signals and Hilbert spectrum  $H(\omega, t)$  have the following relationship:

$$S(\omega,t) = \left|H(\omega,t)\right|^2 / 2 \tag{2.6}$$

As described above, based on the temporal-frequency feature of the HHT, the energy of random process or the samples has been localized. The energy is expanded on the plane of time and frequency, as equation (2.6), then, the local power spectral density (local feature of the temporal-frequency) is got. The simple relationship between Hilbert spectrum obtained from IMF component of signals and the local power spectral density makes the local spectral density estimation very convenient by HHT. Hu et al<sup>[2]</sup> proposed the local spectrum density of the earthquake ground motion with the help of HHT estimation approach and also used short time Fourier transform (STFT) and Multi-filter to estimate the local spectrum density for the four earthquake waves. By comparison, it can be found out that HHT estimation approach for the local spectrum density has better accuracy in time domain and frequency domain, meanwhile its calculating speed is apparently faster than that of the other two approaches. So it is concluded that HHT estimation is an efficient approach to estimate the local spectrum density for non-stationary earthquake ground motion.



## 3. ORTHOGONAL HHT APPROACH

Analysis above is based on the hypothesis that IMF got by EMD could re-compose original signal and there are orthogonality among IMF components. Therefore we can use HHT as an available approach of temporal-frequency analysis to estimate the local spectral density of non-stationary earthquake ground motion. But from the characteristic of temporal-frequency analysis and basic theory we know that the frequency spectrum of signals will have crossover item. The frequency spectrum density of two signals' summation is not the sum of frequency spectrum density of two signals<sup>[3]</sup>.

Strong earthquake ground motion as complicated signal is consist of many different frequency components. There are many crossover items in its frequency spectrum. The local temporal-frequency characteristic of HHT makes it possible to become a good approach in estimating the local spectrum of non-stationary signal. However, through further research, we find the EMD approach proposed by Huang et al can't ensure strict orthogonality in theory, and only indicate approximately orthogonality among each IMF in numerical value. Actually from some numerical examples we can find the degree of orthogonality is relatively imprecise. Leakage of energy will be happened if we directly use HHT to estimate the local spectral density of non-stationary earthquake ground motion, and then the local spectral density estimated by HHT cannot be regarded as the time-dependent spectral density of original earthquake signals. So the approach using HHT to estimate local spectral density of non-stationary earthquake waves is only suitable to some special seismic waves<sup>[2]</sup>. Based on this reason, this study will use orthogonal HHT to estimate the local spectral density of non-stationary earthquake ground motion. The problem of energy leakage will be solved wonderfully, and OHHT can be used as a high-efficiency estimation approach of time-dependent power spectrum.

Huang et al developed EMD approach, and defined overall orthogonal index IOT and the orthogonal index  $IO_{jk}$  between two IMFs. While there is totally orthogonality among each IMF component, the value of IOT and  $IO_{jk}$  should be zero. Due to conventional HHT has some problems, EMD don't have orthogonality in theory. The orthogonal index among each IMF component is common in magnitude degree of  $10^{-2}$  to  $10^{-3}$ . From the following example, we can notice that: using EMD to decompose earthquake time-history signals, the error will lead to quite seriously energy leakage. In order to ensure there is no energy leakage while decomposing, we must make sure that IMF components decomposed by EMD have strict orthogonality.

Through orthogonal treatment to intrinsic mode functions got by EMD<sup>[4]</sup>, we can obtain completely orthogonal IMF components as follows:

(1) Applying EMD to original signals, *n* initial IMF components  $c'_1(t), c'_2(t) \cdots c'_n(t)$  and final residue  $r_n(t)$  can be obtained. Last IMF component  $c'_n(t)$  can be written as  $c_1(t)$ , i.e.  $c_1(t) = c'_n(t)$ .  $c_1(t)$  can be named the first step orthogonal IMF component of time-history signal X(t);

(2) In order to get the second orthogonal IMF component of time-history signal X(t), we should remove all  $c_1(t)$  components included in  $c_{n-1}$ , as:

$$c_{2}(t) = c_{n-1}(t) - \beta_{21}c_{1}(t)$$
(3.1)

Where,  $\beta_{21}$  is called the orthogonal coefficient between  $c'_{n-1}(t)$  and  $c_1(t)$ .  $c_2(t)$  is called the second orthogonal IMF component of time-history signal X(t). In order to obtain  $\beta_{21}$ , we can multiply  $c_1(t)$  to

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both sides of the equation (3.1) and then make time integral. Making use of the orthogonality between  $c_2(t)$  and  $c_1(t)$ , we obtain  $\beta_{21}$ , as:

$$\int_{0}^{T} c_{1}(t) c_{2}(t) dt = \int_{0}^{T} c_{n-1}(t) c_{1}(t) dt - \beta_{21} \int_{0}^{T} c_{1}^{2}(t) dt = 0$$
(3.2)

$$\beta_{21} = \frac{\int_0^T c_{n-1}(t)c_1(t)dt}{\int_0^T c_1^2(t)dt}$$
(3.3)

expressed in discrete form, as:

$$\beta_{21} = \frac{\left\{c_{n-1}^{'}\right\}\left\{c_{1}\right\}^{T}}{\left\{c_{1}\right\}\left\{c_{1}\right\}^{T}} = \frac{\sum_{i=1}^{N} c_{n-1,i} c_{1i}}{\sum_{i=1}^{N} c_{1i}^{2}}$$
(3.4)

(3) Just as the same approach described above, removing orthogonal IMF components, included in the (n-j)th original IMF component of time-history signal X(t), from the first one to the jth one, we can get the (j+1)th orthogonal IMF component  $c_{j+1}(t)(j=1,\dots,n-1)$  of time-history signal X(t). Therefore,

$$c_{j+1}(t) = c_{n-j}(t) - \sum_{i=1}^{j} \beta_{j+1,i} c_i(t)$$
(3.5)

In order to obtain  $\beta_{j+1,i}$ , we multiply  $c_k(t), (k \le j)$  to both sides of the equation (3.5) and make time integral. Noticing orthogonality between  $c_k(t), c_i(t)(i \ne k)$  and  $c_{j+1}(t)$ , we can obtain:

$$\int_{0}^{T} c_{j+1}(t) c_{k}(t) dt = \int_{0}^{T} c_{n-j}(t) c_{k}(t) dt - \sum_{i=1}^{j} \beta_{j+1,i} \int_{0}^{T} c_{k}(t) c_{i}(t) dt = 0$$
(3.6)

when i = k, we can obtain  $\beta_{i+1,i}$ , as:

$$\beta_{j+1,i} = \frac{\int_0^T c_{n-j}(t) c_i(t) dt}{\int_0^T c_i^2(t) dt}$$
(3.7)

Expressed in discrete form, as:

$$\beta_{j+1,i} = \frac{\left\{c_{n-j}\right\}\left\{c_{i}\right\}^{T}}{\left\{c_{i}\right\}\left\{c_{i}\right\}^{T}} = \frac{\sum_{m=1}^{N} c_{n-j,m}^{'} c_{i,m}}{\sum_{m=1}^{N} c_{i,m}^{2}}$$
(3.8)

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After compute above, X(t) is decomposed to the following form, as:

$$X(t) = \sum_{j=1}^{n} c_{j}^{*}(t) + r_{n}(t) = \sum_{j=1}^{n} a_{j}c_{j}(t) + r_{n}(t)$$
(3.9)

In which:

$$a_j = \sum_{i=j}^n \beta_{i,j} (j = 1, 2, \dots, n), \quad \beta_{i,j} = 1 (i = j).$$

So, each component of  $c_j(t)(j=1,2,...,n)$  is completely orthogonal to another one, and among  $c_j^*(t)(j=1,2,...,n)$  got through linear transform from  $c_j(t)(j=1,2,...,n)$ , every component is also completely orthogonal to each other. Thus, the signal X(t) is decomposed to the sum of n-orthogonal IMF components  $c_j^*(t)(j=1,2,...,n)$  and final residue  $r_n(t)$ . The process of obtaining orthogonal IMF component does not change the original sifting process(EMD), and the orthogonal process is a mathematics reconstruction process of original IMF. So, the orthogonal process has strict orthogonality, at the same time, it can keep properties of IMF.

For orthogonal IMF components  $c^*(t)$ , using content in section 2, we can get local power spectral density reflecting the temporal-frequency characteristic of original random signals.

#### 4. Local spectral density of earthquake recording estimated by OHHT

#### 4.1. The Analysis of The Time-dependent Characteristic of Local Spectral Density

Three famous seismic recording are estimated by orthogonal HHT. The first one is EI Centro N-S acceleration recording, the second one is acceleration recording of Taft earthquake, the last one is acceleration recording of Kobe earthquake in Japan. In order to contrast with HHT estimation approach<sup>[2]</sup>, we also use STFT and Kameda Multifilter to estimate the local spectral density of these three earthquake ground motion recordings.

Multifilter in this paper adopt the estimation expressions of sample local spectral density which are proposed by  $Kameda^{[5]}_{1}$ , as:

$$G(t,\omega_0) \approx \frac{2\beta\omega_0^3 r_{\omega 0}^2(t)}{\pi}$$
(4.1)

$$r_{\omega 0}^{2}(t) = y^{2}(t) + \frac{\dot{y}^{2}(t)}{\omega_{0}^{2}}$$
(4.2)

In these two expressions, y(t),  $\dot{y}^2(t)$  is non-stationary random comparatively displacement and comparatively velocity of the oscillator respectively,  $\beta$  is damp coefficient,  $\omega_0$  is natural frequency of oscillator. Where  $\beta = 0.05$ .  $\omega_0$  is respectively changed according to equal interval chosen to be 3 from 3 to 150 and equal interval chosen to be 0.5 from 0.5 to 150, so we use  $\Delta \omega = 3$  and  $\Delta \omega = 0.5$  to estimate local spectral density of these three waves.

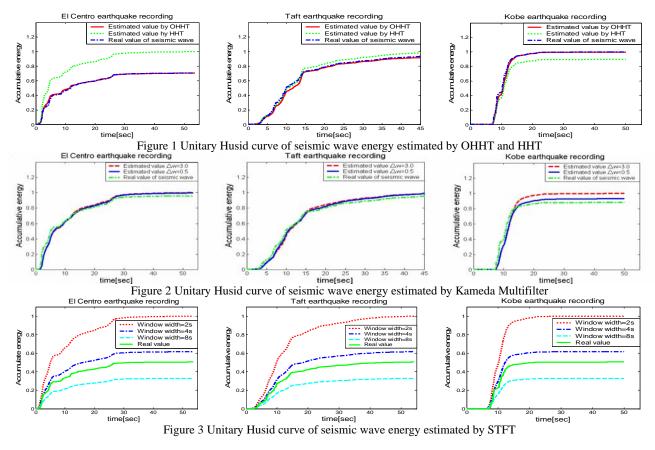


The estimation expression of local spectral density by short time Fourier transform (STFT)<sup>[6]</sup> is given as:

$$S(\omega,t;w) = E\left|\int_{-\infty}^{\infty} w(t-\tau) X(\tau) e^{-i\omega\tau} d\tau\right|^{2}$$
(4.3)

In this expression, window function is Gauss window, and we respectively use window width as 2 seconds, 4 seconds, and 8 seconds to estimate local spectral density of these three waves.

Accumulating every- moment energy and all of before, we can get energy Husid diagram<sup>[7]</sup> of local spectral density estimated by different approach. By contrasting this Husid diagram and the energy Husid diagram of actual seismic waves, we can quantitatively test the accuracy of seismic non-stationary amplitude in local spectral density estimated by different approach.

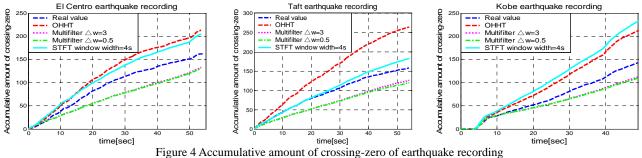


From Fig.1, we can observe the simulating value in Husid curves obtained from the local spectral density estimated by OHHT coincides with the true value of the seismic waves quite well, and the total energy of simulating value is identical to the total energy of true value. We can conclude that there is no energy leakage when OHHT approach is used to estimate local spectral density and the time distribution of seismic energy estimated by OHHT has high accuracy. Estimating these three waves by HHT all have some energy leakage, moreover, the energy leakage of EI Centro wave arrives more than 30%. From Fig.2, we can observe that the energy leakage of Multifilter is less than HHT. However, the accuracy of Multifilter is less than OHHT and calculating time of Multifilter is much more than OHHT estimation, so Multifilter is not an efficient approach to estimate local power spectral density. From Fig.3, we can observe that the energy estimated by STFT is far away from real value and due to the uncertain principle of the spectrum, it is impossible to get high resolution rates in time domain and frequency domain at the same time, so STFT is not a good approach used to estimate local power spectral density.



## 4.2. The Frequency Local Characteristic Analysis of Local Spectral Density

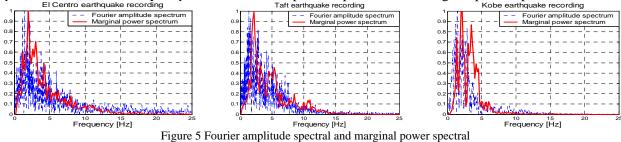
The target of the crossing-zero rates <sup>[8]</sup> may approximately reflect the characteristic of time-varying of frequency included by earthquake waves. The more the crossing-zero rates is, the more high frequency components there are in the earthquake waves. For stationary random process, the crossing-zero rates equals to the average frequency rates. The index of amount of accumulative crossing-zero is used to qualitatively reflect the frequency-varying characteristic of local spectral density. We pay more attention to check the shape of the curves which reflects the amount of accumulation, and we know the curves reflecting the amount of accumulative crossing-zero is non-stationary can not be straight lines. The amount of accumulative amount of reflect frequency changing feature of local spectral density, shown as Fig.4. From the curves of accumulative amount of crossing-zero, we can observe that OHHT estimation approach reflects the frequency feature of these three non-stationary seismic waves quite well. However, the three curves of the accumulative of crossing-zero by the Multifilter are all approximate straight line or broken line, so it is hard to precisely reflect the frequency local characteristics of seismic waves by Multifilter.



With the local power spectrum estimated, we can define marginal power spectrum, as:

$$S_m(\omega) = \int_0^T S(\omega, t) dt$$
(4.4)

The marginal power spectrum provides an energy measure to each frequency, expressing the accumulative energy corresponding to each frequency in the whole time. From the contrast between marginal power spectrum and Fourier amplitude spectrum, we can examine the frequency-varying characteristic of the estimated local spectral density. Because we don't compare their amplitudes and only focus on the distribution of their frequencies, we make unitary disposal to Fourier amplitude spectrum and marginal power spectrum, as:



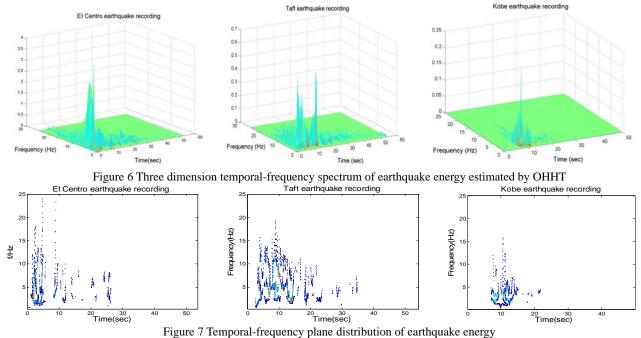
From Fig.5, we can observe that the range of frequency distribution of marginal power spectrum and Fourier amplitude spectrum are conformable, which also indicates that the estimated local spectral density by OHHT can reflect the characteristic of frequency-varying very well.

### 4.3. The Applications of Local Spectral Density Estimated by OHHT

# The 14<sup><sup>th</sup></sup> World Conference on Earthquake Engineering October 12-17, 2008, Beijing, China



As one of the applications of local spectral density of non-stationary earthquake ground motion, the local spectral density got from above discussion can be used to study the energy distribution and alter situation of energy of earthquake wave very conveniently. We still use above-used three earthquake recording samples as example, Fig.6 and Fig.7 gives three dimension surface figures and two dimension isoline respectively, which are the instantaneous acceleration energy temporal-frequency distribution of three earthquake ground motion samples.



From Fig.6 and Fig.7 we can observe the energy of El Centro seismic wave mainly centralize between 1.1 seconds and 28 seconds, and energy distribution does not vary stationary with time. The energy distribution does not simply change from small to large and then become small again, but presents many energy amplitudes. There are some large energy amplitudes near 2 seconds, 4.5 seconds, 7 seconds, 11.6 seconds and 25.5 seconds. The maximal energy amplitude emerges in 1.94 seconds and it is more than 3.5 times to other energy amplitudes. We can also find there are many small energy amplitudes between 11.6 seconds and 25.5 seconds. Meanwhile, the figures show that the energy contained in the earthquake ground motion whose frequencies below 15Hz occupy a majority of the whole earthquake energy. The energy of earthquake ground motion is the largest when its frequency is near 2.7Hz. The energy of earthquake will decrease step by step with the increase of frequency.

The energy of Taft wave mainly centralize between 2 seconds and 15.4 seconds. There are some prominent energy amplitudes near 3.5 seconds, 6.8 seconds, 10 seconds, 14 seconds and the maximal energy amplitude emerges in 14 seconds. After that time the energy amplitude and duration of earthquake ground motion will decrease rapidly. The components of earthquake ground motion whose frequency below 10Hz contain a majority of earthquake energy and the energy of earthquake ground motion whose frequency is near 2.3Hz arrives the maximum. The energy of earthquake will decrease step by step with the increase of frequency.

The energy of Kobe wave mainly centralize between 7.5 seconds and 15.5 seconds. The maximal amplitude emerges in 10.76 seconds. Besides, near 8.6 seconds, 12 seconds, 13 seconds, there also emerge large energy amplitude. A large proportion of energy of earthquake ground motion was contained in the ground motions components below 5Hz. The energy of earthquake ground motion whose frequency is near 2.3Hz arrives the maximum. The energy of Kobe wave will also decrease step by step with the increase of frequency.

From the contrast of these three earthquake waves, we can see that the energy distribution varying with time of



El Centro wave is quite large. The temporal range of energy centralizes between 26.9 seconds. The energy rapidly arrives at maximum in a very short time and then decreases gradually. There are still some prominent energy amplitudes. However, the range of energy distribution of Taft wave is little than that of El Centro, The temporal range of energy centralizes between 13.4 seconds and the beginning part of energy increases gradually with time. Different from El Centro wave whose energy decrease gradually, the energy of Taft wave attenuates rapidly after arriving the largest energy amplitude at 14 seconds. The energy distribution range along with time of Kobe wave is the least and energy only centralize between 7 seconds and 15 seconds. Besides, from the relationship between energy and frequency, we can conclude that in the three seismic waves, low-frequency motion components occupy the highest percentage in Kobe wave; that of Taft wave takes the second place, and that of El Centro wave is the least.

Acceleration energy varying and distributing rule of these three seismic waves was given. We can also make baseline offset correction to acceleration record and then the time-history of displacement is obtained by integral. The displacement energy varying rule of earthquake waves can be analyzed using displacement local power spectrum obtained by the approach given above. So we can conclude that using local spectral density to study the energy varying of earthquake waves can reflect the temporal-frequency characteristic of earthquake ground motion, making it more convenient to become the bases of aseismic analysis and design of structure.

## 5. THE SIMULATION OF SEISMIC WAVE BY OHHT POWER SPECTRUM

Using special target spectrum to simulate earthquake ground motion process is an important research direction in earthquake engineering. The usually used target spectrum is response spectrum or power spectrum according with some actual earthquake ground motion records. Because response spectrum can only reflect the process of earthquake ground motion indirectly and can not exactly express the temporal-frequency non-stationary characteristic of earthquake ground motion, which is only suitable for amplitude non-stationary processes. In order to actually reflect the non-stationary characteristic of earthquake ground motion, we can obtain the earthquake ground motion samples more according with the temporal-frequency characteristic of original earthquake recordings. So the paper will research on the simulation of non-stationary earthquake ground motion by OHHT time-dependent power spectrum.

Scanlan and Sachs<sup>[9]</sup> proposed using trigonometric series and evolutionary spectrum to simulate the analysis expression of seismic wave, as:

$$\ddot{x}_{g}(t) = \sum_{k=1}^{n} \sqrt{2f(t, \omega_{k})\Delta\omega} \cos(\omega_{k}t + \Phi_{k})$$
(5.1)

In order to simulate the temporal-frequency non-stationary characteristic of earthquake recordings, where  $f(t, \omega_k)$  is time-dependent spectrum,  $\Delta \omega = (\omega_k - \omega_{k-1})$  is the frequency interval,  $\Phi_k$  denotes independent random phase angles distributed uniformly between 0 and  $2\pi$ . Analyzing seismic wave by commonly used Multifilter or STFT has poor precision. Besides, if very small  $\Delta \omega$  was used in order to improve precision while we estimate time-dependent spectrum, a great lot of time should be spent and it is difficult to satisfy the demand of simulating many seismic waves at a short time. This paper uses OHHT power spectrum as time-dependent spectrum to simulate seismic waves. Using OHHT can estimate the time-dependent spectrum quickly and accurately which can totally reflect the temporal-frequency non-stationary characteristic of seismic waves. The precision of OHHT is apparently better than STFT and Multifilter, moreover, the analysis speed of OHHT is also faster than STFT and Multifilter. OHHT makes it possible to simulate a lot of non-stationary seismic waves.

We use El Centro recording and a Landers recording as a target to simulate earthquake ground motion. Fig.8 shows the recording of El Centro and three simulating samples. Fig.9 includes Landers earthquake recording

# The 14<sup><sup>th</sup></sup> World Conference on Earthquake Engineering October 12-17, 2008, Beijing, China



and three simulating samples.

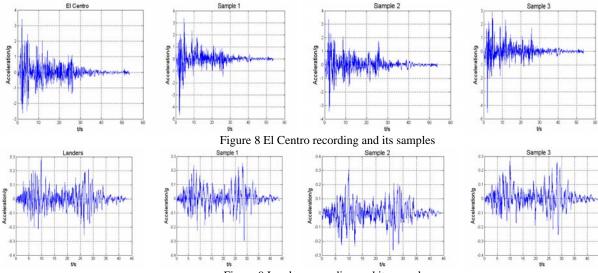
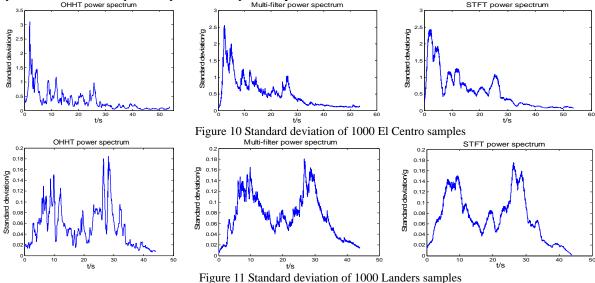


Figure 9 Landers recording and its samples

Fig.8 and Fig.9 show that generated sample accelerate processes of these two recordings can both reflect the trend of amplitude changing along with time, which is the same as original earthquake recordings. Especially the amplitude and distribution of two groups of acceleration peaks in Landers recording are all embodied exactly. The amplitude of peaks of each sample may be different from that of original recordings, but we can study precision of simulated amplitude of peaks based on statistics. At the same time, we use Multifilter power spectrum and STFT power spectrum as target spectrum to simulate seismic waves, and to produce 1000 samples respectively. This paper, as space is limited, does not give the sample curves. Fig.10 and Fig.11 show the standard deviation of 1000 sample processes produced by OHHT simulating power spectrum, Multifilter spectrum, and STFT power spectrum respectively.



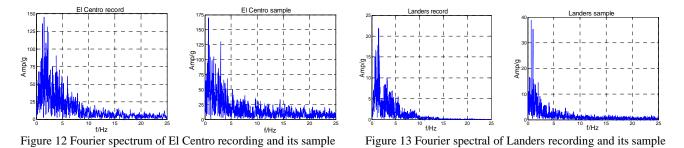
For each specific sample process of the same seismic recording, the amount of peaks, the amplitude of peaks, and the position of the peaks are of great difference. Take the amplitude of peaks as example, to El Centro, for sample processes estimated by OHHT power spectrum, the peaks in three sample processes (Fig.8) are 2.09, 1.63, 1.46 times of its standard deviation respectively; for sample processes by Multifilter power spectrum, the peaks in 3 sample processes are 2.20, 3.00, 2.69 times of its standard deviation; for sample processes by STFT power spectrum, the peaks in 3 sample processes are 2.72, 1.93, 2.99 times of its standard deviation. To Landers, as it has two groups of peaks, for sample processes by OHHT power spectrum, the first

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group of 3 sample processes (Fig.9) are 2.38, 2.14, 1.83 times of its standard deviation, and the peaks in the second group are 2.17, 1.84, 1.82 times of its standard deviation; for sample processes by Multifilter power spectrum, the peaks in the first group of 3 sample processes are 3.55, 2.12, 2.52 times of its standard deviation, and the peaks in the second group are 3.34, 2.08, 2.56 times of its standard deviation; for sample processes by STFT power spectrum, the peaks in the first group of 3 sample processes are 2.96, 2.32, 2.28 times of its standard deviation. From the scattering extent of the peaks in the sample processes, it can be conclude that generated sample ground motion processes are quite good.

From above, we can observe that the ground motion processes generated by orthogonal HHT power spectrum as target spectrum and trigonometric series can accurately reflect the amplitude non-stationary characteristics of original earthquake recordings.



Besides, from Fig.8 and Fig.9, we can also observe directly that the samples frequency changing trend along with time is accordance with original recordings. Fig.12 and Fig.13 are Fourier amplitude spectrums of the two recordings and a random simulating sample respectively. From these two figures, we can observe that the frequency spectrum characteristic of samples is quite close to that of the original recordings. In order to better research the temporal-frequency non-stationary characteristics of strong earthquake recordings and samples, the Hilbert spectrum of the two earthquake recordings and their samples in temporal-frequency plane are given respectively in Fig.14 and Fig.15, and then energy distribution in temporal-frequency plane can be researched. From these figures, we can notice that Hilbert temporal-frequency spectrum distributions of samples are quite the same to the original recordings, and samples can accurately reflect the temporal-frequency non-stationary characteristics of original strong earthquake recordings.

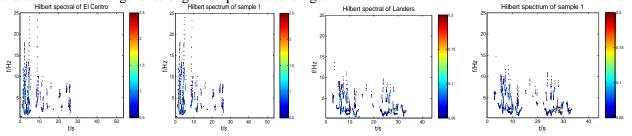


Figure 14 Hilbert spectrum of El Centro recording and its sample 1

Figure 15 Hilbert spectrum of Landers recording and its sample 1

## 6. CONCLUSIONS

This paper introduces the method of HHT and OHHT for earthquake data analysis and proposes approach estimating local spectral density of earthquake ground motion by HHT and OHHT. It reveals the following:

1. Because there is no strict orthogonality among IMF components decomposed by conventional EMD, leakage of energy may be happened if we directly use HHT to estimate the local spectral density of non-stationary earthquake ground motion. The paper proposes the orthogonal HHT method avoiding leakage of energy to estimate local spectral density of seismic wave. OHHT is a universal approach and is suitable for all non-stationary earthquake signals.



2. Using local spectral density estimated by OHHT to study the energy varying of earthquake waves can reflect the time-frequency characteristic of earthquake ground motion, making it more convenient become the bases of aseismic analysis and structure design.

3. This paper simulates non-stationary seismic waves through OHHT power spectrum and triangle progression. Orthogonal HHT power spectrum can wonderfully reflect the non-stationary characteristic of original earthquake recordings no matter in time domain or in frequency domain. So when we use OHHT power spectrum as target spectrum to produce samples, the amplitude and frequency of these samples are non-stationary. Any time-dependent spectrum of sample process is not always accord with target spectrum, but strictly accord in statistics meaning. Indicated from the calculating examples, the approach in this article not only can satisfy the non-stationary macroscopic characteristic of seismic wave, but also has good imitation accuracy on the numerical value, in this way it can supply a lot of sample wave which has homology statistic characteristic for some strong earthquake recordings, and it is also advantageous to the analysis of structure random time-history respond.

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