A REGRESSION MODEL FOR ESTIMATING POWER SPECTRAL DENSITY FUNCTION OF GROUND ACCELERATION

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ABSTRACT:
The purpose of this study is to develop a new regression model for power spectral density function (PSDF) of ground acceleration by using the strong motion data of the K-NET and the KiK-NET in Japan. The PSDF is simply expressed as the PA model, proposed by authors before, by using three parameters: the mean square value of the ground acceleration, the predominant frequency, and the shape factor. The regression analyses for the PA-model parameters are carried out based on the maximum likelihood method with respect to the explanatory variables: the distance from the fault to the site, the magnitude, the focal depth, and the averaged shear-wave velocity at the site. The obtained PSDF regression model can express well the averaged tendency, in quality and in quantity, of the observed PSDF. Furthermore, the efficiency and the applicability of the proposed model are demonstrated through the estimation of the earthquake input energy based on the random vibration theory.

KEYWORDS:

1. INTRODUCTION

For the last several decades, many types of regression models have been proposed for strong-motion parameters such as peak ground motion\(^1,2,3\), response spectrum\(^4\), and so on in reply to the increasing needs for engineering prediction of strong motion characteristics. Especially in this decade, in Japan, dense observation networks such as the K-NET\(^5\) and the KiK-NET\(^6\) provide plenty of strong-motion data that enables to advance these kinds of studies\(^7\). Based on these databases, this study aims to develop a new regression model with engineering applicability for power spectral density function (PSDF) of ground acceleration.

In the field of earthquake engineering, the Kanai-Tajimi model has been widely used to express PSDF of earthquake ground acceleration. However, this model sometimes induces overestimations of the spectral values at low frequencies because of its restriction on the zero-frequency spectral value. To overcome this shortcoming, we proposed a simple PSDF model, named as PA model, which corresponds to the ‘Pseudo-Acceleration’ response of a single-degree-of-freedom (SDOF) system under a white noise excitation\(^8,9\). A PA model is simply described by three parameters: the mean square value of the ground acceleration, the predominant frequency, and the shape factor corresponding to the damping factor of the SDOF system. In this study, regression analyses are carried out for these PA-model parameters with respect to the parameters characterizing the earthquake source, the propagation path, and the local site condition so that a new regression model is developed for the PSDF of ground acceleration. The PA model has a valuable advantage as an input motion model for random response analysis. The applicability of the obtained regression model is demonstrated, therefore, through the estimation of the earthquake input energy based on the random vibration theory.

2. POWER SPECTRAL DENSITY FUNCTION MODEL

The PA model\(^8,9\) used in this study to express the PSDF \(S(\omega)\) of ground acceleration \(A(t)\) is given in the form:
where $\omega_g$ is the predominant angular frequency, $\zeta_g$ is the shape factor, and $\sigma_s^2$ is the mean square value of $A(t)$. $S(\omega)$ has one peak at around $\omega = \omega_g$ and $\zeta_g$ controls the sharpness of the peak. The area under the $S(\omega)$ curve equals to $2\pi \sigma_s^2$. Based on the random vibration theory, the earthquake input energy to a SDOF system with a natural angular frequency $\omega_s$ and a damping factor $\zeta_s$ under an excitation characterized by $S(\omega)$ is analytically derived as

$$E(\omega_s) = \frac{2\gamma(\zeta_s' + \zeta_s) \sigma_s^2 T_d}{\omega_g \omega_s}$$

$$F(\gamma) = \gamma^4 + 4\zeta_s\zeta_s' \gamma^3 + 2(2\zeta_s^2 + 2\zeta_s'^2 - 1)\gamma^2 + 4\zeta_s\zeta_s' \gamma + 1$$

where $T_d$ is the effective duration time of the ground motion.

### 3. REGRESSION MODEL

#### 3.1. Regression Variables and Database

For regression analysis, four explanatory variables are selected: the magnitude $M$ (Japan Meteorological Agency magnitude), the fault distance $R$ (the hypocentral distance or the shortest distance between the site and the fault if available), the focal depth $D$, and the averaged shear-wave velocity $V$ of the site (velocity averaged over the top 30 m of the sediments). The response variables are the PA-model parameters: $\sigma_s$, $\omega_g$, and $\zeta_g$ for the PSDF of the vector combination of two horizontal components, and additionally $T_d$ for the evaluation of the earthquake input energy.

The K-NET and the KiK-NET records from 1996 to 2008 are used to make the sample database of the explanatory variables and the response variables. Sample records are selected so as to satisfy the following conditions: (1) all of the explanatory and the response variables are available, (2) $M \geq 5.0$, (3) the maximum horizontal acceleration $\geq 50$ cm/s, (4) each of the earthquakes has more than 5 records, and (5)
each of the sites has more than 5 records. The selected 1539 records contain 90 earthquakes and 187 sites. Selected earthquakes and sites locations, the $M - R$ distribution, and the $V$ distribution are shown in Fig. 1 to Fig. 3, respectively.

The effective duration time $T_d$ is determined by the following procedure: from the squared acceleration time history $A^2(t)$ smoothed by moving average with an interval of 2.0 sec, find two points of time on both side of the maximum peak at which the level firstly falls down below the 1 percent of the maximum level, and take the difference between the two time points as shown in Fig. 4. The PSDF and the mean square value $\sigma^2$ are calculated from this portion of the time history with duration $T_d$. The PA-model parameters $\omega_0$ and $\zeta_0$ are determined so as to minimize the log-residual between the PSDF of the observation record and of the PA model. The PSDF of the observation is smoothed along the frequency with 1-dB width, and 20 log-discretized frequencies from 0.1 Hz up to 10 Hz are used to estimate the residual. Fig. 5 shows a sample of PSDFs and the fitted PA model. Fig. 6 shows the distribution of root-mean-square (RMS) residuals of all samples versus $\sigma_d$. The 95-percentile of the residuals is about 0.64.

### 3.2. Maximum Likelihood Method

The regression model employed in this study is of the following form:

$$
\log_{10} Y_0 = a_i \log_{10} R_{ij} + a_i R_{ij} + a_i M_j + a_i D_j + a_6 \log_{10} V_j + a_6 + \epsilon_{ij}
$$  \hspace{1cm} (3.1)

where $Y_0$ represents one of the response variables: $\sigma_d$, $\omega_0$, $\zeta_0$, and $T_d$. The subscripts $i$ and $j$ refer to the earthquake and the site, respectively. $\epsilon_{ij}$ is a random variable with zero mean value corresponding to the residual for the record of the $i$th earthquake observed at the $j$th site. The first term on the right-hand-side of Eqn. (3.1) diverges to infinity at $R_{ij} = 0$. To utilize this regression model in the near-fault region, it is needed to insert some near-fault term into the argument of the logarithmic function. As the trial analysis with a near-fault term did not give any improvement of regression as far as for the samples selected in this study, the functional form given by Eqn. (3.1) is used as is. The second term on the right-hand-side is used only for $\sigma_d$; it is not used for $\omega_0$, $\zeta_0$, and $T_d$ in order to avoid the trade-off with the first term.

The covariance between the residual $\epsilon_{ij}$ and $\epsilon_{kl}$ is supposed to have the following form:
\[
\text{Cov}\{\varepsilon_{i}^{*} \cdot \varepsilon_{K}\} = \begin{cases} 
\sigma_{\varepsilon}^{2}, & l = K \text{ and } j \neq \ell \\
\sigma_{\varepsilon}^{2}, & l \neq K \text{ and } j = \ell \\
\sigma_{\varepsilon}^{2} + \sigma_{s}^{2} + \sigma_{r}^{2} = \sigma^{2}, & l = K \text{ and } j = \ell \\
0, & l \neq K \text{ and } j \neq \ell 
\end{cases}
\] (3.2)

where \(\sigma_{\varepsilon}^{2}\), \(\sigma_{s}^{2}\), and \(\sigma_{r}^{2}\) are inherent variances in earthquakes, in sites, and in records, respectively. By the maximum likelihood method\(^{10}\), these variances are determined so as to maximize the likelihood of the samples.

3.3. Regression Result

The obtained variances and the resultant coefficients \(a_{i}\) with standardized ones are listed in Table 1.

<table>
<thead>
<tr>
<th>(\sigma_{A})</th>
<th>(a_{1})</th>
<th>(a_{2})</th>
<th>(a_{3})</th>
<th>(a_{4})</th>
<th>(a_{5})</th>
<th>(\sigma_{2}^{2}/\sigma_{s}^{2})</th>
<th>(\sigma_{2}^{2}/\sigma_{r}^{2})</th>
<th>(\sigma_{2}^{2}/\sigma_{r}^{2})</th>
</tr>
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<tbody>
<tr>
<td>(-7.520E-01)</td>
<td>(-1.072E-03)</td>
<td>(3.016E-01)</td>
<td>(4.935E-03)</td>
<td>(2.264E-02)</td>
<td>(6.807E-01)</td>
<td>(0.211)</td>
<td>(0.225)</td>
<td>(0.248)</td>
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<tr>
<td>(+0.73)</td>
<td>(+0.21)</td>
<td>(+0.78)</td>
<td>(+0.50)</td>
<td>(+0.01)</td>
<td>(+0.17)</td>
<td>(+0.09)</td>
<td>(+0.34)</td>
<td>(+0.53)</td>
</tr>
<tr>
<td>(\omega_{g})</td>
<td>(1.504E-01)</td>
<td>(-1.653E-01)</td>
<td>(8.376E-04)</td>
<td>(5.755E-01)</td>
<td>(6.64E-01)</td>
<td>(0.178)</td>
<td>(+0.09)</td>
<td>(+0.34)</td>
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<tr>
<td>(+0.17)</td>
<td>(+0.50)</td>
<td>(+0.10)</td>
<td>(+0.43)</td>
<td>(+0.18)</td>
<td>(+0.09)</td>
<td>(+0.34)</td>
<td>(+0.36)</td>
<td>(+0.51)</td>
</tr>
<tr>
<td>(\zeta_{g})</td>
<td>(-3.284E-02)</td>
<td>(3.219E-01)</td>
<td>(-2.410E-03)</td>
<td>(3.301E-01)</td>
<td>(-2.866E-00)</td>
<td>(0.399)</td>
<td>(+0.12)</td>
<td>(+0.36)</td>
</tr>
<tr>
<td>(-0.02)</td>
<td>(+0.52)</td>
<td>(-0.15)</td>
<td>(+0.13)</td>
<td>(+0.36)</td>
<td>(+0.18)</td>
<td>(+0.09)</td>
<td>(+0.34)</td>
<td>(+0.36)</td>
</tr>
<tr>
<td>(T_{d})</td>
<td>(2.771E-01)</td>
<td>(1.344E-01)</td>
<td>(-7.287E-04)</td>
<td>(-1.581E-01)</td>
<td>(5.520E-01)</td>
<td>(0.155)</td>
<td>(+0.21)</td>
<td>(+0.18)</td>
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<tr>
<td>(+0.36)</td>
<td>(+0.46)</td>
<td>(-0.10)</td>
<td>(-0.13)</td>
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<td>(+0.18)</td>
<td>(+0.09)</td>
<td>(+0.34)</td>
<td>(+0.36)</td>
</tr>
</tbody>
</table>

Figure 7. Regression curve for \(\sigma_{A}\)  
Figure 8. Regression curve for \(T_{d}\)  
( normalized to \(M = 7.0\), \(D = 10 \text{ km}\), \(V = 300 \text{ m} / \text{s}\) )

Figure 9. Regression curve for \(\omega_{g}\)  
Figure 10. Regression curve for \(\zeta_{g}\)  
( normalized to \(R = 50 \text{ km}\), \(D = 10 \text{ km}\), \(V = 300 \text{ m} / \text{s}\) )
From the Table 1, it is observed that $\sigma_A$ and $T_d$ depend on the distance $R$ and on the magnitude $M$ in the manner as empirically expected. On the other hand, the predominant angular frequency $\omega_g$ depends not only on the shear-wave velocity $V$ but also, more strongly, on $M$. Although it can also be observed that the shape factor $\zeta_x$ has strong dependence on $M$, its standard deviation is relatively large as compared with other three response variables. The regression curves for $\sigma_A$, $T_d$ (versus $R$), $\omega_g$, and $\zeta_x$ (versus $M$) are shown in Fig. 7 to 10, respectively.

As the error distributions are not strongly distorted, the regression method seems to give appropriate results for the regression models. To examine the resultant fitness of PSDF, the log-residuals are plotted in Fig. 11, in the same manner as Fig. 5. The cumulative deviation of $\sigma_A$, $\omega_g$, and $\zeta_x$ raise the 95-percentile of the residuals to 0.94, which is larger than that of the sample set shown in Fig. 5. Fig. 12 and 13, for examples, illustrate PSDFs at NIG020 and NIG025 during the 2004 Niigata-ken Chuetsu earthquake ($M = 6.8$). The RMS residuals for NIG020 (Fig. 12) and NIG025 (Fig. 13) are 0.304 and 0.714, respectively. In both cases, the PA-model described by the regressed parameters seems to express well the average tendency, in quality and in quantity, of the observed PSDF.

4. DISCUSSION

The main reason why to use the PA model for PSDF owes to its valuable feature as an input motion model for random response analysis. For example, as mentioned before, the earthquake input energy can be expressed in closed form as given by Eqn. (2.2). The energy-equivalent velocity spectra corresponding to the same cases as in Fig. 12 and 13 are shown in Fig. 14 and 15, respectively.

Furthermore, the qualitative and quantitative relations of the earthquake and site parameters such as the distance, the magnitude, the focal depth, and the shear-wave velocity to the earthquake input energy can be examined directly by using this model as shown in Fig. 16 to 19. These figures show the energy-equivalent velocity spectra for various combinations of the earthquake and site parameters.
The spectral level increases with the decrease of the distance $R$, when the spectral shape does not change significantly. The increase of the magnitude $M$ leads to the increase of the spectral level and the shift of the peak to longer periods. It simultaneously makes the spectral shape flatter because of the increase of $\zeta_g$. The decrease of the shear-wave velocity $V$ has the same influences in quality except for the flattening effect. The increase of the focal depth $D$ also leads to the increase of the spectral level, but its influence is relatively small.

5. CONCLUSION

A new regression model for PSDF of ground acceleration has been developed by using the strong-motion
The data of the K-NET and the KiK-NET in Japan. The PSDF of ground acceleration is expressed by the PA model, which can be described with three parameters: the mean square value of the ground acceleration $\sigma_A^2$, the predominant frequency $\omega_g$, and the shape factor $\zeta_g$. The regression analyses for these PA-model parameters and the effective duration time $T_e$ have been carried out based on the maximum likelihood method with respect to the explanatory variables: the distance $R$, the magnitude $M$, the focal depth $D$, and the averaged shear-wave velocity $V$. The obtained regression model can express well the average tendency, in quality and in quantity, of the PSDF of the observed ground motion. Furthermore, the applicability of the proposed regression model has been demonstrated through the estimation of the earthquake input energy based on the random vibration theory. It has been shown that the proposed model can be used efficiently not only in engineering prediction of strong ground motion but also in the direct estimation of the structural response.

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REFERENCES


