INVESTIGATION OF THE EFFECT OF THE GROUND MOTION CHARACTERISTICS ON THE $R_y-\mu$ RELATION FOR THE INELASTIC RESPONSE OF SDOF STRUCTURES

I. Taflampas$^1$ and I.N. Psycharis$^2$

$^1$ Civil Engineer, Laboratory for Earthquake Engineering, National Technical University of Athens, Greece
$^2$ Assoc. Professor, School of Civil Engineering, National Technical University of Athens, Greece
Email: taflan@central.ntua.gr, ipsych@central.ntua.gr

ABSTRACT:

In this study, the effect of the ground motion frequency content, especially the period of the directivity pulse in near field records, on the relationship between the reduction factor $R_y$ and the ductility demand $\mu$ is investigated. A large sample of strong motion records showing near field characteristics of forward and backward directivity, is examined. The regimes, in which the known assumptions of equal displacement and equal energy are valid, are re-evaluated taking into account the period of the directivity pulse contained in the ground motion. The results show that, for near field records with directivity effects, the thresholds of these regions depend on the period of the directivity pulse, which is related to the magnitude of the earthquake. This observation is valid for ground motions with intense and weak forward directivity and backward directivity effects. This response implies that the equal displacement rule ($R_y=\mu$), which is incorporated in most earthquake design regulations for structures with a period larger than $T_c$, with $T_c$ being the threshold between the constant acceleration and the constant velocity regions on the response spectrum, might underestimate significantly the ductility demand for structures with long periods. In this analysis, a new method for deriving the pulse duration in case of weak forward and for backward directivities is proposed and a new limit for the regime, in which the equal displacement assumption applies, is defined.

KEYWORDS: Near field, ductility, inelastic response, reduction factor, behaviour factor.

1. INTRODUCTION

According to the Force Based Seismic Design (FBSD), which is adopted in most seismic codes, the seismic load that is applied to a Single-Degree-Of-Freedom (SDOF) structure is derived by dividing the elastic spectral acceleration by a reduction factor, $R$. In Eurocode 8 (EC8), this factor is referred to as behaviour factor and is denoted by $q$. The code reduction factor $R$ takes into account the energy dissipation due to the inelastic response and the so-called overstrength, denoted by $R_s$. It can be defined as $R=R_y-R_s$, in which the term $R_y$ (yield or ductility reduction factor) is related to the energy dissipation and is defined as the ratio of the inertia force that would be developed to the equivalent elastic structure over the yield strength of the inelastic system.

For an effective seismic design, the ductility demand for the design earthquake should be within the limits of the collapse prevention performance criterion. The ductility demand, $\mu$, is defined as the ratio of the maximum displacement over the yield displacement. Since the seismic loads are determined by dividing the elastic ones with the reduction factor, $R$, an adequate relationship between $R_y$ and $\mu$ is necessary. This issue has been addressed for many years by the research community and various approximate $R_y-\mu-T_a$ relationships between the reduction factor, $R_y$, the ductility, $\mu$ and the period $T_a$ of the structure have been proposed.

Most seismic codes adopt the Newmark and Hall (1982) approach, which is based on the $R_y-\mu-T_a$ relationships proposed by Veletsos and Newmark (1982), given by the following expressions:
where $T_a$, $T_b$, and $T_c$ are characteristic periods separating spectral regions and $T_{c'}$ is defined by the relation:

$$T_{c'} = \frac{2\mu - 1}{\mu} \cdot T_c$$

(1.2)

In the constant acceleration region, it is usually assumed that the same energy is absorbed in the elastic and the inelastic systems. This assumption results in a larger maximum displacement for the inelastic system than the elastic one. In the contrary, in the region $T_n > T_c$, the assumption of equal displacements between the elastic and the inelastic systems is made, which results in $R_y = \mu$.

More recent studies (Krawinkler and Nassar, 1992, Vidic et al, 1994, Miranda and Bertero, 1994) present alternative $R_y - \mu - T_n$ relationships, based on statistical results for large samples of earthquake records. For example, Krawinkler and Nassar (1992) propose the relationship:

$$R_y = [c(\mu - 1) + 1]^{1/c}$$

(1.3)

where $c$ is a function of the period of the structure, $T_n$, given by the following expression:

$$c(T_n) = \frac{T_n^a}{1 + T_n^b} + \frac{b}{T_n}$$

(1.4)

For elastic-perfectly plastic systems, the coefficients $a$ and $b$ obtain the values: $a=1$ and $b=0.42$. It is interesting to note that this relation contains only the period of the elastic system and is independent of the frequency content of the ground motion.

Conversely, Chopra and Chintanapakdee (2001) emphasize the different characteristics between near and far field earthquake records. For this reason, they consider different values for $T_a$, $T_b$, and $T_c$ in each case. Based on statistical results for a selected sample of ground motions, they propose the values: $T_n=0.025$ sec, $T_b=0.22$ sec and $T_c=0.42$ sec for far field earthquakes and $T_n=0.04$ sec, $T_b=0.35$ sec and $T_c=0.79$ sec for near field records. It should be noted that, for the records considered in their study, the orientation of the near field ones was normal to the fault trace with prominent forward directivity phenomena. In such cases, response spectra show a wide region of constant spectral acceleration and a narrow region of constant spectral velocity. The period $T_c$ defines the threshold between these regions and, according to the study, obtains values less than 1.0 sec, in general.

In a recent study on near field records, Mavroeidis et al (2004) associate the $R_y - \mu - T_n$ relationships with the duration of the directivity pulse, $T_p$, i.e. the pulse period. This period is related to the earthquake magnitude and the source mechanism. The authors propose the following modification of the relationships of Veletsos and Newmark (Eqn. 1.1) in order to take under consideration the pulse duration, $T_p$: 
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\[ R_y = \begin{cases} 
1 & \text{for } T_n/T_p < \left(\frac{T_n}{T_p}\right)_b \\
\sqrt{2\mu - 1} & \text{for } \left(\frac{T_n}{T_p}\right)_b < T_n/T_p < \left(\frac{T_n}{T_p}\right)_c \\
\mu & \text{for } T_n/T_p > \left(\frac{T_n}{T_p}\right)_c 
\end{cases} \quad (1.5) \]

where, similarly to Eqn. 1.2,

\[ \left(\frac{T_n}{T_p}\right)_b = \frac{\sqrt{2\mu - 1}}{\mu} \left(\frac{T_n}{T_p}\right)_c \quad (1.6) \]

The values of the normalized periods \((T_n/T_p)_a\), \((T_n/T_p)_b\), \((T_n/T_p)_c\), are related to the earthquake magnitude. The parameter \((T_n/T_p)_b\) is set equal to 0.35 for events of medium magnitude and equal to 0.20 for medium to large earthquakes, while \((T_n/T_p)_c=0.75\).

For the application of the above relationships, the value of the pulse period \(T_p\) is needed. Empirical expressions, relating the value of \(T_p\) to the moment magnitude, \(M_w\), of the earthquake, are given by several researchers. Such a relation was proposed by Rodriguez-Marec (2000), based on a large sample of earthquakes. The mean value is given by the expression:

\[ \ln T_p = -8.33 + 1.33 \cdot M_w \quad (1.7) \]

and the standard deviation is 0.54.

In the present paper, a sample of earthquake records is analyzed, in order to evaluate the above-mentioned \(R_y\)-\(\mu\)-\(T_n\) relationships for near field records containing directivity effects. A new threshold for the equal displacement regime is proposed, associated with the period of the pulse, \(T_p\) and the near field characteristics of the ground motion.

2. GROUND MOTIONS CONSIDERED IN THE ANALYSES

The two horizontal components of 27 earthquake recordings (54 records in total) were used in this study. They were selected from the COSMOS and PEER databases with moment magnitudes ranging from 6.2 to 7.3, with the exception of the San Salvador, 1986 earthquake that was of magnitude \(M_w=5.6\). Most of the records were recorded on sites characterized as soil and alluvia, three recordings were obtained on soft rock and another three on hard rock deposits. The distance from the fault trace varies between 1.2 and 13.7 km. The data sample includes well-known earthquakes such as the Northridge, USA, 1994, the Kobe, Japan, 1995 and the Chi-Chi, Taiwan, 1999 events.

Most records show forward directivity effects with the exception of the earthquakes: Imperial Valley (El Centro station), USA, 1940, Morgan Hill (Halls Valley station), USA, 1984 and Landers (Joshua Tree station), USA, 1992 that are associated with backward directivity phenomena (Mavroeidis and Papageorgiou, 2003). In most recordings, the normal to the fault components show strong directivity effects (e.g. component TAB-074 of the Tabas, Iran, 1978 earthquake, records E02-230 and E04-230 of the El Centro Array, Imperial Valley, USA, 1979 earthquake), while the parallel to the fault components are associated with weak directivity effects. In some cases, prominent directivity effects are evident in both components, as in Erzincan, Turkey, 1992 earthquake, in which the orientation, with respect to the fault trace, of both instruments was close to 45 degrees.

For each record, the velocity and the displacement response spectra for 5% damping were derived and the predominant period associated with each spectrum was determined from the corresponding peak spectral value, as shown in Figures 1a-b, 2a-d and 3a-d. These are characteristic periods for near field earthquakes and are denoted by \(T_{SV}\) and \(T_{SO}\) respectively. Also, inelastic analyses were performed for SDOF structures with varying period, \(T_n\),
and for several values of $R_y$. However, in the following only the case of $R_y=3$ since this is a common value for RC structures. From these analyses, the ductility demand was calculated and the corresponding ductility spectrum for each record was drawn (Figures 1c, 3e-f).

3. PRESENTATION OF THE RESULTS

3.1. Records with prominent forward directivity

According to Mavroeidis et al (2004), the response spectra of near field records with prominent forward directivity show a wide region of constant spectral acceleration and a narrow region of constant spectral velocity. As a result of the latter, the velocity and the displacement response spectra have similar predominant periods. Also, the predominant period of the velocity spectrum, $T_{SV}$, can be used to determine the period $T_p$ of the directivity pulse (Alavi and Krawinkler, 2000).

An example of a record with prominent forward directivity is the component ERZ-000 of the Erzincan, Turkey, 1992, earthquake ($M_w=6.6$). In Figures 1a and 1b, the velocity and the displacement spectra for 5% damping are presented, while in Figure 1c, the ductility demand spectrum for $R_y=3$ is shown. The predominant period of the velocity spectrum is $T_{SV}=2.3$ sec (Figure 1a) and the predominant period of the displacement spectrum is $T_{SD}=2.5$ sec (Figure 1b). For $M_w=6.6$, Eqn 1.7 gives $T_p=1.57$ sec and if the standard deviation is added, $T_p=2.3$ sec, which is equal to $T_{SV}$.

From Figure 1c it can be observed that for structures with period larger than approximately 1.3 sec the equal displacement rule is valid and $R_y=\mu$. For structures with a smaller period, the ductility demand increases exponentially as the period decreases. According to Mavroeidis et al (2004), $R_y=\mu$ for $T_n/T_p>(T_n/T_p)_c$ (see third of Eqns. 1.5). For the ERZ-000 record, $T_p=2.3$ sec and for $(T_n/T_p)_c=0.75$, the third of Eqns. 1.5 gives $R_y=\mu$ for $T_n>1.7$

![Figure 1](image-url)  

Figure 1. ERZ-000 component of Erzincan, Turkey, 1992 earthquake: (a) velocity spectrum for $\zeta=5\%$; (b) displacement spectrum for $\zeta=5\%$; (c) ductility demand spectrum for $R_y=3$. 
sec, which is consistent with Figure 1c. For smaller periods, Mavroeidis et al. (2004) propose the relationship 
\[ R_y = \sqrt{2 \mu - 1} \] 
for \((T_n/T_p)_b < T_n/T_p < (T_n/T_p)_c\) (see second of Eqns. 1.5). In this region, the relation between \(R_y\) and \(\mu\) can be written as \(\mu = (1 + R_y^2) / 2\) and thus, Eqn. 1.6 gives:

\[ \left( T_n/T_p \right)_c = \frac{2 R_y}{1 + R_y^2} \left( T_n/T_p \right)_c \]  \hspace{1cm} (3.1)

For \((T_n/T_p)_b = 0.75\) and \(R_y = 3\), Eqn. (3.1) results in \((T_n/T_p)_c = 0.45\). Taking \((T_n/T_p)_b = 0.20\) (medium to large earthquake) and \(T_p = 2.3\) sec, the above relation gives \(\mu = 5\) for \(0.5\) sec < \(T_n < 1.0\) sec. In Figure 1c, this value of ductility is confirmed only for structures with a period around \(T_n = 1.0\) sec. For smaller periods, the ductility demand increases exponentially and attains values significantly larger than \(\mu = 5\).

In general, for the analyzed records with prominent forward directivity the results show a varying ductility demand in the region \((T_n/T_p)_b < T_n/T_p < (T_n/T_p)_c\). For many ground motions, the ductility demand was smaller than that predicted by the second of Eqns. 1.5, but there were cases, as the ERZ-000 record, for which the ductility demand was larger than the predicted. In most cases, however, Eqns. 1.5 were in the safety side.

### 3.2. Records with weak forward directivity

The forward directivity effect is also present in ground motion components with orientation different than the vertical to the fault trace. According to Baker (2007), in these records the directivity pulse is less prominent in comparison to the vertical to the fault component. The velocity and the displacement response spectra have diverging predominant periods. The predominant period of the velocity spectrum, \(T_{SV}\), is affected by the high frequency content of the record and, thus, is smaller than the period of the directivity pulse, which is associated with lower frequencies and coincides with a local peak in the long period region of the velocity spectrum. As a consequence, \(T_{SV}\) cannot be used to define \(T_p\), as in the case of records with prominent forward directivity.

The different characteristics of records with prominent and weak forward directivities are shown in Figure 2, in which the velocity and the displacement spectra of the two horizontal components (TAB-074 and TAB-344) of the Tabas, Iran, 1978 (\(M_w = 7.1\)) earthquake are shown. The component TAB-074 was recorded in a direction almost normal to the fault trace and shows all the characteristics of prominent forward directivity: \(T_{SV}\) and \(T_{SD}\) are similar (see Figures 2a and 2c), with values equal to \(4.7\) sec and \(5.3\) sec, respectively. Considering that \(T_p \approx T_{SV}\) (Alavi and Krawinkler, 2000), one can set \(T_p \approx 5.0\) sec, which is close to the value of \(5.2\) sec that is obtained using Eqn. 1.7 plus a standard deviation. Conversely, the component TAB-344 shows weak directivity phenomena: the predominant period of the velocity spectrum is close to \(1.0\) sec, quite smaller than the period of the pulse, \(T_p\).

It is important to note that, for both components, the displacement spectra have similar predominant periods equal to \(T_{SD} \approx 5.0\) sec (see Figures 2c and 2d), a value close to \(T_p\). Also note that the velocity spectrum of the weak directivity component TAB-344 shows a local peak at \(T_n = 5.7\) sec, i.e. close to the predominant period of the displacement spectrum.

As mentioned above, Eqns. 1.5 were derived using records with prominent forward directivity. Here, we extend these relations to records with weak forward directivity components. In such cases, however, the period \(T_p\) should be set equal to \(T_{SD}\), the predominant period of the displacement response spectrum, and not equal to \(T_{SV}\). Thus, the equal displacement rule \((R_y = \mu)\) can be applied for \(T_p > 0.75 - T_p\). For smaller periods, the ductility demand, \(\mu\), increases and becomes larger than \(R_y\), although it reduces again, in general, in the region close to the predominant period of the velocity spectrum, \(T_{SV}\). Thus, the assumption of equal energy absorption \(R_y = \sqrt{2 \mu - 1}\) can be applied in the range \((T_n/T_p)_b < T_n/T_p < (T_n/T_p)_c\). Note that the right threshold is transferred from \((T_n/T_p)_c\) to \((T_n/T_p)_c\) (compare with the second of Eqns. 1.5).
Figure 2. TAB-074 (left column) and TAB-344 (right column) components of Tabas, Iran, 1978 earthquake: (a) and (b) velocity spectra for $\zeta=5\%$; (c) and (d) displacement spectra for $\zeta=5\%$.

An example of this behaviour is shown in the left column of Fig. 3, in which the velocity and the displacement response spectra and the ductility demand for $R_y=3$ are shown for the component KAR-000 of the Gazli, USSR, 1976 earthquake ($M_w=6.8$). The predominant periods of the spectra are $T_{SV}=1.0$ sec and $T_{SD}=4.4$ sec, thus, it can be set $T_p=4.4$ sec. Note that Eqn. 1.7 gives $T_p=3.5$ sec (a standard deviation is added), i.e. a smaller value. For $R_y=3$ and a medium to large event, the normalized periods in Eqn. 1.5 are: $(T_n/T_p)_b=0.2$, $(T_n/T_p)_c=0.75$ and $(T_n/T_p)_c'=0.45$.

Then, for $T_p=4.4$ sec it comes out: $\mu=3.0$ for $T_n>3.3$ sec (equal displacement region) and $\mu=5.0$ for $0.9$ sec $< T_n < 2.0$ sec (equal energy region). Figure 3e shows that the equal displacement regime ($\mu=3.0$) can indeed be assigned to $T_n>3.3$ sec, but the equal energy region ($\mu=5.0$) must be extended to the right, up to $T_n=3.3$ sec, as proposed above.

### 3.3. Records with backward directivity

The records with backward directivity are characterized by the time sequence of many pulses of the ground velocity, associated with the fault rupture sub-events. Thus, the strong motion duration is quite long, but the ground velocity is smaller compared to the forward directivity. The long duration results in large spectral amplifications in both velocity and displacement spectra. The predominant periods of these spectra, $T_{SV}$ and $T_{SD}$, are quite different, as in the case of weak forward directivity. Again, the period of the pulse, $T_p$, is associated with the predominant period of the displacement response spectrum, $T_{SD}$. Thus, Eqs. 1.5 can be applied with the modifications proposed above for records with weak forward directivity, namely: $T_p$ should be derived from the predominant period of the displacement response spectrum, $T_{SD}$, and the right threshold of the equal energy regime should be moved to $(T_n/T_p)_c'$ instead of $(T_n/T_p)_c$.

An example is presented in the right column of Figure 4 for the ELC-180 component of the Imperial Valley, USA, 1940 earthquake ($M_w=6.2-6.4$). The record contains a large number of velocity cycles with small amplitude and Mavroeidis and Papageorgiou (2003) acknowledge it as a case of backward directivity. The predominant periods
Figure 3. KAR-000 component of Gazli, USSR, 1976 earthquake (left column) and ELC-180 component of the Imperial Valley, USA, 1940 earthquake (right column): (a) and (b) velocity spectra for $\zeta=5\%$; (c) and (d) displacement spectra for $\zeta=5\%$; (e) and (f) ductility demand spectra for $R_y=3$.

of the velocity and the displacement spectra are $T_{SV}=1.0$ sec and $T_{SD}=2.4$ sec. Thus, the period of the pulse can be set equal to $T_p=2.4$ sec. Eqn. 1.7 gives $T_p=2.05$ sec with the standard deviation added, i.e. again a smaller value. The normalized periods in Eqns. 1.5 are: $(T_n/T_p)_b=0.2$, $(T_n/T_p)_c=0.75$ and $(T_n/T_p)_c'=0.45$. Consequently, application of Eqns. 1.5 for $T_p=2.4$ sec results in $\mu=3.0$ for $T_n>2.0$ sec (equal displacement region) and $\mu=5.0$ for $0.5$ sec $<T_n<1.2$ sec (equal energy region). As shown in Figure 3f, the prediction is quite good for the equal displacement regime ($T_n/T_p>2.0$ sec), but the equal energy assumption ($\mu=5.0$) should be extended approximately up to 1.6 sec. The proposed transfer of the right threshold of the equal energy regime to $(T_n/T_p)_c'$ corrects this error.

4. CONCLUSIONS

In this analysis, the regions of validity of the $R_y-\mu-T_n$ relationships (Eqns. 1.5) proposed by Mavroeidis et al (2004) are examined for near field records with various types of directivity. The results show that for records with
prominent forward directivity, these relations give reasonable results. In this case, the period $T_p$ can be derived from the predominant period of the velocity response spectrum, $T_{SV}$. For records with weak forward directivity and records with backward directivity, these relations should be applied with two modifications: the pulse duration, $T_p$, should be derived from the predominant period of the displacement response spectrum, $T_{SD}$, and the right threshold of the equal energy regime should be extended from $(T_p/T_n)c'$ to $(T_p/T_n)c$.

It is evident from the analysis that, for near field records with directivity effects, the region in which the ductility demand is larger than $R_y$ (equal energy regime) depends on the period, $T_p$, of the directivity pulse. For records with weak or backward directivity, the right threshold of this region is close to $0.75 \cdot T_p$, which means that the equal displacement rule might not be valid even for long-period structures, if $T_p$ is long. This effect might be suppressed if a statistical process is applied to the results of a large sample of records. For example, the relationships proposed by Chopra and Chintanapakdee (2001), although they consider different thresholds for near field and far field earthquakes, do not take under consideration the pulse period $T_p$, and, thus, might underestimate the ductility demand in the long period regime. The same observation holds for the conclusion drawn by Garcia and Miranda (2006) that the ductility demand is not affected by the magnitude of the seismic event. The results presented here show that the $R_y-\mu-T_n$ relationships depend on the duration of the directivity pulse, which is associated with the magnitude of the earthquake (Eqn. 1.7).

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