Study on Elasto-plastic Response Spectra for Isolated Structures

W.F. He[1] W.G. Liu[2] X.D. Huang[1] X. Yang[2] and D.Huo[3]

[1]*Postdoctor, Shanghai University, Shanghai, China*
[2]*Professor, Dept. of Civil Eng., Shanghai University, Shanghai, China*
[3]*Professor, School of Civil and Arch. Eng., Beijing University of Technology, Beijing, China*
Email: howunfu@yahoo.cn

ABSTRACT :

Based on the theory of elastic-plastic response spectrum and the relationship between displacement ductility and equivalent damping ratio of the non-linear restoring force model, the elasto-plastic response spectrum of isolated structure is established according to the results of single-mass elasto-plastic analysis. Further comparison of Chinese code and elasto response spectrum, the results showed that the value of the elasto-plastic acceleration response spectra in the long cycle of the spectrum, damping attenuation coefficient is larger. Based on the results of single mass elasto-plastic analysis, the elasto-plastic response spectrum formula of isolated structure is established.

KEYWORDS: Isolated structure, elasto-plastic, response spectrum

1. INTRODUCTION

Seismic isolation is an old design idea, proposing the decoupling of a structure or part of it, or even of equipment placed in the structure, from the damaging effects of ground accelerations. One of the goals of seismic isolation is to shift the fundamental frequency of a structure away from the dominant frequencies of earthquake ground motion and the fundamental frequency of the fixed base superstructure. The other purpose of an isolation system is to provide an additional means of energy dissipation, thereby reducing the transmitted acceleration into the superstructure. A variety of isolation devices including rubber bearings (with and without lead core), frictional/sliding bearings and roller bearings have been developed and used practically for aseismic design of buildings during the last 20 years (Kelly, JM., 1986, Jangid, R.S., 1995).Among the various base isolation systems, the lead–rubber bearings (LRB) had been used extensively in New Zealand, Japan and United States. The LRB consists of alternate layers of rubber and steel plates with one or more lead plugs that are inserted into the holes. The lead core deforms in shear providing the bilinear response (i.e. adds hysteretic damping in the isolated structure) and also provides the initial rigidity against minor earthquakes and strong winds (Tyler, R.G., 1984). The first building isolated by the LRB was the William Clayton building in Wellington, New Zealand completed in 1981 and followed by other buildings in several countries. The buildings isolated with LRB performed very well during the 1994 Northridge and 1995 Kobe earthquakes confirming the suitability of LRB as abase isolator (Asher, J.W., 1997, Nagarajaiah, S., 2000, Nagarajaiah, S., 2001).

Several seismologists have suggested that base-isolated buildings are designed by response spectrum method. The response spectrum theory had been developing since last century. Specially, in 1940s, Many significant earthquake records were got in American (such as El-Centro, 1940, May 18) (Hu, Y. X., 1988). The long period response spectrum led to considerable interest by there searchers; recently several studies of the long period response spectrum have been reported (Lin, Y.Y., 2007). In China, a few scholars (Yu Haiying and Zhao Xiao zhi (Zhang, X.Z., 2004), Guo Mingzhu and Chen Houqun (Guo, M.Z., 2003), Wang Yayong(Hu, R.R., 2002)) had conducted studies and researches on the response spectrum. Zhou Yongnian (Zhou, Y.N., 2004) had studied the long period part of design response spectrum. Liang Xiaohua (Liang, X.H., 2004) presented the long period response spectrum based on the average response spectrum according to statistical analysis method. All of the above listed studies focused on the field of traditional aseismic structure. Related to the response spectrum of isolated structure, the author in Ref. (He, W.F., 2008) found that the response spectrum in long period part of design response spectrum of China code is too large by time history analysis and shaking table test, so it could not accurately predict the structure seismic response for long period isolated structure. Further, the reason of over deviation is that the China aseismic code is on the basis of elastic response spectrum. Although the elastic response spectrum could effectively represent the structure elastic response under the small earthquake action, it can not correctly calculate the inelastic displacement of isolated structure, owing to the isolated layer being in the yield stage under the medium and large earthquake action. While elasto-plastic response spectrum can reflect the behavior and situation after structure yielding, so it is often used as a method to analyze the nonlinear elastic response.

Here, the elasto-plastic response spectrum of isolated buildings is investigated. The specific objectives of the study are (i) to study the performance of the elasto-plastic response spectrum of isolated structures by statistical studying under the 80 ground motions for various site classes, (ii) to the difference obtained by comparing Chinese code and elasto response spectrum, (iii) to the elasto-plastic response spectrum expressions obtained by nonlinear regression analyses are proposed.

2. ELASTO-PLASTIC RESPONSEE SPECTRUMS BASED ON EQUIVALENT DAMPING RATIO

2.1. Computation model

Figure 1(a) shows the general buildings supported by the isolation system. For the present study, the LRB consisting of alternating layers of steel shims and rubber with central lead plug are considered as isolation device. The LRB is very stiff in vertical direction and flexible in horizontal direction (due to presence of steel shims and rubber). The horizontal flexibility and damping characteristics of the LRB provides desired isolation effects in the system.

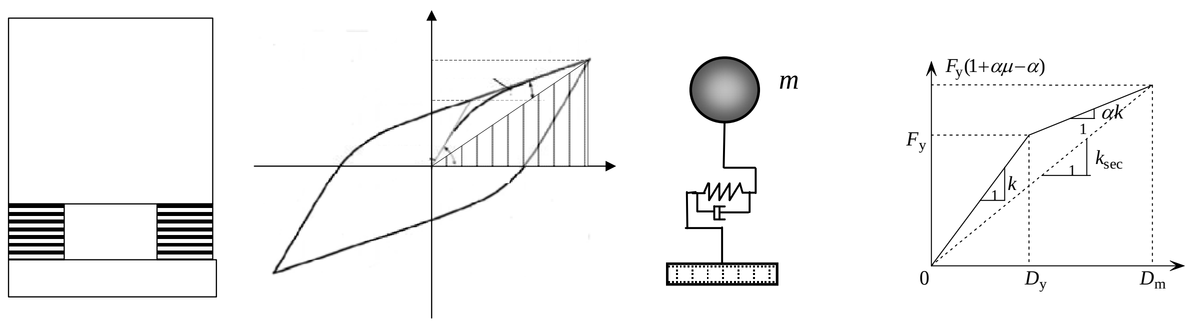

Figure 1 The stiffness and damping of the equivalent single mass

For the present study, the force-deformation behavior of the LRB is expressed by the Wen's equation (Wen, Y.K., 1976) (see in figure. 1(b)). The restoring force of the LRB is expressed by:

$$R(x,\dot{x}) = \alpha \frac{F_y}{d_y} x + (1-\alpha)\frac{F_y}{d_y} Z \tag{2.1}$$

$$\dot{Z} = A\dot{x} - \beta|\dot{x}||Z|^{n-1} Z - \gamma\dot{x}|Z|^n \tag{2.2}$$

where α is the ratio of post-yielding stiffness to initial stiffness; F_y, d_y is the yield force and yield displacement of isolated bearing; $x, \dot{x}$ is the displacement and velocity of isolated bearing; Z is the hysteretic component, a function of the time history of x; the quantities A, γ, β and n are "loop parameters", which control the shape and magnitude of the hysteresis loop and can be observed by test. For lead rubber bearing, the quantities A=0.1, γ=0.5, β=0.5 and n=1.3, respectively.

2.2. Dynamic equation and calculation steps

Figure 1(c) shows the simplified SDOF(single-degree-of-freedom) model of general buildings supported by the isolation system. This was studied earlier in Refs. (Tyler, R.G., 1984, Zhang, X.Z., 2004, Yi, W.J., 2006). The governing equations of motion of the isolated bridge model are expressed by:

$$m\ddot{x} + c\dot{x} + q(x) = -m\ddot{x}_g(t) \tag{2.3}$$

where m is the mass of superstructure; c is the damping coefficient; $q(x)$ is the hysteretic force; $\ddot{x}_g$ is the acceleration of ground motion; x, $\dot{x}$, $\ddot{x}$ is the displacement, velocity and acceleration response of structure.

Some dimensionless parameters are expressed as:

$$u = \frac{x}{x_{\mathrm{y}}}, \quad \eta = \frac{F_y}{m \cdot \max\left|\ddot{x}_g(t)\right|}, \quad \tau(t) = \frac{\ddot{x}_g(t)}{\max\left|\ddot{x}_g(t)\right|}$$

where F_{y} is the yield force of structure, $\max\left|\ddot{x}_g(t)\right|$ is the peak acceleration of ground motion.
Substituting u, η and $\tau(t)$ into Eq. (2.3) gives:

$$\ddot{u} + \frac{4\pi}{T}\xi\dot{u} + \frac{4\pi^2}{T^2}q(u) = -\frac{4\pi^2}{T^2\eta}\tau(t) \tag{2.4}$$

where $q(u)$ is the hysteretic force, whose the yield force has been normalized; $\tau(t)$ is the waveform of earthquake wave that is the spectrum characteristics; T is the structure equivalent period. The hysteretic model is assumed to the Bouc-Wen model.

The ductility demand coefficient can be defined as:

$$\mu = \max|u| = \left|x_{\max}\right| / x_{\mathrm{y}} \tag{2.5}$$

where $\left|x_{\max}\right|$ is the maximum displacement of superstructure; x_{y} is the yield displacement.

The period of equivalent elastic system can be calculated by equivalent stiffness k_{sec}, which is the secant stiffness corresponding to the maximum displacement of isolated layer (see in figure 1(d)). The equation of equivalent period is simplified as:

$$T_{\mathrm{eq}} = T_0\sqrt{\frac{\mu}{1-\alpha+\alpha\mu}} \tag{2.6}$$

where T_0 is the elastic period of inelastic SDOF system; α is the post-yielding stiffness coefficient.

For SDOF system, according to Bouc-Wen hysteretic model and post-yielding stiffness coefficient α, the equivalent damping ratio can be obtained as follows:

$$\xi_{\mathrm{eq}} = \xi + \hat{\xi}_{\mathrm{eq}} = \frac{2}{\pi}\frac{(\mu-1)(1-\alpha)}{\mu(1+\alpha\mu-\alpha)} \tag{2.7}$$

where ξ is the initial elastic damping ratio.

From Eq. (2.5) and (2.7), the relationship of equivalent damping ratio ξ_{eq} and ductility demand coefficient μ of SDOF can be established. Assigned a value to ξ_{eq}, ductility demand coefficient μ can be calculated by Eq. (2.7), as a target parameter, to compare with the ductility demand coefficient of every mass obtained under different periods of SDOF system. Finally, the elatio-plastic response spectrum can be established, the ordinate is ductility demand coefficient μ and the abscissa is the period.

The elasto-plastic response spectrum corresponding to a specific equivalent damping ratio for each site condition are calculated by the following steps:

(1). To assign a value to equivalent damping ratio ξ_{eq}, and to calculate the ductility demand coefficient μ.

(2). To give an initial value to ductility coefficient μ, and calculate the response of acceleration, velocity and displacement of each mass by using the method of time history analysis, until the calculated ductility coefficient μ is equivalent to the ductility demand coefficient got by step.1.

(3). To calculate the responses of every mass, and then connect these response values corresponding to these points with line, an elasto-plastic response spectrum can be obtained.

2.3. Selection of earthquake waves

The 4 kinds of earthquake waves are selected in accord to the 4 kinds of sites, referring to Chinese anti-seismic code. Corresponding to every site, there are 20 earthquake waves chosen in this paper, and each wave is normalized with the peak acceleration of ground motion.

3. RESULTS OF NUMERICAL ANALYSIS

3.1. The long period response spectrum

According to the steps of inducing the elasto-plastic response spectrum based on equivalent damping ratio, a computer program wrote with Matlab is used for elasto-plastic time history analysis of single mass model. By the analysis of isolated with the range of period is 0.4sec-6sec generally meet the demands of isolated structure. The equivalent damping ratio is selected from 5% to 40% at 10% intervals meets the engineering demands.

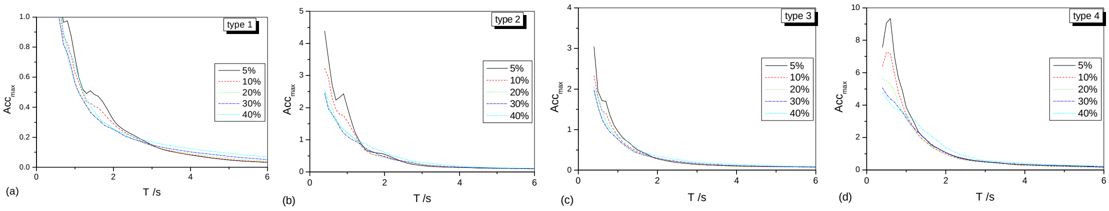

Figure 2 The elasto-plasticity acceleration response spectrum on 4 types of site

The elasto-plastic acceleration response spectrum on 4 types of sites under the different equivalent damping ratio (5%, 10%, 20%, 30% and 40%) are showed in figure 2.

Figure 2 shows the change of elasto-plastic acceleration response spectrum under the different equivalent damping ratio and sites. The acceleration response is large when the period is short, with the increase of period, it becomes smaller. While the period of range is 1-2sec, the acceleration response decreases rapidly, but if period is greater than 2sec, it diminishes slowly and gradually. From the figure 2, it can be seen that the acceleration response reaches maximum for short part of period when the equivalent damping ratio is 5%, and when the equivalent damping ratio increases, the acceleration response reduces clearly. On the contrary, the acceleration response goes up with the increase of equivalent damping ratio for long part of period.

3.2. Comparison of the acceleration response spectrum and the Chinese codes

Figure 3 shows a comparison of elasto-plastic acceleration response spectrum on 4 types of sites and acceleration response spectrum in Chinese anti-seismic code. From the figure 3, it can be seen that the acceleration response suggested by the code is smaller than the elasto-plastic acceleration response for short part of period on the 4 types of sites; and for long part of period, it is quite contrary. Further more, as to short part of period, it is illuminated that the value of acceleration response is relatively small according to anti-seismic code, and the result of calculation is not safety. However, it is relatively large and too conservative for long part of period.

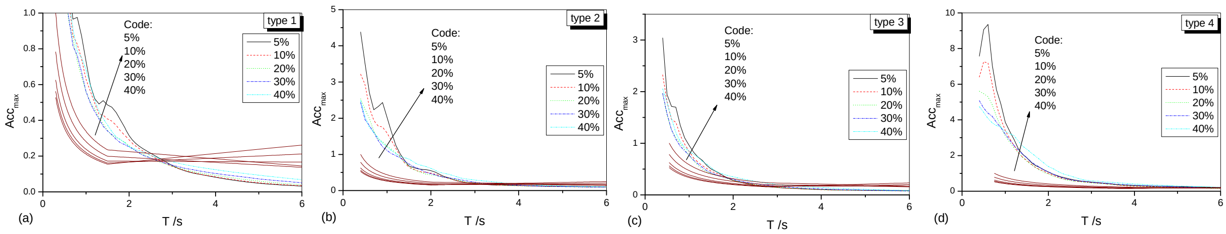

Figure 3 Comparison of the elasto-plasticity acceleration response spectrum and the codes

3.3. Formula of elasto-plastic response spectrum

From the result of analysis, it can be seen that using the acceleration response spectrum of anti-seismic to calculate the response of isolated buildings will bring in relatively large deviation, because this spectrum is mainly suitable for non-isolated building. In order to solve this problem, an elasto-plastic response spectrum should be established for isolated structures. Meanwhile, the equation form of elasto-plastic response spectrum is in line with the code so that it is convenient for engineering designers to use. But the attenuation curve of response spectrum in the long part of period is not segmented, just to adjust the attenuation index. The modified response spectrum curve for long period can be seen in figure.4.

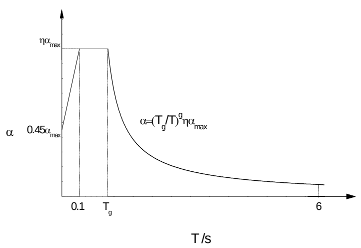

Figure 4 Modified acceleration design spectrum

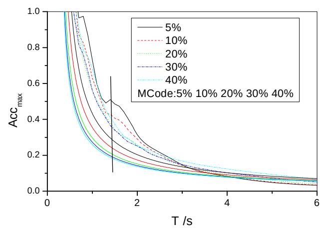

Figure 5 Comparison elasto-plasticity acceleration response spectrum and Modified acceleration

where $\gamma = 0.9 + \dfrac{0.05-\zeta}{0.5+5\zeta}$, the damping coefficient: $\eta = 1 + \dfrac{0.05-\zeta}{0.06+1.7\zeta}$.

4. CONCLUSIONS

The analytical model is a single-mass model and the Bouc-Wen hysteretic model is adopted for isolation layer, In addition, 80 waves is selected for dynamic analysis of the structure, according to 4 types of sites classified by Chinese anti-seismic code. From the relationship of equivalent damping ratio and displacement in inelastic hysteretic model, the elasto-plastic response spectrum based on equivalent ratio has been proposed through this study. The results of seismic response analysis are summarized as follows:

There is a small difference between the elasto-plastic acceleration response spectrums of isolated structure on 4 types of sites. While the period of range is 1-2sec, the acceleration response decreases rapidly, but if period is greater than 2sec, it diminishes slowly and gradually. And that the acceleration response goes up with the increase of equivalent damping ratio for long part of period. In addition, there is a great difference between the displacement response spectrums on 4 types of sites.

Through the comparison of elasto-plastic acceleration response spectrum and acceleration suggested by Chinese code for seismic design of buildings, it can be seen that the acceleration response of code is smaller than the elasto-plastic acceleration response for short part of period on the 4 types of sites; and for long part of period, it is quite contrary. Further more, for short part of period, it is illuminated that the value of acceleration response is relatively small according to anti-seismic code, and the result of calculation is not safety. However, it is relatively large and too conservative for long part of period.

The equation of elasto-plastic response spectrum for isolated structure has been proposed in the paper. The equation form of elasto-plastic response spectrum is in line with the code so that it is convenient for engineering designers to use. But the attenuation curve of response spectrum in the long part of period is not segmented, just to adjust the attenuation index.

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