SEISMIC RELIABILITY ANALYSIS AND OPTIMIZATION OF WATER SUPPLY NETWORK

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Abstract:
A water supply system is vulnerable in earthquake. The hydraulic distribution in post-earthquake is modeled by the hydraulic analysis of the water supply system with leakage. The pipe leakage area is used to describe the damaged state of a buried pipe in earthquake. Based upon the buried pipe’s seismic analysis, a calculating method for the pipe leakage area is put forward.

The pipe leakage areas in earthquake are random parameters. The seismic reliability of the water supply system is defined as the probability that the system provides a given service after earthquake. The paper presents the first-order-second-moment method to solve the seismic reliability.

The seismic designs should satisfy the reasonable and economical requirements for the water supply system in the normal state. Considering the requirements for the water supply system in the seismic state and the normal state, an inset optimization model is presented. The seismic reliability is selected as the system layout object. The average optimal flow velocity is selected as the restriction in the pipe diameter optimization.

Different elements have the different contribution to the system seismic reliability. Two optimization models are put forward. One is used for maintenance and the other is used for reconstruction after earthquake.

Keywords: water supply network, reliability, optimization, seismic

Introduction
A water supply system is vulnerable in earthquake. It is proved in Northridge Earthquake (1994) and Kobe Earthquake (1995). It is non-economical and impossible to ensure the water supply network perfectly surviving from the seismic damage. Therefore, some of the seismic damages have to be considered in the seismic design of the water supply system.

The seismic capability of the water supply system is achieved by means of the seismic design of the pipes, tanks, pumps and towers in china. In the seismic design procedure, all pipes are designed according to the same seismic standards though the main pipes are more important than the branch pipes in earthquake. It is more efficient to improve the seismic capability of the system by the increase of the seismic reliability of the key pipes than the increase of the seismic standards of all the pipes. In order to efficiently improve the seismic reliability of the system, it is necessary to confirm the key pipes in earthquake.
The key pipes are the ones that cause many nodes out of function once they are damaged in earthquake. In the other words, the key pipes are the ones that make more contributions to the seismic capability of system than other pipes. The influenced area of the damaged pipes will be different assuming that the damaged pipes can be controlled (e.g. closed) or uncontrolled (e.g. work with leakage) in the earthquake. Assuming that the damaged pipes can be controlled in earthquake, the connective analysis method can be used to calculate the influenced area of the damaged pipes. Otherwise, the system flow analysis with the damaged pipes has to be done. It is time-consuming to detect and close the damaged pipes except for the serious damaged pipes because most pipes are buried in soil. Therefore the damaged pipes (except for the serious damaged pipes) are in an uncontrolled state in earthquake. The system has to work with leakage, which leads to the changes of its hydraulic distribution after earthquake. This causes some nodes out of work. The hydraulic distribution after earthquake is modeled by a hydraulic analysis of the water supply system with leakage. The pipe leakage area is used to describe the damaged state of a buried pipe in earthquake. Based upon the buried pipe’s seismic analysis, a calculating method for the pipe leakage area is firstly put forward.

1. Leakage area estimation model of damaged pipes in earthquake

There are many vulnerability models or fragility curves to estimate the seismic damage rate of the buried pipes that are based upon the history seismic damage data, such as Eguchi model (1983), HAZUS model (NIBS, 1997) and ALA recommended pipe vulnerability functions (2001). The seismic pipe-damages can be further classified into two categories, leak and break. Kitaura et al (1996) suggested that for cast iron pipes 15% of the pipe damage be breaks and 85% leaks in the non-liquefied field. For ductile iron pipes, 4% of the pipe damage is breaks and 96% leaks in the non-liquefied field. In the liquefied field, breaks and leaks are half and half. The suggestion was based on the Kobe (1995) pipe seismic damage data. Define the non-damage probability as $P_1$, the light damage probability as $P_2$, and the serious damage probability as $P_3$. $P_1$, $P_2$, and $P_3$ satisfy

$$P_1 = 1 - P_2 - P_3$$

In the liquefied field:

$$P_2 = P_3 = 1 - e^{-RR \times L / 2}$$

In the non-liquefied field:

$$P_3 = 1 - e^{-0.15 \times RR \times L}$$

$$P_2 = 1 - e^{-0.85 \times RR \times L}$$

Where $RR$ is the pipe seismic damage rate that can be estimated by ALA recommended pipe vulnerability functions (2001) and $L$ is the length of a pipe.

In order to measure the different damage modes of pipes for further analysis, Ballantyne et al (1990) and Hwang et al (1998) estimated that the average leakage area was equivalent to 3% of the pipe cross section area and the break area is equal to the pipe cross section area ($A$). In fact, the leakage area of the damaged pipe is a random value in the range of 0 and $A$. The relationship of the damage mode and the leakage area of pipe is shown in Fig.1.
Here, $A_L$ is the leakage area of pipes, $\alpha_1, \alpha_2, \alpha_3$ and $\alpha_4$ are the limits. In the normal state, the water supply network will lose more than 3%~5% water, which is unavoidable. It is universal that the water supply network wastes 10%~15% water. Water volume is the direct ratio to the pipe cross-section approximately. Therefore, it is reasonable to assume $\alpha_1 = 0.03$, $\alpha_2 = 0.06$, $\alpha_3 = 0.12$, $\alpha_4 = 0.15$. The mean value of the pipe leakage area can be expressed as

$$E \left( \frac{A_L}{A} \right) = P_1 \cdot \frac{\alpha_1}{3} + P_2 \left/ (\alpha_3 + \alpha_4 - \alpha_1 - \alpha_2) \right. \left[ \frac{\alpha_1^2 + \alpha_2^2 - \alpha_1^2 - \alpha_2^2 - \alpha_3 \alpha_2 + \alpha_3 \alpha_4}{3} \right]$$

$$+ \frac{P_3}{(2 - \alpha_3 - \alpha_4)} \left[ \frac{3 - \alpha_3^2 - \alpha_4^2 - \alpha_3 \alpha_4}{3} \right]$$

(5)

The variance of the leakage area is

$$\sigma^2 \left( \frac{A_L}{A} \right) = P_1 \cdot \frac{\alpha_1^2}{6} + P_2 \left/ (\alpha_3 + \alpha_4 - \alpha_1 - \alpha_2) \right. \left[ \frac{\alpha_1^3 + \alpha_2^3 - \alpha_1^3 - \alpha_2^3}{6} \right]$$

$$+ \frac{P_3}{(2 - \alpha_3 - \alpha_4)} \left[ \frac{4 - \alpha_3^3 - \alpha_4^3}{6} \right]$$

(6)

In this paper, Function (5) and Function (6) can be simplified as

$$E \left( \frac{A_L}{A} \right) = 0.01P_1 + 0.09P_2 + 0.567P_3 \quad \sigma^2 \left( \frac{A_L}{A} \right) = 0.00015P_1 + 0.0045P_2 + 0.3849P_3$$

2. Seismic Reliability of Water Supply Network

The limit state equation for the nodal service of the seismic damaged water supply network can be expressed as

$$G_i = H_i(A_L) - H_{i, \text{min}}$$

(7)

Where $H_i$ is the $i^{th}$ nodal pressure that is the nonlinear equation of the pipe leakage area. $H_{i, \text{min}}$ is the minimum allowable pressure. $H_{i, \text{min}} = 10$ meters in the paper, which is determined by the requirements for the fire fighting.
When $G_i > 0$, the node is in the reliable serving state. When $G_i = 0$, the node is in the limit state. When $G_i < 0$, the node is in the service failure state. The system seismic serviceability can be expressed as

$$P_i = P(G_i > 0)$$  \hspace{1cm} (8)

The seismic reliability index of the water supply network is

$$\beta_i = \frac{u_{G_i}}{\sigma_{G_i}}$$  \hspace{1cm} (9)

When the minimum allowable head $H_{i,\text{min}}$ is regarded as a determined value, Equation (14) can be rewritten as

$$\beta_i = \frac{u_{H_{i}(A_L)}}{\sigma_{H_{i}(A_L)}}$$  \hspace{1cm} (10)

The water supply network in earthquake satisfies the mass balance theory, the energy conversation law, Hazen-Williams function and the leakage model. The nodal formulation of the network in earthquake can be expressed as

$$F(H', A_L) + \left\{ \frac{Q}{Q^L(A_L)} \right\} = 0$$  \hspace{1cm} (11)

Where $H' = [H_1, H_2, \ldots, H_n, H_{L,1}, \ldots, H_{L,n}]^T$, $A_L = [A_{L,1}, \ldots, A_{L,n2}]^T$, the nodal demand flow $Q$ of the water supply network in the post-earthquake may be different from the values in the normal states because the nodal demand flow in the post-earthquake requires the adjustments according to the seismic standards of the water supply network.

If the network solution exists at the mean values of the pipe damage areas, the nodal pressures of the damage area can be approximately regarded as the mean values of nodal heads. Meanwhile, the standard deviation of the nodal pressures can be gained from the standard deviation of the pipe damage area by using the first-order Taylor Expansion.

$$F(H', \overline{A}_L) + \left\{ \frac{Q}{Q^L(\overline{A}_L)} \right\} + \left( \frac{\partial F}{\partial H'} + \frac{\partial Q^L}{\partial H'} \right) \cdot \Delta H' + \left( \frac{\partial F}{\partial A_L} + \frac{\partial Q^L}{\partial A_L} \right) \cdot \Delta A_L = 0$$  \hspace{1cm} (12)

Since the hydraulic network is balanced at the mean values of the pipe damage area, the first term plus the second term in Eq.(12) is zero. Thus, Eq. (12) can be also expressed as

$$H' = \overline{H'} - \left( \frac{\partial F}{\partial H'} + \frac{\partial Q^L}{\partial H'} \right)^{-1} \cdot \frac{\partial Q^L}{\partial A_L} \cdot (A_L - \overline{A}_L) = 0$$  \hspace{1cm} (13)

Defining $B = -\left( \frac{\partial F}{\partial H'} + \frac{\partial Q^L}{\partial H'} \right)$, $C = \frac{\partial Q^L}{\partial A_L}$, $\hat{H} = \overline{H'} - B^{-1}C\overline{A}_L$, Eq. (13) can be written as

$$H = \overline{H} + B^{-1}C \cdot (A_L - \overline{A}_L)$$  \hspace{1cm} (14)
The deviation of the node pressure is

\[ \sigma_{H_i}^2 = \sum_{j=1}^{n_1} \sum_{k=1}^{n_2} C_{ij} C_{ik} \text{Cov}(A_{ij}, A_{ik}) \]  

(15)

The seismic reliability of the node in the water supply network is

\[ \beta_i = \frac{H_i - H_{\text{min}}}{\sigma_{H_i}} \quad (i = 1, 2, \ldots, n) \]  

(16)


There are many factors that affect the seismic reliability of the system, such as the topological structure, pipe joint types, pipe material, pipe diameter, etc. During the planning period, the topological optimization is the most effective method to improve the seismic capability of the system. The topological optimization imbeds the element design. In order to avoid the contradiction between the seismic design and the design for the regular working state of the water supply network, the seismic reliability is selected as the system layout object. It proves that all the values of the flow velocity for the reasonable and economical water supply system in the normal state are in the limited value range. The optimal flow velocity is selected as the restriction in the pipe diameter optimization. The embedding optimization model is

**P1:** minimize \( \sum \gamma_{ij} \cdot \cos(t_{ij}, d_{ij}) \)  

subject to \( \beta_{\text{min}} \geq \beta_0 \)

**P2:** min. Cost \( = \sum_{i=1}^{N_{\text{pipe}}} (a_1 + a_2 \cdot D_i^{\gamma_j}) \cdot L_i \)  

subject to \( V_{\text{min}} \leq V_i \leq V_{\text{max}} \), \( d_{\text{min}} \leq D_i \leq d_{\text{max}} \), \( (i = 1, 2, \ldots, N_{\text{pipe}}) \)

Where \( \gamma_{ij} = 0 \) or 1. When \( \gamma_{ij} = 0 \), there are no pipes between node \( i^{th} \) and \( j^{th} \). When \( \gamma_{ij} = 1 \), there is one pipe between node \( i^{th} \) and \( j^{th} \). \( l_{ij} \) and \( d_{ij} \) are respectively the pipe length and the pipe diameter that are determined by the model P2. \( \beta_{\text{min}} = \min(\beta_1, \beta_2, \ldots, \beta_n) \) is the function of \( \gamma_{ij} \) and \( D_i \).

\( \beta_0 \) is the expected nodal seismic reliability index of the water supply network. \( a_1, a_2, a_3 \) are the constants. \( N_{\text{pipe}} \) is the number of the total pipes in the topology that is determined by model P1. \( V \) \((m/s)\) is the flow velocity, which is the function of the pipe diameter. The lower and upper limits of the flow velocity present the range of the average economical flow velocity. \( L_i \) and \( D_i \) are the length and the diameter of the \( i^{th} \) pipe.

There is little chance to build a new water supply network. For the network maintenance, the change of joint types is the most effective method to improve the pipe seismic capability. The pipe seismic optimization model is
P3: \[ \text{minimize } \sum \psi_i \cdot \cos t(l_i, d_i, s_i) \]  

Subject to \[ \beta_{\text{min}} \geq \beta_0 \]  

Here, \( \psi_i = 0 \) or 1. When \( \psi_i = 0 \), there is no change for the \( i^{th} \) pipe. When \( \psi_i = 1 \), it means changing the joint types of \( i^{th} \) pipe. \( l_i \) (meter) is the length of the \( i^{th} \) pipe. \( d_i \) (meter) is the diameter of the \( i^{th} \) pipe. \( s_i \) (meter) is the buried depth of the \( i^{th} \) pipe in soil. The larger \( l_i, d_i \) and \( s_i \) are, the bigger the investment for the pipe seismic reconstruction is. \[ \beta_{\text{min}} = \min(\beta_1, \beta_2, \cdots, \beta_n) \] is the function of \( \psi_i \). \[ \cos t(l_i, d_i, s_i) = c_0 \cdot l_i \cdot s_i \cdot d_i^c \] calculates the investment for changing the joint types of a pipe. \( c_0 \) and \( c_i \) are the constants.

It is reasonable to give the top priority to the reconstruction of the elements that are of more help to improve the whole system reliability. The element-approximately-sensitive-degree method is put forward to measure the contribution of a pipe to the seismic capability of the water supply network. The element-approximately-sensitive-degree is

\[ e_i = \frac{R_{\text{sys}}(r_1, r_2, \cdots, r_{i-1}, r_i, r_{i+1}, \cdots, r_n) - R_{\text{sys}}(r_1, r_2, \cdots, r_{i-1}, r_i^{'}, r_{i+1}, \cdots, r_n)}{C_i} \]  

\( R_{\text{sys}}(r_1, r_2, \cdots, r_{i-1}, r_i, r_{i+1}, \cdots, r_n) \) is the seismic capability of the water supply network which can be measured by \( \beta_{\text{min}} \) or the system serving area or the system serving population after earthquake. \( r_i \) is the seismic reliability before the \( i^{th} \) element is reconstructed. \( r_i^{'}, \) is the seismic reliability after the \( i^{th} \) element is reconstructed. \( r_i \) and \( r_i^{'}, \) can be measured by the seismic damage ratio. \( C_i \) is the investment for the reconstruction of the \( i^{th} \) element.

It is efficient and economical for the improvement of the seismic capability of the system to reconstruct the pipes of which \( e_i \) are larger than other pipes.

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