THE SIMPLIFIED ELASTO-PLASTIC ANALYSIS MODEL OF REINFORCED CONCRETE FRAMED SHEAR WALLS

Norikazu ONOZATO¹, Yukichi KANEHIRA², Hiroyuki MATSUDA³ and Makoto MOCHIZUKI⁴

SUMMARY

In this paper, the load-displacement relations of reinforced concrete shear walls are determined by analysis. The simplified elasto-plastic analysis models that the author proposed to analyze the maximum strength are used for the analysis. The models are simple as compared with other elastio-plastic analytic models. In order to determine load-displacement relations using these models, the stress-strain relations of the struts are improved through many experimental results. The struts constitute the wall panels of shear walls. Therefore, the stress-strain relations of struts are associated with the compressive strengths of concrete, and the displacement of shear walls. And those relational equations are determined. By using these relational equations for the simplified elasto-plastic analysis model, the load-displacement relations of shear walls are analyzed accurately.

INTRODUCTION

In the studies to analyze the maximum strength of reinforced concrete framed shear walls (it is abbreviated to shear walls henceforth), it is possible to analyze accurately using the approach by the macro models adapting the limit analysis method. Also in the studies to analyze the load-displacement relations of shear walls, the load-displacement relations to the maximum strength are analyzed accurately. However, the load-displacement relations after the maximum strength are not analyzed exactly. In the structural design of buildings, it is important to evaluate the load-displacement relations of earthquake resisting elements exactly. Since shear walls are resisting the great seismic force particularly, they are very important.

¹ Lecturer, Kogakuin University, Tokyo, Japan
² Kogakuin University, Tokyo, Japan
³ Graduate student, Kogakuin University, Tokyo, Japan
⁴ Emeritus Professor, Kogakuin University, Tokyo, Japan
In order to analyze the maximum strength of shear walls, Fig. 1 shows the simplified elasto-plastic analysis model. The authors showed it in reference [1]. In the reference, the analysis results of the maximum strength of 518 specimens of Japan analyzed by the models are shown. The analysis accuracy is as follows. In examine of the value of the experimental value / analysis value of the maximum strength, the mean value

Table 1 Data of specimens

<table>
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<tr>
<th>No.</th>
<th>Specimen</th>
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<th>D (mm)</th>
<th>N/mm²</th>
<th>p_h</th>
<th>h′</th>
<th>t</th>
<th>σ_y</th>
<th>σ_p</th>
<th>N</th>
<th>σ_p / N</th>
<th>T</th>
<th>σ_y / σ_p</th>
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<th>eRU</th>
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**Table 1 Data of specimens**

- **No.**: Experiment number
- **Specimen**: Sample name
- **b (mm)**: Width of column
- **D (mm)**: Depth of column
- **N/mm²**: Longitudinal reinforcement ratio
- **p_h**: Yield strength of longitudinal reinforcement
- **h′**: Height of wall
- **t**: Thickness of wall
- **σ_y**: Yield strength of hoop reinforcement
- **σ_p**: Hoop reinforcement ratio
- **N**: Axial force
- **σ_p / N**: Vertical reinforcement ratio of wall
- **T**: Axial force
- **σ_y / σ_p**: Horizontal reinforcement ratio of wall
- **eRM**: Height of point of contraflexure
- **eRU**: Axial force
- **eRx**: Displacement at Qexp
- **eRy**: Displacement of Qexp in downward region

**References**

[2] Strength of 518 specimens of Japan analyzed by the models are shown. The analysis accuracy is as follows.
[3] In the reference, the analysis results of the maximum strength of 518 specimens of Japan analyzed by the models are shown. The analysis accuracy is as follows.
is 1.013, a standard deviation is 0.124, and coefficient of variation is 0.124. The models are more accurate than other ones to analyze the maximum strength of shear walls. And in the models proposed as elasto-plastic analysis models, they are very simple models. Hereafter, the simplified elasto-plastic analytic models analyze the load-displacement relations of shear walls.

**SPECIMENS FOR ANALYSIS**

The specimens chosen for analysis are 55 among authors' experiments. Those experiments were executed from 1987 to 1997. Those results are shown in Reference [2] - [10]. The data of all specimens are shown in Table 1. The loading of the displacement increment repeated in the direction of plus and minus controlled by displacement acted on all specimens. Thus, the number of samples is 110 including plus and minus loads. Furthermore, the limit displacement shown in Table 1 is the displacement at the strength falling to 80% of the maximum strength. In Japan, the limit displacement is generally used to estimate the deformation capacity of earthquake resisting elements.

**ANALYTIC MODELS**

The simplified elasto-plastic analytic models were assumed as follows to analyze the maximum strength of shear walls. Multi-story framed shear walls are the objects to analyze by the models. Therefore, the upper beam and footing beam of shear walls are substituted for rigid bodies in the models. The wall panel is substituted for inclined multiple struts and the tension members of the vertically and horizontally direction. Struts are concrete wall plates, and tension members are reinforcing bars. The details of the stress-strain relations of struts are mentioned later. Tension members are perfect elasto-plasticity, which has the stiffness and strength of reinforcing bars and does not resist the compressive force. Rigid body-spring models substitute for the columns of each side. The springs, which connect rigid bodies, are the combination of axle springs and a shear spring. Axle springs substitute for the stiffness and strength of concrete and longitudinal bars. The stress-strain relations of the concrete are the perfect elasto-plasticity with which secant stiffness is used for elastic stiffness and the compressive strength is used for yield strength. The stress-strain relations of the reinforcing bars are the perfect elasto-plasticity with which yield strength was used for the maximum strength. The sectional area of an axle spring is the half of a column cross section, and the position is the center of gravity of the longitudinal bar in the half of a column depth. However, axle springs have only the properties of reinforcing bars in the state of the tensile force. The substitution length is equal to the length of a rigid element. A shear spring is substituted for the shear stiffness of concrete equal to the length of a rigid element. Struts, tension members, axle springs, and shear springs are connected to the surface of rigid elements. Fig. 2 shows the hysteresis rule of a strut, a tension member, an axle spring, and a shear spring.

As examples of analysis, Fig.3 shows the load-displacement relation curves of an experiment and analysis. Also, the analysis result of other specimens is almost the same. Though analysis values are almost the same as experimental values about the maximum strength, the relation between a load and displacement is not the same. In the stress-strain relations of Fig.3, especially struts control the load-displacement relation of a shear wall. The stress-strain relations of the other elements seldom affect the load-displacement relations of
shear walls. Therefore, in order to determine the load-displacement relations of shear walls, it is necessary to reconsider assumption of the stress-strain relation of the struts currently used in the analysis of the maximum strength. Furthermore, the inclination angle $\theta$ of struts shown in Fig.1 is the direction of the principal stress of the wall panels at the maximum strength of shear walls. The direction of the principal stress is determined as follows. When struts change various inclination angles $\theta$ and the maximum strength is analyzed, principal stress is the angle which obtains the greatest maximum strength. However, this approach requires many calculations. The authors proposed the macro models [11] applying a limit analysis method, in order to determine the maximum strength other than the simplified elasto-plastic analysis models. Macro models were constructed based on elasto-plastic analysis models, and it excels in determining the inclination angle $\theta$ of struts. Hereafter, the inclination angles $\theta$ determined by macro models are used for an angle of inclination of the struts of the simplified elasto-plastic analysis model.

**STRESS-STRAIN RELATION OF STRUTS**

**Stress-strain relation of the struts for the maximum strength analysis**

The stress $\sigma$-strain $\varepsilon$ relations of the struts used for the analysis of the maximum strength of shear walls apply the equation of Popovics [12] shown in an equation (1).

$$\sigma = \frac{n \cdot \varepsilon / \varepsilon_0}{n - 1 + (\varepsilon / \varepsilon_0)^n} \times \sigma_B$$  \hspace{1cm} (1)

Where, $\varepsilon_0$: Strain at the maximum stress, $n$: Experiment constant, $\sigma_B$: Unconfined compressive strength.

It is confirmed that the equation of Popovics is flexible including a downward region of stress-strain curves of concrete. Although the equation of Popovics means the character of cylinder concrete, it does not mean the character of the concrete of the wall panels of shear walls. Therefore, when this equation is used to consider the biaxial stress states of wall panels, the concrete compressive strength $\sigma_B$ is multiplied by 0.63 (Fig.2). 0.63 is an effectiveness factor of the wall panel concrete. It is determined by regression analysis from the experimental data of 36 specimens, which occurred slip failure all over the wall. The authors showed it in reference [13]. Also, it was observed in the experiment that failure of the wall panels after the maximum strength is very brittle. And it is not tough as the equation of Popovics shows. Therefore, the downward region of this equation was corrected. The correction is a simple method of setting the limit strain $\varepsilon_u$, as shown by Fig.2. As mentioned above, the maximum stress was thoroughly compared with experimental results. However, strain $\varepsilon_0$ the maximum stress, the experiment constant $n$, and the limit strain $\varepsilon_u$ were not thoroughly compared.

**Effect of the compressive strength of concrete**

In order to determine exact $\varepsilon_0$, and $n$ and $\varepsilon_u$, the stress-strain relations of struts are examined by the experimental result. First, it is examined to determine $\varepsilon_0$. The specimens for the examination must be chosen from the specimens that have comparatively small limit displacement for the following reasons. The wall panels of specimens with great limit displacement are acted by many repeated loads until they achieve limit displacement. Because repeated loads deteriorate wall panels, it is thought that the stress-strain relation of the struts of specimens changes with the sizes of limit displacement. Therefore, to examine strain of struts should be targeted at specimens that are acted by limited repeated loads, and they are specimens with limited limit displacement. Then, the limit displacement of the target specimens is provided under $8 \times 10^{-3}$rad. In Table 1, the number of the target specimens is 12.
In each specimen, the strain $\varepsilon_0$ at the maximum strength with the highest suitability is determined by comparing with the envelope curves of an experiment and analysis. The envelope curves of the analysis are obtained by the analysis changing the value of $\varepsilon_0$ of the stress-strain curved line of struts. However, there are cases that the maximum strength of specimens is difficult to read correctly by the scattering in an experimental value. To consider such cases, the displacement at the maximum strength of experiments is determined as an average of displacement of the upturned region and downward region at 90% of the maximum strength on an envelope curve. Fig. 4 shows relations of strain $\varepsilon_0$ of the struts at the maximum strength obtained by such an approach and concrete compressive strength $\sigma_B$. It is understood that $\varepsilon_0$ decreases with the increase in $\sigma_B$. The curved line in Fig. 4 is a regression curve of distribution, and is shown with the following equation.

$$\varepsilon_0 = 0.0065/\sigma_B + 0.0020 \quad (2)$$

The experiment constant $n$ of the equation of Popovics transmutes the configuration of a stress-strain curved line. When the value of $n$ is 1, the property of perfect rigid plasticity is shown. When the value of $n$ is infinity, the property of perfect elasto-brittleness is shown. In each specimen, the suitable $n$ is determined by comparing with the envelope curves of an experiment and analysis. The envelope curves of analysis are obtained by the analysis changing the value of $n$ of the stress-strain curved line of struts. Here, the values of $\varepsilon_0$ determined for each specimen are used as a value of $\varepsilon_0$. Fig. 5 shows relations of $n$ obtained by such an approach and concrete compressive strength $\sigma_B$. It is understood that $n$ increases with $\sigma_B$. The curved line in Fig. 5 is a regression curve of distribution, and is shown with the following equation.

$$n = 0.057\sigma_B + 0.79 \quad (3)$$

The suitable $\varepsilon_u$ is also determined by comparing with the envelope curves of an experiment and analysis. The envelope curves of analysis are obtained by the analysis changing $\varepsilon_u$ value of the stress-strain curved line of struts. Here, the values of $\varepsilon_0$ and $n$ determined each specimen are used as values of $\varepsilon_0$ and $n$. Fig. 6 shows the relation between limit strain $\varepsilon_u$ obtained by such an approach and concrete compressive strength $\sigma_B$. It is understood that $\varepsilon_u$ decreases with the increase in $\sigma_B$ same as $\varepsilon_0$. The curved line in Fig. 6 is a regression curve of distribution, and is shown with the following equation.

$$\varepsilon_u = 0.043/\sigma_B + 0.0022 \quad (4)$$

As for the result above, $\varepsilon_0$, $n$, and $\varepsilon_u$ are derived as a function of compressive strength $\sigma_B$ of concrete, respectively. They are shown in an equation (2), an equation (3), and an equation (4). All the specimens of Table 1 are analyzed using these equations. Fig. 7 shows the relation of the analysis value $\bar{R}_u$ and experimental value $R_u$ of limit displacement. The analysis values of the specimens more than limit deformation $8 \times 10^{-3}$rad are less accurate than the analysis values of the specimens used to determine these equations. Those analysis values are larger than experimental values. As stated previously, when limit
displacement is large, a wall panel is deteriorated under the effect of repeated loads. Therefore, it needs to consider that the stress-strain curved line of struts changes according to the quantity of displacement.

**Effect of degradation by a repeated load**

By using the specimens that exceed limit deformation $8 \times 10^{-3}$ rad, improved $\varepsilon_u$ is determined by same operations as equation (4). However, the samples of limit displacement exceeding $20 \times 10^{-3}$ rad are excluded because the maximum displacement observed in the experiment is $20 \times 10^{-3}$ rad. Fig. 8 shows the relation between $\varepsilon_u/\varepsilon_u$ obtained by this approach and limit displacement $\varepsilon_R_u$. It is understood that the limit strain of struts decreases with the increase in limit displacement. This is considered to be a degradation phenomenon accompanying the increase in displacement. Thus, reduction ratio $\varepsilon_u/\varepsilon_u$ of limit strain is associated by displacement $R$. It is shown with the following equation as a regression curve of distribution of Fig. 8.

$$\varepsilon_u/\varepsilon_u = \min(0.094, \varepsilon_R_u^{-0.48}, 1)$$

$$\varepsilon_u = \min(0.094, \varepsilon_R_u^{-0.48}, 1)(0.043/\sigma_B + 0.0022)$$  (6)

Fig.9 changes an equation (4) to an equation (6), and shows the result of having reanalyzed all the specimens. Furthermore, Fig.10 shows the relation of the analysis values $\varepsilon_R_m$ and experimental values $\varepsilon_R_m$ of the displacement at the maximum strength. The analysis accuracy of Figs.9 and 10 has the value of an experimental value / analysis value as follows. A mean value is 1.008 from 1.034, and a standard deviation is 0.221, and coefficient of variation is 0.219 from 0.214. Accurate analyses of the load-displacement relations of shear walls are enabled by the simplified elasto-plastic analysis models.

**Simple calculation of the inclination angle of struts**

In reference [2], authors calculated 518 specimens using the macro model. And the close correlation between the inclination angle $\theta$ of a strut and the aspect ratio $h'/\ell$ of a wall panel became obvious. An equation (7) shows the relation.

$$\theta = \tan^{-1}(0.72 h'/\ell + 0.40)$$

Furthermore, reference [2] shows the equation (7) can be used for the inclination angle $\theta$ of the struts of the elasto-plastic analysis for the maximum strength. Fig.11 shows relation of experimental values and the analysis values that used $\theta$ of an equation (7). Since there is no great difference in the analysis accuracy of Figs.9, 10, and 11, the practicality of an equation (7) is clear.
The analysis accuracy of the simplified elasto-plastic analysis models was verified using the experimented specimens by the authors. And it was shown sufficient analysis accuracy among these specimens. However, all of these specimens are the experiments of the same loading. Different load doesn't guarantee sufficient accuracy. Especially dynamic load and one-way load may not be unable to calculate an experimental value correctly. However, it is expected that dynamic load and one-way load have larger strength and displacement than static load. The number of repeated loads is more than an experiment of other researchers because experiments of the authors are repeated twice each for the increment of 1×10⁻³ rad to displacement 10×10⁻³ rad. Therefore, it is expected that the limit displacement of the authors' experiment is smaller than other experiments. Thus, the analysis approach determined by the authors' experiments gives safety evaluation to other experiments.

CONCLUSION

Simplified elasto-plastic analysis models for analyzing the load-displacement relations of shear walls were shown. The analysis result of the model was able to catch the load-displacement relations of 55 specimens in sufficient accuracy. The analysis accuracy of the model is controlled by the stress-strain relation of struts, and the feature of struts can be summarized as follows.
1) The strain $\varepsilon_0$ at the maximum strength of a strut decreases with the increase in compressive strength $\sigma_B$ of concrete. The relational equation is shown as an equation (2).
2) In order to use the equation of Popovics for the stress-strain curved line of struts, the experiment constant $n$ increases with the increase in concrete compressive strength $\sigma_B$. The relational equation is shown as an equation (3).
3) In order to use the equation of Popovics for the stress-strain curved line of struts, it is necessary to prepare limit strain $\varepsilon_u$. The value of limit strain $\varepsilon_u$ decreases with the increase in concrete compressive strength $\sigma_B$ and displacement $R$ of a shear wall. The relational equation is shown with an equation (6).
REFERENCES


