



EQUIVALENT LINEARIZATION IN EARTHQUAKE RESPONSE ANALYSIS OF PRESTRESSED REINFORCED CONCRETE SYSTEM

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SUMMARY

A hysteresis model for prestressed concrete members, PC model, was proposed where the unloading stiffness was degraded as a function of the prestress moment ratio defined as the ratio of the flexural capacity by the prestress tendon to the total capacity. Reduction of the hysteretic energy of the model with the prestress moment ratio was verified with experimental data. Nonlinear earthquake response analyses of single-degree-of-freedom systems with the PC model were carried out. The responses were compared with those estimated by an equivalent linearization method adopted in the revised Building Code of Japan. The estimation method had been verified for reinforced concrete models, where the secant stiffness to the maximum response and the substitute damping after Shibata and Sozen were used. The equivalent linear estimation generally underestimated the nonlinear responses of the PC model and the error increased with the increase of the prestress moment ratio. This is because the stiffness proportional damping is assumed in the nonlinear analysis while the viscous damping is assumed to be constant in the code specified linearization. This may be pointed out as an essential source of error in the current estimation, which becomes obvious in the cases with small hysteretic energy and large degrading stiffness, such as of the PC model. A new and rational definition of the equivalent damping was proposed in consideration of the degrading stiffness, from which a good correlation was obtained generally between the estimated and the calculated responses of the PC model.

INTRODUCTION

It has been pointed out that prestressed concrete (PC) members shows obvious pinching behavior in the hysteresis relations under cyclic loading. The pinching shape varies depending on the prestress ratio, i.e., the ratio of the flexural capacity provided by the prestressed tendon to the total capacity in the section. If the prestress ratio is higher, then the pinching becomes more obvious so that the hysteretic energy dissipation under cyclic loading becomes relatively smaller than that of typical reinforced concrete members with stable behavior. Therefore, the nonlinear displacement responses of the prestress concrete structures to earthquakes would be larger than those of reinforced concrete structures due to the pinching behavior, although the effects on the response have not yet been investigated quantitatively.

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The objective of this study is to make a hysteresis model for numerical response analysis idealizing the pinching behavior of prestressed concrete members named as PC model, to quantify the effects of the pinching characteristics on the maximum displacement responses of the prestressed concrete structures by numerical simulation using the model, and to interpret the analytical results theoretically. Nonlinear earthquake response analysis is carried out by single degree of freedom system with the PC model. The responses are compared with those of the system with a hysteresis model of reinforced concrete members called as RC model and are analyzed with the system parameters. Also, the responses are estimated from an equivalent linear system, by which the effects of the system parameters on the responses can be interpreted theoretically.

HYSTERESIS MODEL OF PRESTRESSED CONCRETE MEMBERS

Hysteretic energy dissipation of prestressed concrete members under cyclic loading after yielding is generally smaller than that of reinforced concrete members, because the unloading stiffness from the maximum inelastic deformation tends to degrade relatively more in the prestressed concrete members due to the axial load supplied by the prestressed tendon. The flexural capacity provided by the elastic tendon shows theoretically nonlinear elastic behavior as in the case of a member under constant axial load. The hysteresis model¹⁾ considering this property has been idealized [Hayashi, Okamoto and Otani, 1995]. In this study, the hysteresis rules for the PC model was coded for a response computer program, where the details were completed and added to complete the rule under any possible loading paths. The hysteresis characteristics of the model are described as follows in comparison with the hysteresis model of reinforced concrete members known as Takeda model²⁾ [Takeda, Sozen and Nielsen, 1970], which is called here as RC model

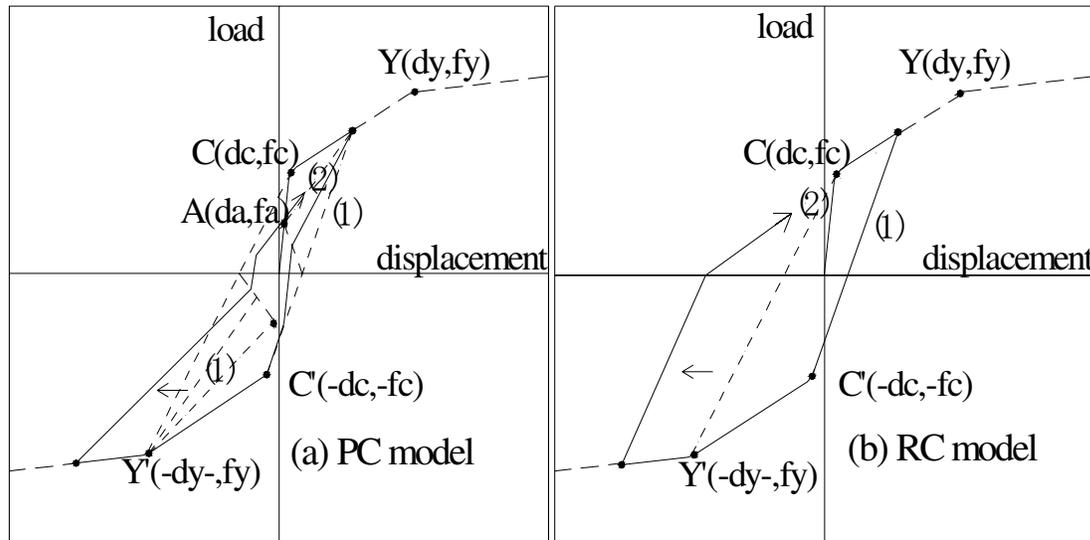


Figure 1 Hysteresis rules of PC model and RC model

Hysteresis of a PC member can be specified from the member section and the prestress tendon. Typical hysteresis rules under cyclic loading are compared for the PC and RC models in Figure 1. The hysteresis shapes of the PC model are mainly characterized by the prestress moment ratio λ , which is defined as the ratio of the incremental flexural capacity by the prestress tendon to the total flexural capacity of the section. Therefore the factor λ can be calculated for a PC member section as a function of the ratio of the amount of the prestressed steel tendon to the amount of the total steel bars in the member section. The

index λ is zero for a reinforced concrete member, unity for a fully prestressed concrete member without reinforcing bars, and between zero and unity for a partially prestressed concrete member.

The hysteresis shape may also be affected by the averaged compressive stress in concrete introduced by the prestressing tendon, because the cracking strength becomes higher by the prestress. The prestress level also affects the unloading path. Therefore, another factor F_a is used as the index to prescribe the hysteresis rules of the PC model, which is the increment of the cracking moment by the prestress. A point corresponding to this force level in the elastic stiffness is defined as Point A as shown in Figure 1.

- 1) The unloading stiffness K before and after yielding from peak response points in the PC model is degraded from that of the unloading stiffness K_c of the RC model using a prestress effect factor λ' , as shown in the following equation (1):

$$K = \lambda' K_a + (1 - \lambda') K_c \quad (1)$$

K_a is initially given as the stiffness from the peak response point toward the point A at the force level of F_a in the side of the peak point before yielding. If the peak is beyond yielding, K_a is degraded using the ductility factor μ and the degrading exponential coefficient γ' , similarly to the degrading rule of the unloading stiffness in the RC model after yielding, as given by the equation (2):

$$K_a = \frac{F_y - F_a}{D_y - D_a} \times \left(\frac{D_m}{D_a} \right)^{-\gamma'} \quad \text{after yielding} \quad K_a = \frac{F_m - F_a}{D_m - D_a} \quad \text{before yielding} \quad (2)$$

d_a : displacement in point A d_y : displacement in yielding d_m : displacement in maximum response
 F_a : force in point A F_y : force in yielding F_m : force in maximum response

These hysteresis indexes λ' and γ' are given as functions of the prestress moment ratio λ . so that the unloading stiffness of the model would fit close to that observed in past tests of prestressed concrete specimens with various sections and prestress levels¹⁾. The relations drawn from the tests are shown in Figure 2.

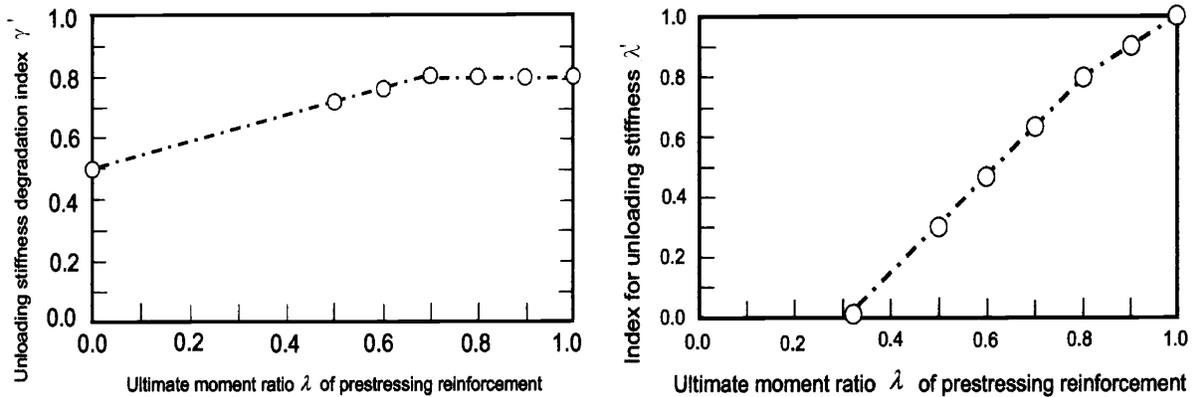


Figure 2 The relation between factors λ' , γ' in PC model and factor λ for PC member section³⁾

The index γ' value starts from 0.5 for non-prestressed RC members and is linearly increased up to the maximum value of 0.8 for partially prestressed at the prestressed moment ratio of 0.7 to the full prestress

level. The index λ' also is taken as 0 up the prestressed moment ratio λ is 0.3 and is increased with λ . Because the unloading hysteresis rules of the PC model are based on those of the RC model, the shape is almost identical for the two models when λ is zero. The pinching behavior of the PC model gradually becomes obvious with the increase of the prestress moment ratio λ .

2) The reloading rules after zero crossing to the opposite directions are slightly different between the two models. If the cracking occurs in both directions, the reloading stiffness of the RC model goes toward the yielding point in the opposite direction. On the other hand, the reloading stiffness of the PC model goes toward the last maximum response point.

The overall hysteretic relations between the horizontal earthquake induced force and displacement in a structure results in summation of the hysteretic responses of all elements in the structure. In this analysis model, a typical prestress concrete beam section is assumed, and the hysteresis model of the beam member is supposed to be the overall structural response against the horizontal force. The sectional properties, especially the prestress moment ratio, are systematically varied. Stiffness before yielding and that after yielding in the skeleton curve suppose one-thirds and 1% of elastic stiffness each. Cracking force supposes one-thirds of yielding force.

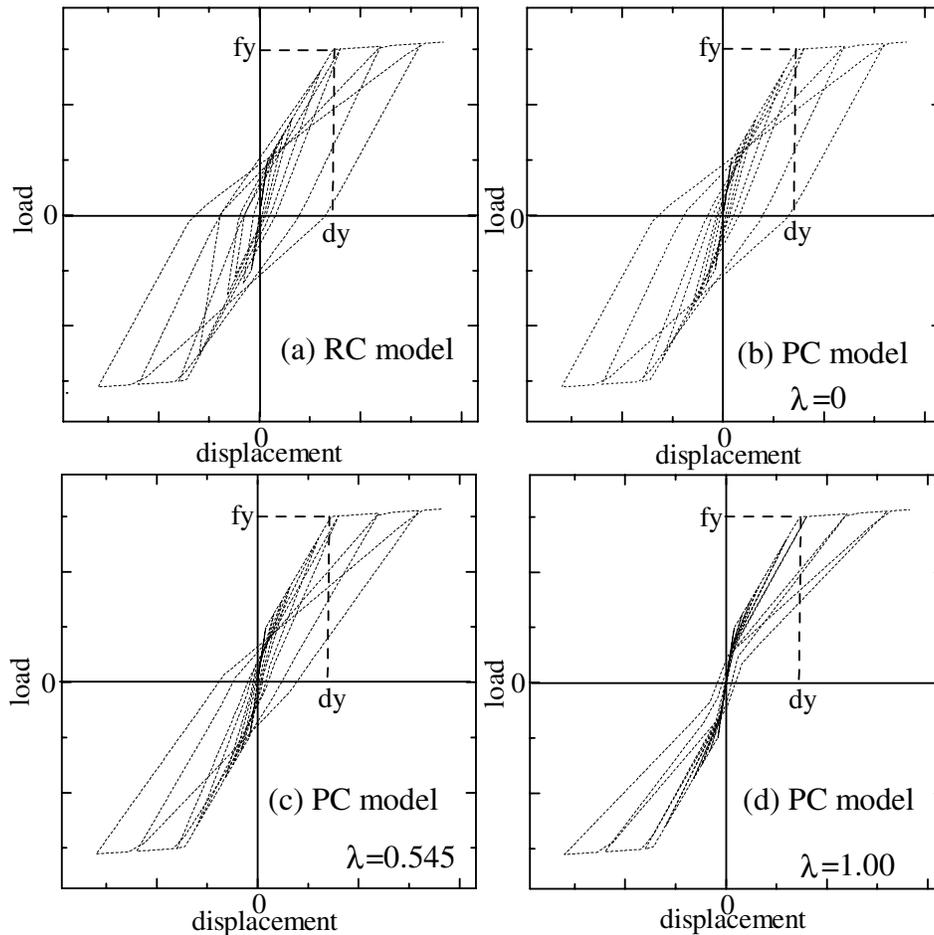


Figure 3 Hysteresis shapes of RC and PC model against cyclic loading

THE PC MODEL AND TESTS UNDER STATIC CYCLIC LOADING

The hysteresis rules of the PC model are verified by static analysis under cyclic loading. Hysteresis shapes of the PC model and the RC model under the same loading sequence are compared as shown in Figure 3. A typical beam section was designed as a reinforced concrete member. The prestress is introduced parametrically into the PC member sections, by changing the amount of prestress tendon and the main bars keeping the yielding moment equal to the original level of the RC section for each analysis model. In the figure, the typical hysteretic shapes are shown for the cases when the prestressed moment ratio λ , is varied as 0, 0.54, 1.0 as result.

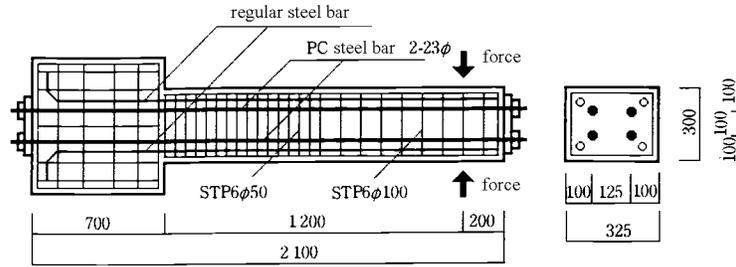


Figure 4 The test specimen of the experiment ¹⁾

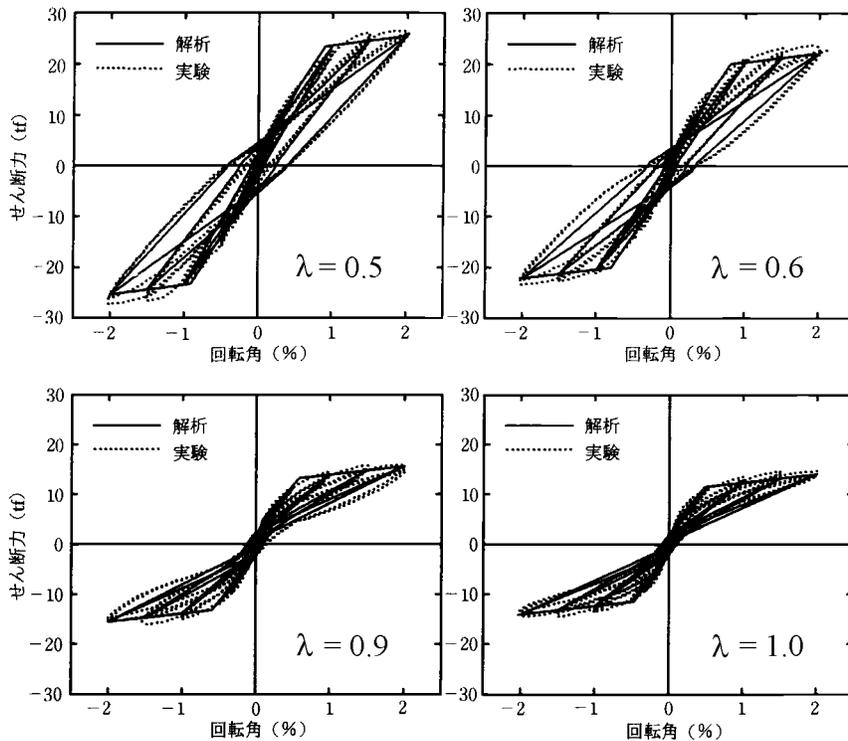


Figure 5 The experiment result and PC model against static cyclic loading ¹⁾

Hysteresis energy dissipation capacity in the PC model is higher when λ was small. It decreases with the increase of λ following the rules as described as above. The PC model when λ is zero shows almost the same hysteretic shape with the RC model except around cracking force level because of rule (2). On the other hand, PC model with λ of 1.0 shows very small hysteretic energy dissipation with nearly origin

oriented unloading stiffness and degrading linear stiffness to the secant stiffness to the peak response point.

In the past research¹⁾, the original PC model has been verified through the experiments on PC beam members, the prestress moment ratio λ being varied from 0.5 to 1.0 as shown in Figure 4. The hysteresis shapes of the original PC model with solid lines are compared with those from the tests with dotted lines in Figure 5. Also comparing to those of Figure 5, the hysteresis model of the PC model in this research for response analysis is proved being based on the original rules of the PC model and is similar to those from the test of PC members under reversed cyclic loading.

EFFECTS OF THE HYSTERETIC BEHAVIOR ON THE EARTHQUAKE RESPONSE

The hysteresis rules of the PC model is coded and added into a computer program SDF⁴⁾, by which parametric nonlinear time-history earthquake response analysis is carried out for the single-degree-of-freedom system with the hysteresis relations of the PC model and the RC model. The objective is to quantify the effects of the pinching behavior, such as of the PC model, on the maximum response displacement. An artificial earthquake motion BCJ-L2 is used in this case for the input base motion to examine the effect of the hysteretic behavior as independently as possible. The digital data is distributed from the Building Center of Japan for the design of special structures such as high-rise buildings, which requires time-history response analysis. The maximum velocity is made as 60 kine.

The stiffness and yielding force of the system are varied parametrically. The damping coefficient is taken as 5% of critical damping for the initial stiffness. The damping force is supposed to be proportional to the inelastic tangent stiffness. Here, typical practical cases are shown when the base shear coefficient is taken as 0.3 and the elastic natural period of the system T are 0.7s and 1.0s. T is based on yielding stiffness. The calculated maximum responses to the positive and the negative directions are expressed as the ductility factors, the ratios to the yield deformations, and shown in relation to the prestress moment ratio λ in Figure 6.

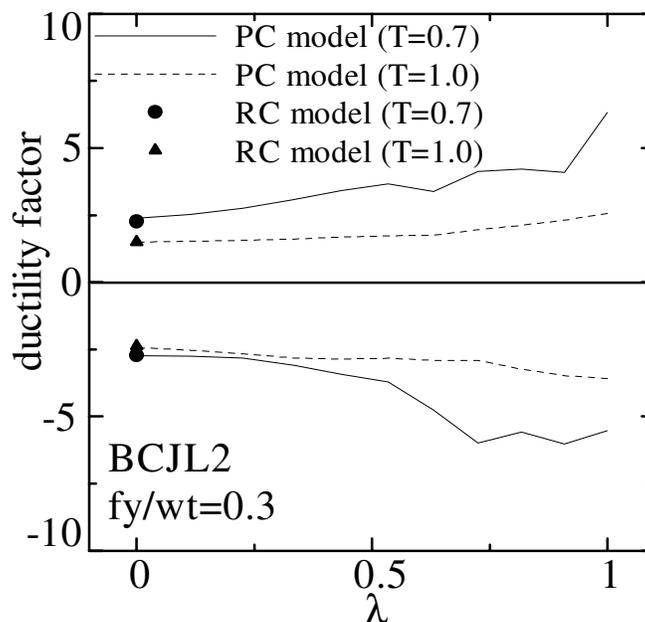


Figure 6 Effect of the prestress moment ratio on the maximum responses of PC model under an artificial earthquake motion

The maximum response is larger in PC model, especially in the cases with higher prestress moment ratio λ , than in RC model. It is conceivable because hysteresis energy dissipation is smaller in PC model. The responses are almost same with those of the RC model when λ is zero or small. The difference between the responses of the two models is remarkable in the cases when λ was larger than 0.5 and $T=0.7$. In other words, the responses of the two models is not so much different when λ is less than 0.5 in other words. Also in the longer period case, the effect would be small up to larger prestress moment ratio.

ESTIMATION OF EARTHQUAKE RESPONSES OF THE PC MODEL BASED ON EQUIVALENT LINEARIZATION

The effects of the hysteretic behavior on the responses of the PC model under the recorded earthquake motions are generally investigated hereafter with estimation based on equivalent linearization theory.

Equivalent linearization method⁵⁾ is a procedure for estimating nonlinear responses of the structure from an equivalent elastic response. The nonlinear system is represented with an equivalent linear system, namely with equivalent period T_e , and equivalent damping coefficient h_e . T_e is typically reduced in nonlinear hysteresis response, and h_e is taken as the sum of the viscous damping of the elastic system and the equivalent viscous damping h_{eq} , corresponding to the inelastic hysteretic energy dissipation.

Capacity Spectrum Method⁶⁾ (CSM) is used as the equivalent linearization method here for the estimation. As is typically assumed in CSM, the equivalent period T_e is defined based on the secant stiffness between the origin and the maximum response. The equivalent viscous damping h_{eq} , is determined from the hysteretic energy at the maximum ductility level and added to the elastic viscous damping coefficient of 0.05 as given by the following equation (3):

$$h_e = h_{eq} + 0.05 \quad (3)$$

The equivalent viscous damping h_{eq} is basically defined so that the hysteretic energy dissipation in one cycle with the displacement peaks of the maximum response ductility to both directions is equal to the energy dissipated by the viscous damping force of the equivalent coefficient. However, h_{eq} , simply given from the maximum ductility generally overestimate the effect of damping, because the earthquake response is not stationary as assumed as above. Therefore, in CSM, the equivalent damping is often decreased, which is originally named as “substitute damping⁵⁾,” given by the equations (3) and (4) using the degradation factor α :

$$h_{eq} = \frac{1}{4\pi} \frac{\Delta W}{W} \times \alpha \quad (4)$$

In the revised Building Standards of Japan, a new earthquake resistant design procedure is introduced called as the limit capacity design method⁶⁾ where the estimation of inelastic earthquake responses is based on CSM. The equivalent damping coefficient h_{eq} is given as the equation- (5) as function of ductility μ :

$$h_{eq} = \frac{1}{4} \times \left(1 - \frac{1}{\sqrt{\mu}}\right) \quad (5)$$

The above formula is based on the hysteretic energy of a simple degrading model (Clough Model) with the exponential coefficient of 0.5 for unloading stiffness and is equivalent to the case when α is taken as nearly equal to 0.8 in the equation (4). Past research verified that above formula (5) for equivalent

damping gives a fair estimation of nonlinear responses of Takeda model (RC model) with stiffness degrading factor $\gamma' = 0.5$. The degradation factor $\alpha = 0.8$ in capacity spectrum method is selected for the code specification empirically derived from earthquake response analyses.

The estimation from CSM is applied to the calculated nonlinear responses using the PC model to investigate the effects of the hysteric damping on the response as theoretically as possible. At first, equivalent period T_e is calculated by the secant stiffness from the origin to the maximum response calculated from nonlinear response analysis to identify the only the effect of the equivalent viscous damping h_{eq} which is calculated also by the nonlinear response ductility, using the formula- (3), and (4).

The earthquake responses calculated using the PC model and the estimated responses from the equivalent linear system are compared as shown in Figure 7. The PC model is the same as above, which the initial fundamental period is 0.7s, base shear factor is 0.3, and the viscous damping coefficient of 5%. Two different earthquake records are used as input base motions: One is NS component of 1985 Chile earthquake representing typical far-field earthquake motion denoted here as CHILE. The other is NS component of 1995 Hyogoken-Nanbu Earthquake recorded at Kobe observatory of Japan Meteorological Agency representing typical near-field earthquake motion denoted as JMA. The calculated responses are plotted in relation to the prestress moment ratio λ .

As shown in Figure 7(b), the response of the PC model to CHILE is not so much increasing with the prestress moment ratio λ , which is also observed in the estimates from the equivalent linearization with dashed lines. On the other hand, the responses to JMA is apparently increasing with the increase of λ , while the estimate from the equivalent linearization is not increasing with the increase of λ with flat against transition of λ as shown with dashed lines.

The reason for the discrepancy is found after detailed survey due to the essential source of error in the response computation and CSM estimation. The viscous damping is taken as constant coefficient in the CSM equivalent linearization method. However, the stiffness proportional damping is assumed in the time-history nonlinear earthquake response analysis, which is degraded in proportion to the tangent stiffness in inelastic range of response. Therefore the damping force after yielding is much smaller than that in elastic stage. This effect is not so much apparent in the cases of RC model, where the hysteretic damping become prevailing in the inelastic range.

However, the hysteretic energy dissipation doesn't so increase so much with the increase of ductility in the PC model, so that the ratio of energy dissipation by the viscous damping become relatively large to the total energy dissipation including the hysteretic energy. This can be pointed out as the essential source of error in CSM, which is not obvious in RC model but become obvious in the cases of the relatively small hysteretic energy dissipation such as in the PC model. Therefore, the constant value of viscous damping should not be used but should be reduced generally in the estimation for the model with the stiffness proportional damping. In this case, the averaged damping coefficient may be evaluated from the equivalent fundamental period T_e based on the secant stiffness toward the maximum response than the elastic fundamental period T in the equivalent linearization method. If the viscous damping is assumed to be proportional to T_e from the secant stiffness, the equivalent damping coefficient should be modified as given by the equation (6).

$$h_e = h_{eq} + 0.05 \times \frac{T}{T_e} \quad (6)$$

The responses estimated from the modified equivalent linearization as above formula-(6) are also shown with the dotted lines in Figure 7. The estimates from the modified linearization under JMA is increasing

with the increase of λ being close to the responses of the PC model from the time-history analysis. On the other hand, the linearization under CHILE also keeps up similar result with the response from the time-history analysis.

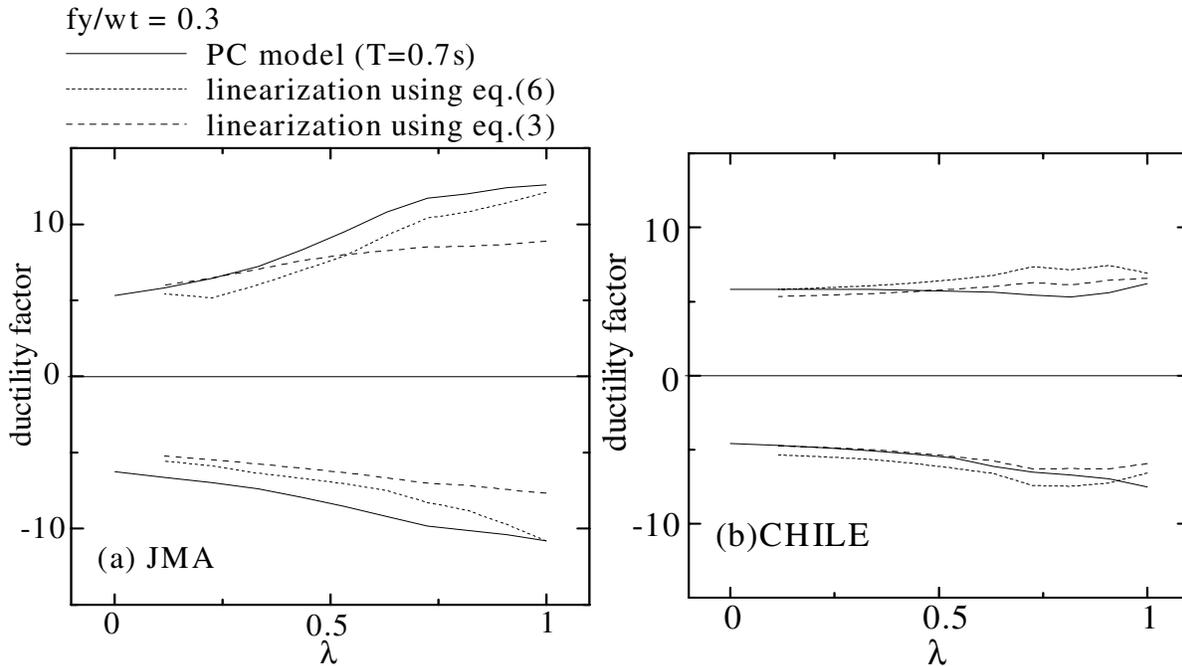


Figure 7 Maximum responses of PC model against JMA and CHILE

To investigate the different response characteristics under the two earthquake motions further, linear response spectra with various viscous damping coefficients are shown in Figure 8. The response to JMA is degrading with viscous damping in the range of the equivalent period, or susceptible to the damping coefficient, so that the equivalent viscous damping should be accurate in the evaluation. On the other hand, the response to CHILE is not affected so much by the viscous damping, or insensitive to the damping, incidentally in the range of the equivalent period. It may be said that the response to CHILE is an exceptional case where the response is insensitive to the damping.

It is found from the response characteristics of the PC model that the CSM estimation so far has a general source of error in the formula of the equivalent viscous damping not only for the PC model but also for the RC model. This phenomenon is recognized obviously because of the peculiar hysteresis shape of PC model with degrading stiffness and small hysteretic energy. In the responses of the RC model, the ratio of hysteresis energy dissipation to the damping energy dissipation becomes relatively large in comparison with those of the PC model. Besides, CSM has been verified in the range of not so much large ductility of reinforced concrete members in Japan. Therefore, the modification of the viscous damping shown here is certainly not so much effective to the equivalent linear response of CSM for the RC model practically.

However, theoretically, equivalent damping calculated by formula- (2) does not conform to the model of the system, where the viscous damping is assumed to be proportional to the inelastic tangent stiffness in nonlinear responses. Although the assumed viscous damping model and coefficient for nonlinear response analysis needs further verification, it may also required to reevaluate appropriate degradation factor α for equivalent substitute damping, based on a rational formula- (6).

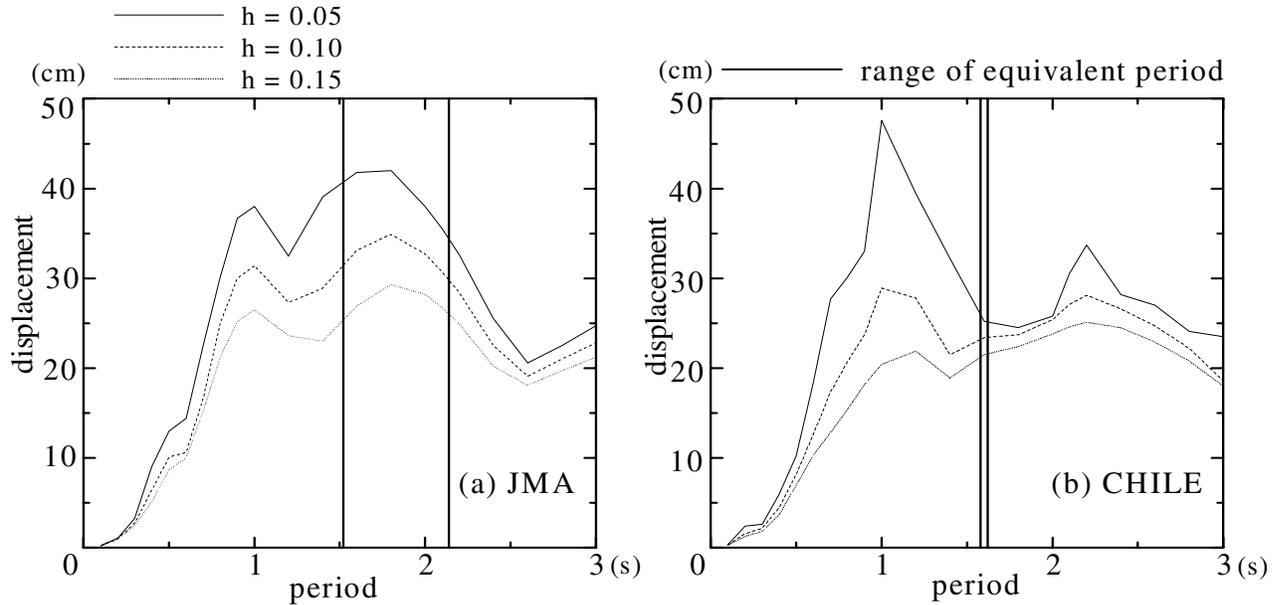


Figure 8 Elastic linear response spectrum of JMA and CHILE with various viscous damping

CONCLUSION

In this paper, the hysteresis model for the prestressed concrete members, PC model, by which nonlinear earthquake response analyses of SDF system are carried out. The response characteristics are compared with those by the model for reinforced concrete, RC model, and with the estimates by the equivalent linearization, CSM, from which the following conclusions may be drawn:

- 1) The maximum displacement response of the PC model is generally greater than that of RC model. The difference of the responses between the two models becomes obvious in the cases when the prestress moment ratio λ is larger than 0.5.
- 2) The current CSM linearization generally underestimates the responses of the PC model with large value of λ . This is because the viscous damping becomes prevailing in the cases of small hysteretic energy dissipation and the equivalent damping in the CSM formula overestimates the viscous damping, which is degraded in proportion to the inelastic tangent stiffness in nonlinear time-history analysis. The current formula of equivalent damping has an essential source of error, which become obvious for the PC model
- 3) A rational modified formula of the equivalent damping is proposed for the equivalent linearization method. Degrading viscous damping corresponding to the secant stiffness is introduced in the evaluating equivalent damping. A fair correlation is obtained by the modified formula in the estimation of the responses of the PC model with small hysteretic energy dissipation.
- 4) Equivalent damping evaluation in the current CSM is not consistent with the model of the stiffness proportional damping assumed in nonlinear response analysis fundamentally. Although it is not so much significant in the estimation for the stable RC model because the hysteretic energy dissipation is prevailing, it might be needed to reevaluate the past empirical factors for substitute damping based on the new formula of equivalent damping.

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