HYSTERESIS MODEL OF STEEL MATERIAL FOR THE BUCKLING
RESTRAINED BRACE CONSIDERING THE STRAIN RATE
DEPENDENCY

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SUMMARY

In this study, in order to clarify the hysteresis behavior of low yield strength steel considering the strain rate dependency, a series of dynamic and quasi-static loading test of low yield strength steel was carried out. The main parameters of the test were strain rate and loading pattern. Experimental results were divided into displacement dependence part and velocity dependence part. By dividing it into two components and evaluating each component separately, it is possible to construct a hysteresis model that is easily used for inelastic response analysis. The displacement dependence part of the hysteresis was modeled in poly-linear model, after it divided into skeleton part, Bauschinger part and skeleton part. On the other hand, the velocity dependence part of the hysteresis was also modeled as a function of the strain rate. The model was reflected the effect of the cumulative strain. Using these models, the hysteresis behavior of low yield strength steel, which receives the dynamic loading, becomes be able to be simply and accurately predicted.

INTRODUCTION

Recently, the buckling restrained brace is used widely as an energy dissipation element. In the buckling restrained brace, mild steel and low yield strength steel are used as yielding steel core. Especially, the low yield strength steel has been developed as a steel material for the damper. One well-known feature of the low yield strength steel is the strain rate dependency. In mild steel, though the initial yield strength rises about 20% at the strain rate of the degree, which arises in the earthquake, the effect of the strain rate is not significant in the whole hysteresis behavior. On the other hand, in low yield strength steel, not only the initial yield strength rises, but also the effect of the strain rate is greatly received for the whole hysteresis behavior. In order to clarify the performance of low yield strength steel as a steel material for damper, it is important to construct the restoring hysteresis characteristic model considering the velocity dependence. In this study, in order to clarify the hysteresis behavior of low yield strength steel considering the strain rate dependency, a series of dynamic and quasi-static loading test of low yield strength steel was carried out. The main parameters of the test were strain rate and loading pattern. Experimental results were divided into displacement dependence part and velocity dependence part. By dividing it into two components and evaluating each component separately, it is possible to construct a hysteresis model that is easily used for inelastic response analysis. The displacement dependence part of the hysteresis was modeled in poly-linear model, after it divided into skeleton part, Bauschinger part and skeleton part. On the other hand, the velocity dependence part of the hysteresis was also modeled as a function of the strain rate. The model was reflected the effect of the cumulative strain. Using these models, the hysteresis behavior of low yield strength steel, which receives the dynamic loading, becomes be able to be simply and accurately predicted.

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rate dependency, a series of dynamic and quasi-static loading test of low yield strength steel was carried out. From the experiment result, restoring force characteristic of low yield strength steel is constructed as shown in Fig.1. The whole behavior, Fig.1-(c), of the low yield strength steel is divided into a displacement dependence part, Fig.1- (a), and a velocity dependence part, Fig.1- (b). By dividing it into two components and evaluating each component separately, it is possible to construct a hysteresis characteristic model that is easily used for analysis.

![Figure 1: Dynamic Behavior of Low Yield Strength Steel](image)

**DYNAMIC AND QUASI-STATIC LOADING TEST OF LOW YIELD STRENGTH STEEL**

In order to construct restoring hysteresis characteristic model of low yield strength steel, a series of cyclic loading test of low yield strength steel is carried out. Parameters of the test are shown in Table 1 and 2. In these tests, strain rate is the most important parameter. Quasi-static loading tests are carried out at 0.01(%/s) strain rate. From the experimental results of quasi-static loading tests, hysteresis model of displacement dependence part is constructed. On the other hand, dynamic loading tests are carried out at the strain rate of 1(%/s), 10(%/s) and 20(%/s). Although the triangle waves are inputted in these tests, dynamic loading test which inputted sign wave is also carried out. In order to evaluate the rise of the stress accompanying the rise of strain rate, the experimental results of dynamic loading tests are compared with the experimental results of quasi-static loading tests.

Shape of the specimen is shown in fig.2, and setup of the specimen is shown in Fig.3. In this test, test portion of the specimen is sandwiched between rigid plates. Those rigid plates prevent buckling of the test portion of the specimen. Teflon sheet for preventing friction is inserted between the rigid plate and specimen. This setup imitated buckling restrained brace.

About the experiment results, the nominal axial stress $\sigma_n$ and nominal strain $\varepsilon_n$ are calculated as follows.

$$\sigma_n = \frac{P}{A} \quad (1)$$

$$\varepsilon_n = \frac{\delta}{L} \quad (2)$$

Where, $P$ is the axial force, $A$ is the original cross-section area of the specimen, $\delta$ is the axial deformation of the specimen, and $L$ is the original length of the specimen, respectively. The true stress $\sigma$ and true axial strain $\varepsilon$ are obtained as follows, based on the assumption of plastic incompressibility.

$$\sigma = (1 + \varepsilon_n) \cdot \sigma_n \quad (3)$$

$$\varepsilon = \ln(1 + \varepsilon_n) \quad (4)$$
Fig. 2 Specimen

The buckling preventing equipment.

Specimen

Fig. 3 Setup of the Specimen

Table 1 Loading history 1

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Loading No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td></td>
<td>Amph. (strain %)</td>
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<td>3.0</td>
<td>4.0</td>
<td>4.0</td>
<td>2.0</td>
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<td></td>
<td>Cycles</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td></td>
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<tr>
<td></td>
<td>Strain</td>
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<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rate</td>
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<td>0.01</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(%/s)</td>
<td>10.0</td>
<td>10.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.0</td>
<td>20.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Loading history 2

| Specimen | Loading No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|----------|-------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
|          | Amph. (strain %) | 2.0 | | | | | | | | | | | | | |
|          | Cycles | 3 |
| LYP_05   | Sin Wave (%/S) | 30 | 30 | 50 | 100 | 30 | 50 | 100 | 10 | 20 | 60 | 70 | 75 | 90 | 30 |
| LYP_06   | Triangle Wave (%/a) | | | | | | | | | | | | | | 0.01 |
MODELLING OF THE DISPLACEMENT DEPENDENCE PART

Decomposition of hysteresis loop
The stress-strain relationship under cyclic loading is divided into skeleton curve, Bauschinger part and elastically unloading part as shown in Fig.4. This decomposition is a useful method of describing the hysteresis behavior of structural steel. The hysteresis model by the decomposition method was proposed in previous researches (Akiyama et.al. 1995, Yamada et.al. 2002). This model is based on the following empirical knowledge.

(a) The shape of skeleton curve is similar to the stress-strain relationship under monotonic loading.

(b) The softening due to Bauschinger effect is observed in Bauschinger part, and the stiffness reduction depends on the accumulative plastic strain in the preceding loading history.

Stress-strain relationships under quasi-static loading are divided as shown in Fig.4, and the properties of skeleton curve and Bauschinger part are examined. Comparison of skeleton curves and result of tensile coupon test is shown in Fig.5. Both correspond well and it turns out that skeleton curve of low yield strength steel can be represented by the stress-strain relationship of tensile coupon test. Therefore, only modeling of Bauschinger part is conducted.

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![Hysteresis Loop](image)

Fig.4 Decomposition of Hysteresis Loop
Modeling of the Bauschinger part

As mentioned above, the previous researches reported that the stiffness reduction in Bauschinger part depends on the accumulative plastic strain in the preceding loading history. In order to investigate it quantitatively, the index, $\Delta \varepsilon_B$, $\sum \varepsilon_S$ and $\alpha_B$ were defined as shown in Fig.6 and the correlations are examined. $\Delta \varepsilon_B$ is plastic strain amplitude in each segment of Bauschinger part. Relationship between $\Delta \varepsilon_B$ and $\sum \varepsilon_S$, which is accumulative plastic strain in the preceding skeleton curve, is shown in Fig.7. $\Delta \varepsilon_B$ is proportional to the $\sum \varepsilon_S$, and the coefficient is approximately 0.125. In this study, each segment of bauschinger part was modeled as bi-linear, which is equivalent to the original segment in the energy dissipation. The initial stiffness of the bi-linear model is equal to the elastic modulus. $\alpha_B$ is obtained as the ratio of elastic limit stress to $\sigma_{BS}$, which is maximum stress in preceding skeleton part. The relation between $\alpha_B$ and $\Delta \varepsilon_B$ is shown in Fig.8. $\alpha_B$ is approximately constant value of 0.85. Hysteresis model constructed as mentioned above is shown in Fig.9 as compared with experimental results. Hysteresis model and experimental results show good correspondence.
Fig. 7 Relationship Between $\Delta \varepsilon_B$ and $\Sigma \varepsilon_S$.

Fig. 8 Relationship Between $\alpha_B$ and $\Delta \varepsilon_B$.

Fig. 9 Comparison of Experimental Results and Hysteresis Model (Quasi-Static)
MODELLING OF THE VELOCITY DEPENDENCE PART

Experimental results of dynamic loading test are shown in Fig.10. These figures show the stress-strain relation relationships for three different strain rates at the first load. Fig.10-(a) is a result in fastest strain rate(20.0%/sec triangle wave). And Fig.10-(c) is the one in slow strain rate. It can be seen that the faster the strain rate is, the maximum stress increases. Moreover, focusing on strain rate in each instant, this tendency is much stronger. In this study, increasing value of the stress by the effect of strain rate is expressed as a function of the strain rate. Thus, the dynamic stress ratio is defined as Eq. (5). Using this coefficient and the experimental results, an examination is carried out.

\[ D.S.R. (Dynamic \ Stress \ Ratio) = \frac{\sigma_{\text{dynamic}}}{\sigma_{\text{static}}} \] (5)

Relationship between the dynamic stress ratio and the strain rate in each instant is shown in Fig. 11. In this figure, the strain rate is shown at absolute value. From this figure, it is proven that there is the correlation between the dynamic stress ratio and the strain rate. These relations are shown by the following equations.

\[ D.S.R. = R_d \cdot \dot{\varepsilon} + 1.0, \quad R_d = 0.1043 \ (0.0(\% / s) \leq \dot{\varepsilon} < 0.1(\% / s)) \] (6)
\[ D.S.R. = R_d \cdot \dot{\varepsilon} + 1.1, \quad R_d = 0.0043 \ (0.1(\% / s) \leq \dot{\varepsilon} < 50.0(\% / s)) \] (7)
\[ D.S.R. = R_d \cdot (\dot{\varepsilon} - 50.0) + 1.315, \quad R_d = 0.00043 \ (50.0(\% / s) \leq \dot{\varepsilon}) \] (8)

In the experiment, it was observed that the dynamic stress ratio tended to decrease with the progression of the deformation. Then, the experimental result was arranged in each half cycle. It is shown in the relationship between the value of \( R_d \) and the cumulative strain \( \sum |\Delta \varepsilon| \) of each every half cycle in Fig.12. The relationship between \( R_d \) and cumulative strain is shown by Eq. (9).

\[ R_d = \frac{0.03}{\sum |\Delta \varepsilon| + 0.5} + 0.005 \] (9)

From these relationships and boundary conditions, the dynamic stress \( \sigma_{\text{dynamic}} \) is expressed by the following equations.

\[ \sigma_{\text{dynamic}} = ((D.S.R. , 0.01 - 1.0) \cdot |\ddot{\varepsilon}| + 1.0) \cdot \sigma_{\text{static}} \ (0.0(\% / s) \leq \dot{\varepsilon} < 0.1(\% / s)) \] (10)
\[ \sigma_{\text{dynamic}} = \left( \frac{0.03}{\sum |\Delta \varepsilon| + 0.5} + 0.005 \right) \cdot |\ddot{\varepsilon}| + 1.1 \cdot \sigma_{\text{static}} \ (0.1(\% / s) \leq \dot{\varepsilon} < 50.0(\% / s)) \] (11)
\[ \sigma_{\text{dynamic}} = \left( \frac{1}{10} \cdot \left( \frac{0.03}{\sum |\Delta \varepsilon| + 0.5} + 0.005 \right) \cdot (|\ddot{\varepsilon}| - 50.0) + D.S.R. , 50.0 \right) \cdot \sigma_{\text{static}} \ (50.0(\% / s) \leq \dot{\varepsilon}) \] (12)

\( D.S.R. , 0.01 \): The dynamic stress ratio at 0.1(\%/s) strain rate in some cumulative strain.
\( D.S.R. , 50.0 \): The dynamic stress ratio at 50.0(\%/s) strain rate in some cumulative strain.
Fig. 10 Stress-Strain Curves in each strain rates

Fig. 11 Relationship between the dynamic stress ratio and the strain rate

Fig. 12 Relationship between $R_d$ and $\sum |\Delta \varepsilon|$
COMPARISON OF EXPERIMENTAL RESULTS AND HYSERESIS MODEL UNDER DYNAMIC LOADING

The dynamic hysteresis model is constructed by including the dynamic stress rise by Eqs. (10), (11), (12) in the hysteresis model of displacement dependence part. Constructed model is shown in Fig.13 as compared with experimental results. Although these results are for a strain rate 10(%/sec), good correspondence is also obtained for other speeds.

![Comparison of Dynamic Hysteresis Model and Experimental Results.](image)

(a) 10(%/s), Amp. 2%  (b) 10(%/s), Amp. 3%

Fig.13 Comparison of Dynamic Hysteresis Model and Experimental Results.

CONCLUSIONS

In this paper, the hysteresis model considering velocity dependence of low yield strength steel is constructed. This model is constituted by displacement dependence part and velocity dependence part. First, a series of cyclic loading test of low yield strength steel is carried out. In these tests, strain rate is the most important parameter. From the experimental results of quasi-static loading test, hysteresis model of displacement dependence part is constructed. This model is constituted by skeleton curve, Bauschinger part and elastically unloading part. Skeleton curve is represented by the stress-strain relationship under monotonic loading. Bauschinger part is modeled bi-linear. On the other hand, velocity dependence part is expressed as a function of the strain rate and cumulative strain. Constructed hysteresis model shows good correspondence with experimental results.

REFERENCES