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DYNAMIC RESPONSE AND COLLAPSE OF COLUMN MODELS

Fuming ZHANG[1]

SUMMARY

As the collapse of reinforced concrete columns is attributed to the loss of vertical bearing capacity accompanied with cumulative increase of vertical deformation, analysis that is able to include the vertical response is indispensable for collapse simulation of building structures. In this paper, the collapse of reinforced concrete column models is studied, using a simplest lumped mass model considering the horizontal and vertical vibrations. Fiber model of plastic hinge for RC columns are applied, and factors deciding column collapse such as P-delta effect and stress deterioration of materials are taken into consideration. It is indicated that column models will collapse at a smaller earthquake input level for a simultaneous input of vertical motion, the ultimate safety of buildings should be evaluated considering the 3-dimensional dynamic behaviors of vertical members.

INTRODUCTION

Recently, very large peak accelerations both at horizontal and vertical directions were recorded during intense near-fault earthquakes and this may give us two research subjects to investigate. Firstly, as the large horizontal peak values may exceed *1g* in PGA and *1m/s* in PGV, it is increasingly necessary to check the ultimate safety and capacity of structures using earthquake loads or records several times larger than considered in seismic design codes. At the same time, analysis should consider the possible strength deterioration and negative stiffness at deformations far beyond the limit of non-negative stiffness restoring force models. Secondly, the large vertical component requires us to investigate the 3D dynamic response of structures, and the 3D restoring force characteristics and the 3D resisting capacities of structures.

Until now, numerous studies were carried out to investigate the effect of vertical ground motion [1], and it was found that vertical motion might increase the axial force of columns and the horizontal response of buildings. However, the frame model is not able to investigate the structural response until building collapse, as the restoring force models are usually applicable only to a relatively small elasto-plastic deformation.

Analytical program by element discretization (fiber model) enables us directly including the strength deterioration from the stress-strain relation of materials [2]. It was applied for investigating the effect of

[1] Research Engineer, EdM-NIED (Earthquake Disaster Mitigation Research Center, National Research Institute for Earth Science and Disaster Prevention), Kobe, Japan. Email: zhang@edm.bosai.go.jp.

vertical motion on the response of reinforced concrete frame buildings, and found that the simultaneous input of vertical motion does increase the damage of concrete elements and the damage of buildings [3]. It is expected that this kind of sophisticated program be applied for a direct discussion of structural response not only within a moderate damage level but also up to collapse.

Before studying the collapse of real structures, simple model and parametric studies may be useful for picking up the main factors deciding structure collapse, and expose the problems and limits of current seismic design. The author investigated the collapse and the effect of vertical motion using simplified 2DOF models, for RC columns assuming constant story deformations and without considering P-delta effect [4], for RC soft story buildings [5][6]. These studies concluded that the phenomenon of collapse might be described as the divergence of vertical and horizontal displacements. And even the horizontal responses may be not affected by the input of vertical motion if the damage is still moderate, the vertical downward displacement will be accumulated and the model will collapse at a smaller earthquake input level for a simultaneous input of vertical motion. The vertical resistance of columns must be insured for avoiding structure collapse.

The significance of vertical vibration may be reasoned from the fact that the collapse of columns is accompanied with the accumulation of compressive axial strain; it was evidenced from numerous static tests of reinforced concrete columns. The stable limit of RC columns for horizontal restoring force was proposed based on the axial deformation [7].

This paper investigates the dynamic response and collapse of columns located at the first story of RC buildings. The analytical model is modified from a popular loading setup installed in Japan where columns subjected to anti-symmetric double curvature flexure deformation. Research is focused on the vertical resistance of RC columns, and horizontal strength and long-term axial stress are two main parameters for investigation. Fiber model of plastic hinge for RC columns are applied, and factors deciding column collapse such as P-delta effect and stress deterioration of materials are taken into consideration. Numerical integration is performed using the correct instantaneous stiffness of elements by predicting the time increment compatible with stiffness variation. For simplicity, only one of the two horizontal earthquake components is considered.

SIMPLIFIED DYNAMIC MODEL FOR REINFORCED CONCRETE COLUMNS

Analytical Model

Safety of building structures is largely indebted to the capacities of vertical members at the first story. Usually, static tests maintain a constant axial force during the lateral loading reversals; here a dynamic analytical model with 2DOF in Figure 1 is assumed for investigating the earthquake response of RC columns, and only one horizontal component is considered. Although the axial stress of columns may be strongly related to the overturning moment [5], this paper discusses the dynamic collapse of columns located at the central part of building frames.

In Figure 1, RC column deforms anti-symmetrically in double curvature flexure, and the deformation will concentrate at the ends of member at large deformation. Response and collapse of the analytical model will be determined not only by the characteristics of the column but also the rigidities k_h and k_v of upper stories at horizontal and vertical directions.

Let EI and EA be the bending and axial elastic stiffness of RC column, then the rigidities of upper stories are assumed as follows in proportional to the height H of the lamped mass.

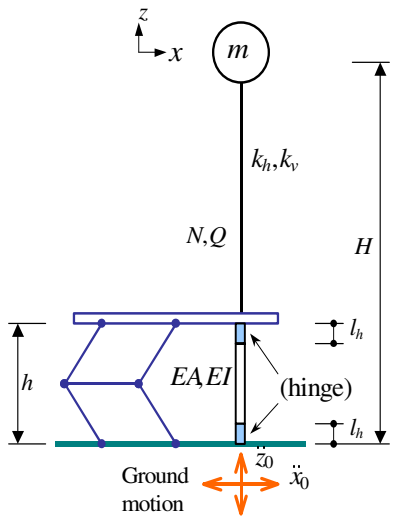


Figure 1. An analytical model for studying the dynamic collapse of RC columns

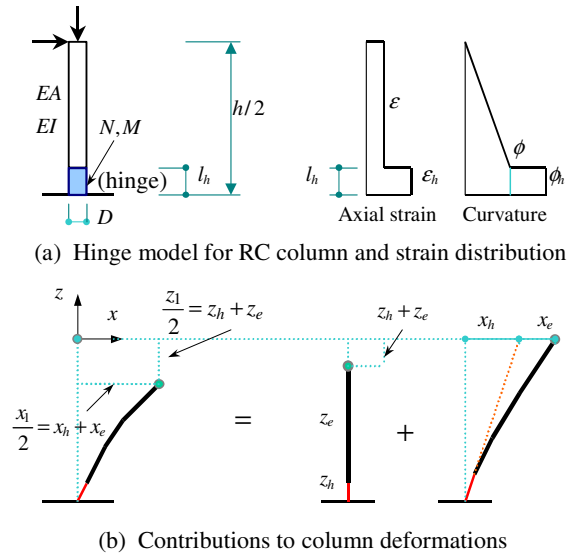


Figure 2. Plastic hinge model and deformations of one half of a column

$$k_h = \frac{12EI}{h^2(H-h)} \tag{1}$$

$$k_v = \frac{EA}{(H-h)} \tag{2}$$

Actually, the value of lamped mass relies on the deformation distribution of the first story and upper stories. Assumption of constant mass could not account the effective mass associated with the deformation concentration at the first story after experienced large deformations.

Plastic Hinge Model for RC Column

Plastic hinge model is applied for RC column with uniform section and reinforcement. In Figure 2, the deformation distribution of one half of a column is assumed as shown in Figure 2(a), and deformation will be concentrated at the plastic hinge after yielding of column. For the region outside the plastic hinge, a uniform axial strain and a triangle curvature distribution are assumed corresponding to the axial force and bending moment. Figure 2(b) shows the components of horizontal and vertical displacements $\{x_1, z_1\}^T$, and they are expressed in incremental formation by Equation (3) as the sums of deformations resulted by plastic hinges and deformations by the region outside of plastic hinge. Here, h is the height of column, l_h represents the length of the plastic hinge, $\{\Delta\phi_h, \Delta\varepsilon_h\}^T$ are the curvature and axial strain at plastic hinges. EI and EA are assumed as constant by using their elastic rigidities.

$$\begin{Bmatrix} \Delta x_1 \\ \Delta z_1 \end{Bmatrix} = \begin{bmatrix} h \cdot l_h & 0 \\ 0 & 2l_h \end{bmatrix} \begin{Bmatrix} \Delta\phi_h \\ \Delta\varepsilon_h \end{Bmatrix} + \begin{bmatrix} \dfrac{(h-2l_h)^3}{12hEI} & 0 \\ 0 & \dfrac{(h-2l_h)}{EA} \end{bmatrix} \begin{Bmatrix} \Delta M \\ \Delta N \end{Bmatrix} \tag{3}$$

Increments of restoring forces $\{\Delta Q, \Delta N\}^T$ of column are related with the stresses $\{\Delta M, \Delta N\}^T$ at column ends by Equation (4), where the second and the third terms account the P-delta effect.

$$\begin{Bmatrix} \Delta Q \\ \Delta N \end{Bmatrix} = \begin{bmatrix} \dfrac{2}{h} & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta M \\ \Delta N \end{Bmatrix} + \begin{bmatrix} 0 & \dfrac{x_1}{h} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta M \\ \Delta N \end{Bmatrix} + \frac{N}{h} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta x_1 \\ \Delta z_1 \end{Bmatrix} \tag{4}$$

Fiber Model at Plastic Hinge

At the possible plastic hinge, column section is divided into discrete elements. Applying the assumption that plane cross section remains plane after deformation, strain of the *ith* element with a distance of s_i from the center of section will be expressed by curvature ϕ_h and axial strain of the section ε_h as follows.

$$\Delta\varepsilon_i = \Delta\varepsilon_h + s_i \cdot \Delta\phi_h \tag{5}$$

The local stiffness matrix $[{}_s k]$ for plastic hinge section is expressed as follows, where E_i, A_i are the tangent modulus and area of discrete elements.

$$\begin{Bmatrix} \Delta M \\ \Delta N \end{Bmatrix} = [{}_s k]\begin{Bmatrix} \Delta\phi_h \\ \Delta\varepsilon_h \end{Bmatrix} = \begin{bmatrix} {}_s k_{11} & {}_s k_{12} \\ {}_s k_{21} & {}_s k_{22} \end{bmatrix}\begin{Bmatrix} \Delta\phi_h \\ \Delta\varepsilon_h \end{Bmatrix} = \begin{bmatrix} \Sigma E_i A_i s_i^2 & \Sigma E_i A_i s_i \\ \Sigma E_i A_i s_i & \Sigma E_i A_i \end{bmatrix}\begin{Bmatrix} \Delta\phi_h \\ \Delta\varepsilon_h \end{Bmatrix} \tag{6}$$

Compatibility and Equilibrium of Column

Substituting Equation (6) into Equation (3), we have the compatibility matrix $[{}_s T]$ between column displacements and deformations at plastic hinge as follows.

$$\begin{Bmatrix} \Delta x_1 \\ \Delta z_1 \end{Bmatrix} = [{}_s T]^{-1}\begin{Bmatrix} \Delta\phi_h \\ \Delta\varepsilon_h \end{Bmatrix} \quad or \quad \begin{Bmatrix} \Delta\phi_h \\ \Delta\varepsilon_h \end{Bmatrix} = [{}_s T]\begin{Bmatrix} \Delta x_1 \\ \Delta z_1 \end{Bmatrix} \tag{7}$$

$$[{}_s T]^{-1} = \begin{bmatrix} hl_h & 0 \\ 0 & 2l_h \end{bmatrix} + \begin{bmatrix} \dfrac{(h-2l_h)^3}{12hEI} & 0 \\ 0 & \dfrac{(h-2l_h)}{EA} \end{bmatrix}[{}_s k] \tag{8}$$

Substituting Equation (7) into Equation (4), we have the stiffness matrix $[{}_c k]$ of column as follows.

$$\begin{Bmatrix} \Delta Q \\ \Delta N \end{Bmatrix} = [{}_c k]\begin{Bmatrix} \Delta x_1 \\ \Delta z_1 \end{Bmatrix} \text{, where } [{}_c k] = \begin{bmatrix} \dfrac{2}{h} & \dfrac{x_1}{h} \\ 0 & 1 \end{bmatrix}[{}_s k][{}_s T] + \begin{bmatrix} \dfrac{N}{h} & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

Equation of Motions and Overall compatibility

Let the displacements and restoring forces at the horizontal and vertical directions to be x, z and Q, N, and initial viscous damping factors h_x, h_z are independent and proportional to their initial elastic circular frequencies ω_x, ω_z, the incremental equation of motions are expressed as follows. Where x_0, z_0 represent ground motions in each direction. Axial force *N* will be minus value at compression.

$$m\begin{Bmatrix} \Delta\ddot{x} \\ \Delta\ddot{z} \end{Bmatrix} + 2m\begin{bmatrix} h_x\omega_x & 0 \\ 0 & h_z\omega_z \end{bmatrix}\begin{Bmatrix} \Delta\dot{x} \\ \Delta\dot{z} \end{Bmatrix} + \begin{Bmatrix} \Delta Q \\ \Delta N \end{Bmatrix} = -m\begin{Bmatrix} \Delta\ddot{x}_0 \\ \Delta\ddot{z}_0 \end{Bmatrix} \tag{10}$$

Displacements of the lamped mass are expressed by Equation (11), and using Equations (9) and (11), the overall stiffness $[K]$ will be expressed by Equation (12).

$$\begin{Bmatrix} \Delta x \\ \Delta z \end{Bmatrix} = \begin{bmatrix} 1/k_h & 0 \\ 0 & 1/k_v \end{bmatrix}\begin{Bmatrix} \Delta Q \\ \Delta N \end{Bmatrix} + \begin{Bmatrix} \Delta x_1 \\ \Delta z_1 \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} \Delta Q \\ \Delta N \end{Bmatrix} = [K]\begin{Bmatrix} \Delta x \\ \Delta z \end{Bmatrix} \text{, where } [K]^{-1} = \begin{bmatrix} 1/k_h & 0 \\ 0 & 1/k_v \end{bmatrix} + [{}_c k]^{-1} \tag{12}$$

Further, using Equations (7),(9) and (12), we have the compatibility matrix $[T]$ between overall displacements and deformations at plastic hinge as follows.

$$\begin{Bmatrix} \Delta\phi_h \\ \Delta\varepsilon_h \end{Bmatrix} = [T]\begin{Bmatrix} \Delta x \\ \Delta z \end{Bmatrix}, \quad \text{where } [T] = [{}_s T][{}_c k]^{-1}[K] \tag{13}$$

DYNAMIC ANALYSIS OF REINFORCED CONCRETE COLUMNS

Constructive Rules of Materials

The stress-strain ($\sigma - \varepsilon$) relations of reinforcement and concrete are assumed as shown in Figure 3. Tension strength of concrete is ignored, and $\mu_c \varepsilon_c$ is the ultimate strain of concrete. For reinforcement, buckling is simplified as a straight line with negative stiffness. Young modulus of reinforcement and concrete are assumed to be $E_s = 210000MPa$, $E_c = 21000\sqrt{F_c / 20MPa}$ respectively.

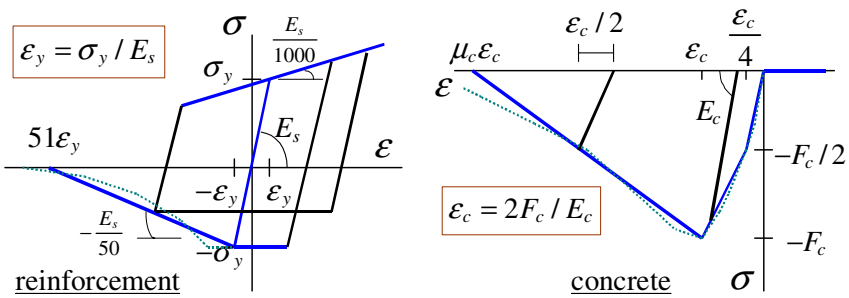


Figure 3. Constructive rule of materials

Non-Iterative Integration Method Using Instantaneous Stiffness

As expressed by Equations (12) and (13), both compatibility matrix $[T]$ and overall stiffness matrix $[K]$ are related with the instantaneous matrix $[{}_s k]$ of plastic hinge. Therefore, response analysis should use the correct values of $[{}_s k]$, i.e., the correct tangent modulus of discrete elements, to ensure the equilibrium and the compatibility conditions.

Within the record time interval Δt_0 of an earthquake wave, Newmark's β method $(\beta = 1/4)$ is applied temporally using an appropriate constant time interval. If a stiffness variation is confirmed for any elements, then the time increment Δt compatible to the first of stiffness variation is calculated for response regression [8]. This variable time increment was made possible by using explicit formulas about overall displacements, and about element strains from the relation of Equation (5). For each element, a quadratic equation for loading and a linear equation for unloading are obtained for solving the time increment Δt compatible to stiffness variation. The correct Δt will be the minimum value tried for all fiber elements.

Earthquake Record and Inputted Earthquake Motions

Vertical (SCS-UP) and one horizontal (SCS052) components of Sylmar-converter Station, January 17, 1994 Northridge earthquake are used, record time interval is $\Delta t_0 = 0.005s$, and duration is 40 second. The peak values of earthquake records are $PGA_{UP} = 5.61m/s^2, PGA_{052} = 6.00m/s^2$ respectively.

In order to obtain the maximum input level leading to column collapse, earthquake records of both directions are multiplied by a same multiplier Ω, and the analysis is to find the minimum of Ω leading to collapse for different analytical parameter combinations. Multiplier Ω is used for convenience of discussion; it may contradict with the fact that the ratio of peak accelerations between vertical and horizontal components as well as the wave forms may vary for different earthquake intensity.

Analytical Parameters

For column of a *n-story* RC building, assuming the height of lamped mass is $H = (2/3)nh$, the story height is $h = 3.5m = 350cm$, the weight of a single story is $W_0 = 40tf = 40000kgf$, then using the long-term axial force ratio n_0 of column defined by Equation (14), the size of section will be decided by Equation (15).

$$n_0 = \frac{mg}{D^2 F_c} = \frac{n \cdot W_0}{D^2 F_c} \tag{14}$$

$$D = \sqrt{\frac{n \cdot W_0}{n_0 F_c}} \tag{15}$$

The number of story $(n = 3,5,7,9)$ and the long-term axial force ratio $(n_0 = 0.1 \ to \ 0.4, with \ a \ pitch \ of \ 0.05)$ are chosen as the analytical parameters. Two input conditions are investigated about, (1) simultaneous input of vertical ground motion, (2) vertical ground motion is not inputted but variable axial force by vertical displacement is considered.

Time history analysis is performed for an input waves that are Ω times of the earthquake records. Computation is terminated when meeting the following conditions assumed for column collapse. After that time, the column will accompany with a divergence of horizontal and/or vertical displacements.

(1) Loss of horizontal resistance when $|x/h| > q_y$. This is a condition for column losing its horizontal restoring force due to P-delta effect, when assuming the fix end having a perfect elasto-plastic moment-rotation relationship under constant axial force.
(2) Loss of vertical resistance due to the failure of core concrete at plastic hinge when $\varepsilon_h < 2\mu_c \varepsilon_c$. This means the axial strain at the center of section exceeds two times of the ultimate strain of concrete material.

For analytical model, initial damping at horizontal as well as vertical directions are taken as $h_x = 0.02$, $h_z = 0.02$, as we have little information until now about the vertical damping. The length of possible plastic hinge is assumed be $L_h = D$, the section is divided into 10 concrete and 2 reinforcement elements.

For a square RC section, reinforcement ratio is assumed be $p_g = 2.0\%$, the yielding strength of reinforcement be $\sigma_y = 350MPa$, the compressive strength of concrete be $F_c = 30MPa$. For the stress-strain ($\sigma - \varepsilon$) relation of materials, the compressive strain of reinforcement when fracture occurred is $\mu_s = 51$, and the ultimate strain of concrete $\mu_c \varepsilon_c$ is assumed as $\mu_c = 10,20$.

The maximum bending moment of a RC section may vary with the variation of axial force. As a rough estimation, horizontal strength coefficient q_y of the model is evaluated approximately by Equation (16) using a constant axial force.

$$q_y = \frac{2}{n_0} \frac{D}{h} \left[\frac{n_0(1-n_0)}{2} + 0.4 p_g \frac{\sigma_y}{F_c} \right] \tag{16}$$

Therefore, if the reinforcement ratio p_g and strength of materials σ_y, F_c are given, q_y is decided only by the long-term axial force ratio n_0 and the number of story *n*. In Table 1, elastic vibration periods are listed for some parameters. Time increment for numeric integration is limited as $\Delta t \le \Delta t_0 / 5 = 0.001s$, that meets the requirement for insuring computation accuracy when using variable time increment [8], i.e., $0.001s$ is smaller than 1/10 of minimum vertical vibration period ($T_z = 0.058s \ for \ n = 3 \ and \ n_0 = 0.1$).

Table 1. Analytical parameters for 3-story and 9-story column models

n	H(m)	mg(tf)	D(m)	n0	qy	Tx (s)	Tz (s)
3	7.0	120	0.632	0.10	0.500	0.321	0.058
3	7.0	120	0.516	0.15	0.309	0.482	0.071
3	7.0	120	0.447	0.20	0.221	0.643	0.082
3	7.0	120	0.400	0.25	0.171	0.803	0.092
3	7.0	120	0.365	0.30	0.138	0.964	0.101
3	7.0	120	0.338	0.35	0.114	1.124	0.109
3	7.0	120	0.316	0.40	0.096	1.285	0.116

n	H(m)	mg(tf)	D(m)	n0	qy	Tx (s)	Tz (s)
9	21.0	360	1.095	0.10	0.866	0.321	0.101
9	21.0	360	0.894	0.15	0.535	0.482	0.123
9	21.0	360	0.775	0.20	0.384	0.643	0.142
9	21.0	360	0.693	0.25	0.296	0.803	0.159
9	21.0	360	0.632	0.30	0.239	0.964	0.174
9	21.0	360	0.586	0.35	0.198	1.124	0.188
9	21.0	360	0.548	0.40	0.167	1.285	0.201

DYNAMIC RESPONSE AND COLLAPSE OF RC COLUMNS

Dynamic response of columns is discussed for $\mu_c = 10$, and $\mu_c \varepsilon_c$ is the ultimate strain of concrete as illustrated in Figure 3.

Maximum Response

In order to find the earthquake input level leading to column collapse, Figures.4 and 5 show the maximum response vs. Ω for 3-story and 9-story models respectively. Ω is a multiplier for Sylmar-converter station records, and started from 0.05 to the value leading to column collapse.

(a) Maximum horizontal displacement

Maximum horizontal displacement $X_{\max}$ (in *cm*) increase with Ω, and at a certain input level, $X_{\max}$ become unstable and increase dramatically with a very small increase of Ω. In order to avoid the divergence of horizontal displacement, $X_{\max}$ must be restricted within some value. Actually, this is the horizontal displacement when column lost its horizontal resistance. This displacement increases with a lower long-term axial force ratio n_0, or a larger horizontal strength coefficient q_y (see relation between n_0 and q_y by Equation (16) and Table 1). In both Figures of 4(a) and 5(a), for $n_0 = 0.1, 0.15, 0.2, 0.3$, the limits of $X_{\max}$ are smaller than $X = 25cm, 20cm, 15cm, 10cm$, they are much smaller than the deformation capacity obtained from considering the P-delta effect of column ($X = q_y h = 175cm, 108cm, 77cm, 48cm$, for 3-story column model).

(b) Minimum vertical displacement

The downward minimum vertical displacement $Z_{\min}$ (in *cm*) increase with Ω and n_0, and at a certain input level corresponding to the stable limit of horizontal displacement, $Z_{\min}$ also become unstable and increase dramatically with a very small increase of Ω. The limit of $Z_{\min}$ is about *-0.5cm* for 3-story column model, and is about -1*cm* for 9-story column model. The difference may be arisen from the elastic deformation outside the plastic hinges.

(c) Maximum vertical displacement

The upward maximum vertical displacement $Z_{\max}$ also increase with Ω. However, for higher long-term axial force ratio $n_0 = 0.3$, there are no tensile deformation occurred until column collapse.

(d) Maximum horizontal restoring force

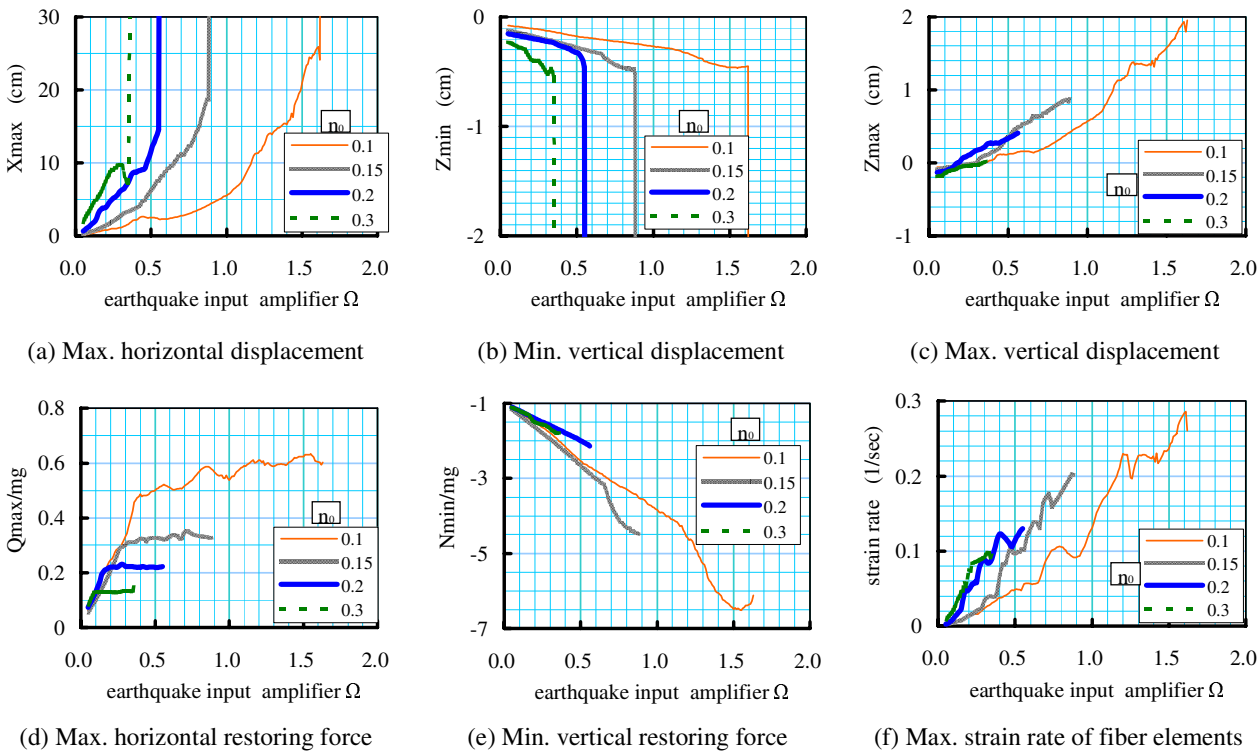
The maximum horizontal restoring force $Q_{\max} / mg$ (normalized by weight *mg*) increase with Ω if it is small and column is within elastic state. For lower long-term axial force ratio $n_0 = 0.1, 0.15$, $Q_{\max} / mg$ increase with Ω even after yielding of column. However, for higher long-term axial force ratio $n_0 = 0.2, 0.3$, $Q_{\max} / mg$ remain constant irrelevant with the increase of Ω.

(e) Minimum vertical restoring force

The compressive minimum vertical restoring force $N_{\min} / mg$ (normalized by weight *mg*) increase with Ω. For lower long-term axial force ratio $n_0 = 0.1, 0.15$, $N_{\min} / mg$ may be as large as 6 or 4. However, if it is normalized by the compressive strength of concrete $D^2 F_c$, then the maximum of $N_{\min} / D^2 F_c$ near collapse will be $N_{\min} / D^2 F_c = n_0 \cdot N_{\min} / mg = 0.5 \leftrightarrow 0.6$ for different n_0. Therefore, the axial compressive force is limited within the compressive force capacity of columns when subjected to large bending deformation.

(f) Maximum strain rate of fiber elements

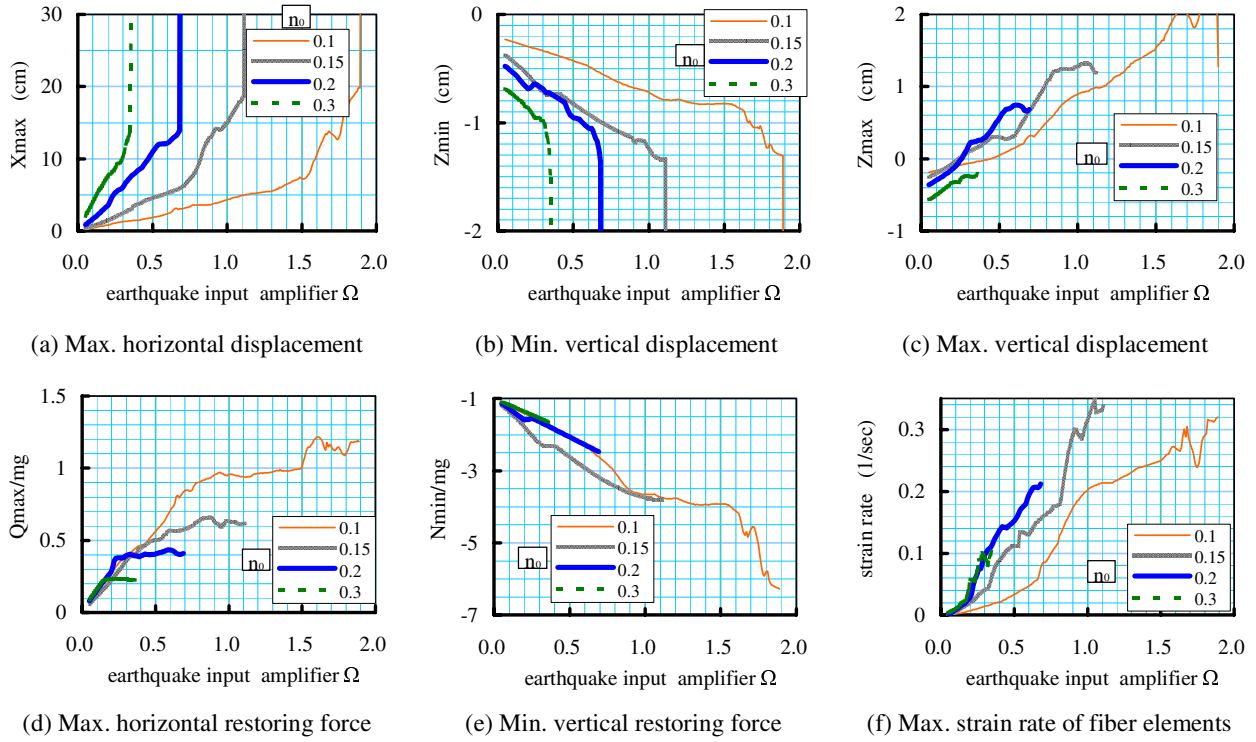
The maximum strain rate of fiber elements at plastic hinges is extremely large, and may be as large as *0.1/s-0.3/s*. This paper has not included the effect of strain rate on the constructive rules of materials.



(a) Max. horizontal displacement (b) Min. vertical displacement (c) Max. vertical displacement

(d) Max. horizontal restoring force (e) Min. vertical restoring force (f) Max. strain rate of fiber elements

Figure 4. Maximum response of 3-story column models under various input amplifier



(a) Max. horizontal displacement (b) Min. vertical displacement (c) Max. vertical displacement

(d) Max. horizontal restoring force (e) Min. vertical restoring force (f) Max. strain rate of fiber elements

Figure 5. Maximum response of 9-story column models under various input amplifier

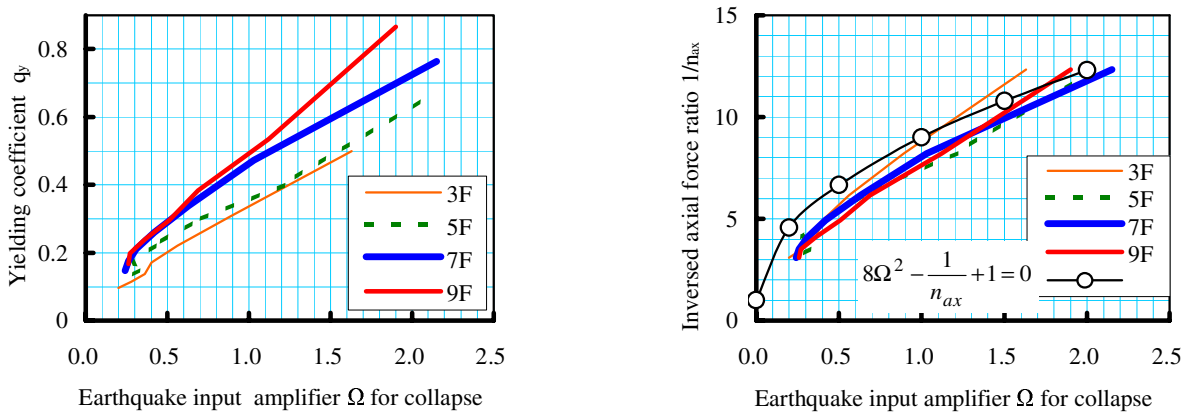
Resisting Capacity of Column Models

The ultimate resisting capacity of structures for preventing collapse may be interpreted as the capacity to resist gravity load, as well as the earthquake actions both at the horizontal and at the vertical directions. In this paper, collapse of column models is defined as the divergence of horizontal or vertical displacements, and as a matter of factor, they occur simultaneously and suddenly with a very small increase of earthquake input level.

In Figure 6(a), the capacity to resist an earthquake input level Ω is plotted against the horizontal strength coefficient q_y. Ω leading to collapse increase with q_y, and is also affected by the number of story. In Figure 6(b), the capacity against collapse for resisting gravity load and earthquake input is plotted together, and there is a strong correlation between the long-term axial capacity ratio n_{ax} with earthquake input Ω, This means that the ultimate capacity of columns should be evaluated considering the vertical resistance. The long-term axial capacity ratio n_{ax} is defined by Equation (17), normalized by axial capacity of RC columns.

$$n_{ax} = \frac{mg}{D^2(F_c + p_g\sigma_y)} = \frac{n_0}{1 + p_g(\sigma_y / F_c)} \tag{17}$$

In Figure 6(b), a curve of quadratic function between Ω and $1/n_{ax}$ is plotted to describe the tendency of their relation. Extremely, the function means RC column could not resist earthquake when gravity load equals to the axial capacity of columns ($\Omega = 0,\ n_{ax} = 1$), and RC column will never collapse if without gravity load ($\Omega = \infty,\ n_{ax} = 0$).



(a) Capacity of columns with a horizontal strength (b) Capacity against gravity load and earthquake actions

Figure 6. Resisting capacity of column models vs. yielding coefficient or long-term axial force ratio

Restoring Force at Horizontal and Vertical Directions

Hysterics at both directions are shown in Figures 7,8 for 3-story model with $n_0 = 0.15$. When $\Omega = 0.88$ in Fig.7, although displacement at the horizontal direction is relative large and degradation of horizontal resistance is accompanied, no divergence of displacements were occurred. However, a slightly increase of input $\Delta\Omega = 0.01$, collapse is occurred at both directions in Fig.8 where $\Omega = 0.89$.

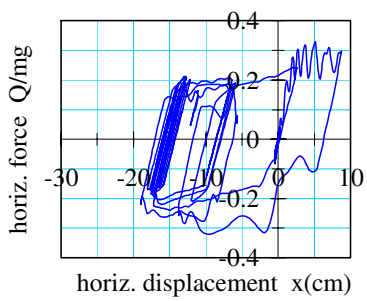
In Figure 8, the horizontal restoring force of (a) is similar to that of bending moment at plastic hinge of (c), the effect of P-delta effect on the horizontal restoring force is small, and the degradation of horizontal resistance might be considered as the result of material deterioration (see Figure 3).

Figure 8(b) and Figure 8(d) show large upward displacement and large upward deformation, and the upward displacement is the result of tensile deformation at plastic hinge. There is no clear hysteretic rule at the vertical direction.

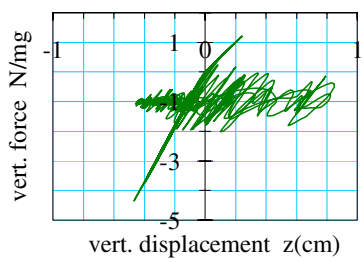
Time History of Displacements

For 3-story model with $n_0 = 0.15$ and $\Omega = 0.89$, Figure 9 shows the time history of (a) horizontal displacement, (b) vertical displacement, (c) axial strain of plastic hinge, (d) vertical restoring force. A sudden increase of displacements in both directions is occurred at the time around $t = 12\,\text{sec}$. Apparently, the divergence of vertical displacement z is a slightly faster than that of horizontal displacement x.

For $\Omega = 0.88$, Figure 10 shows the time history of (a) horizontal displacement, (b) vertical displacement, (c) inputted horizontal acceleration, (d) inputted vertical acceleration. The inputted accelerations were multiplied the Sylmar-converter station records by $\Omega = 0.88$. Although large residual displacements resulted, there is no divergence of displacements at both directions. The large upward vertical displacement coincide with the large horizontal displacement ($t = 4\,\text{sec}$ *and* $t = 5\,\text{sec}$), this may be interpreted as the result of the movement of neutral axis at plastic hinge.

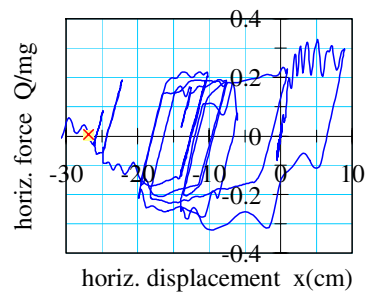


(a) Horizontal restoring force

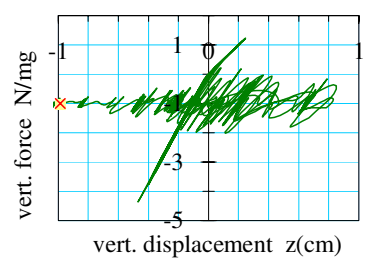


(b) Vertical restoring force

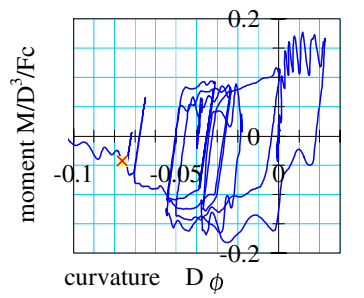
Figure 7. Restoring forces Ω =0.88



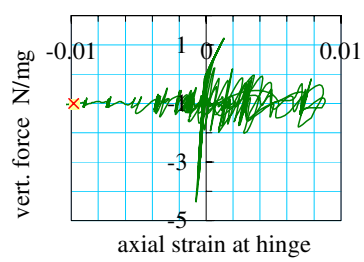
(a) Horizontal restoring force



(b) Vertical restoring force



(c) Restoring moment at hinge



(d) Axial restoring force at hinge

Figure 8. Restoring forces with Ω =0.89

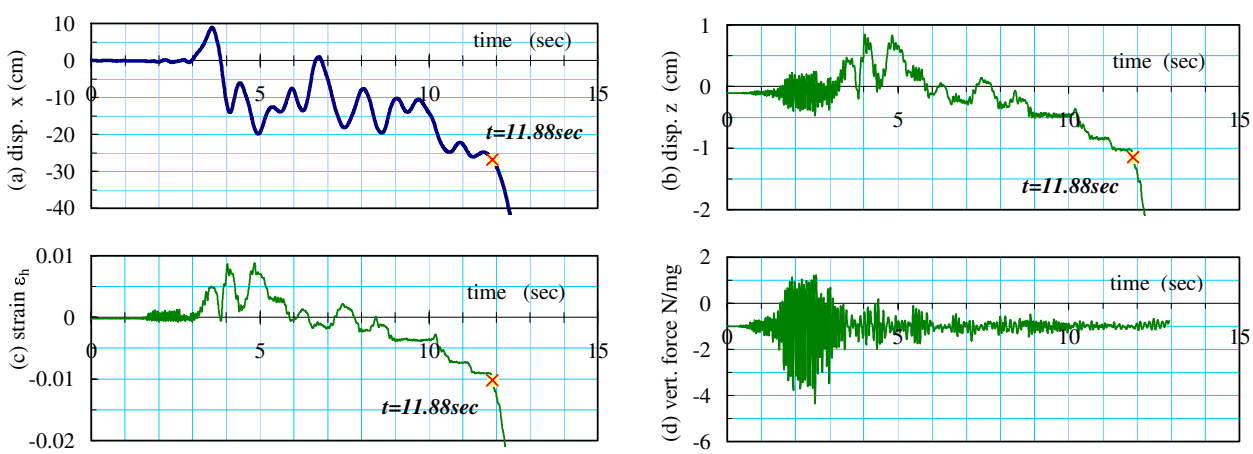


Figure 9. Time history of 3-story model with $n_0 = 0.15$ and Ω =0.89

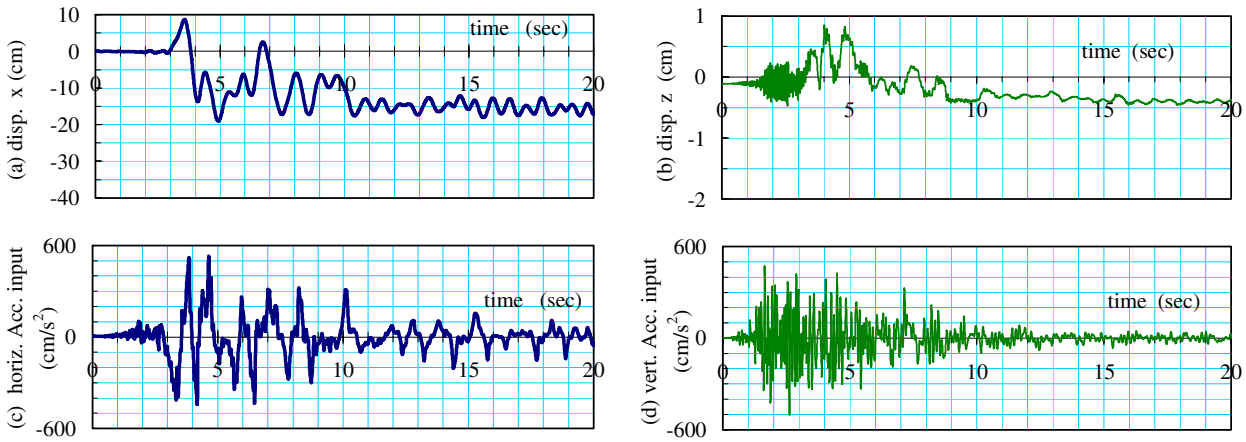


Figure 10. Time history of 3-story model with $n_0 = 0.15$ and input acceleration (Ω=0.88)

INFLUNCE OF VERTICAL MOTION AND VERTICAL VIBRATION ON COLUMN COLLAPSE

Dynamic response of columns and the effect of vertical motion are discussed for $\mu_c = 20$, where $\mu_c \varepsilon_c$ is the ultimate strain of concrete as illustrated in Figure 3. Similar results and conclusions may be obtained for $\mu_c = 10$. Here, "H+V" means the simultaneous input of horizontal and vertical motions; "H" means only the horizontal motion was inputted.

Influence of Vertical Motion on Maximum Response

In Figure 11, maximum responses of 3-story models are plotted against earthquake input multiplier Ω for (a) vertical displacement, (b) vertical compressive force. The simultaneous input of vertical motion results a larger vertical downward displacement and a large vertical compressive force. The minimum vertical force $N_{\min}/mg$ in compression is proportional to Ω. For the analytical parameters discussed, if the vertical displacement excess *–0.4cm* irrelevant to the input of vertical motion, then a divergence of displacements will be initiated and the column model will be collapsed.

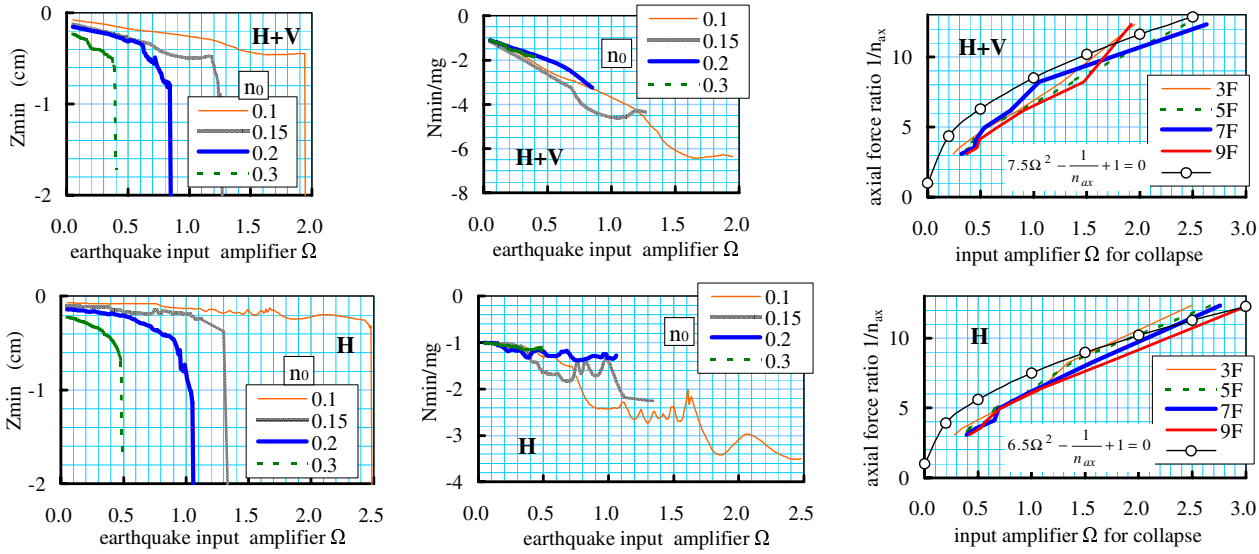
Even the vertical motion is not inputted; a larger vertical compressive force is resulted. However, $N_{\min}/mg$ is not only affected by the input level Ω, but also affected by the long-term axial force ratio n_0. For $n_0 = 0.2, 0.3$, $N_{\min}/mg$ is not less than *–1.4*, i.e., a 40% increase of axial compressive force. However, for $n_0 = 0.1, 0.15$, $N_{\min}/mg$ may be as large as *–2.0 to –3.0*, a result of the large movement of neutral axis at plastic hinge. Vertical vibration occurs even without vertical input.

Influence of Vertical Motion on Resisting Capacity of Columns

With simultaneous input of horizontal and vertical motions, column model will collapse at a smaller input level; averagely the omission of vertical motion will over-estimate the earthquake resistant capacity by 20%. It must point out that the over-estimation may be more significantly if compared with the result using restoring force obtained from the assumption of constant axial force. Again, in Figure 11(c), the capacity against collapse for resisting gravity load and earthquake input is plotted together. Curves of quadratic function between Ω and $1/n_{ax}$ are plotted to describe the tendency of their relation. Here, n_{ax} is the long-term axial capacity ratio.

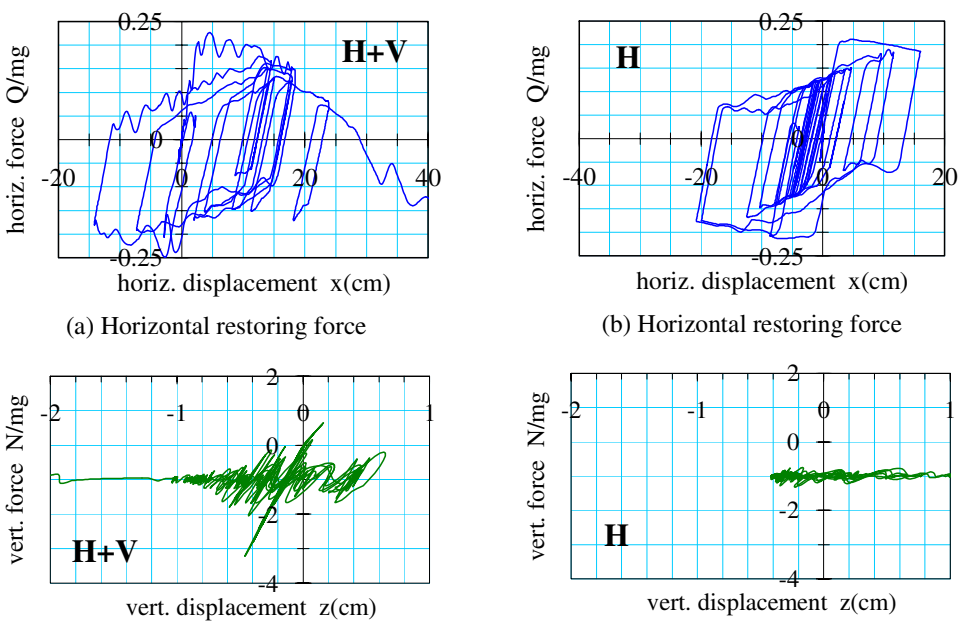
Influence of Vertical Motion on Restoring Forces

Figure 12 shows the restoring forces of a 3-story model with or without vertical motion. The simultaneous input of vertical motion results large deterioration of horizontal restoring force, and a large variation of axial force.



(a) Max. vertical displacement (b) Min. vertical force (c) Resisting capacity

Figure 11. Effect of vertical motion on the maximum response and resisting capacity



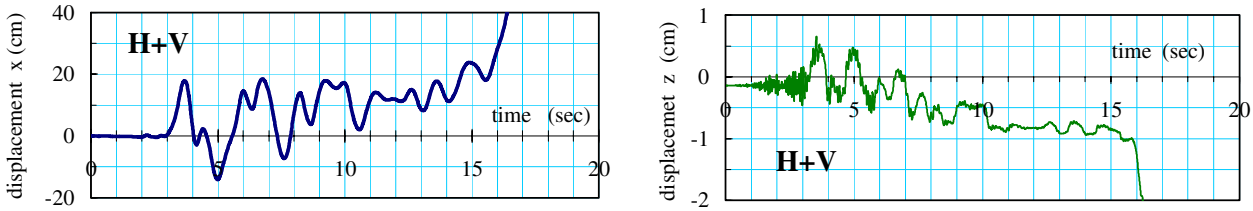
(a) Horizontal restoring force (b) Horizontal restoring force

(c) Vertical restoring force (d) Vertical restoring force

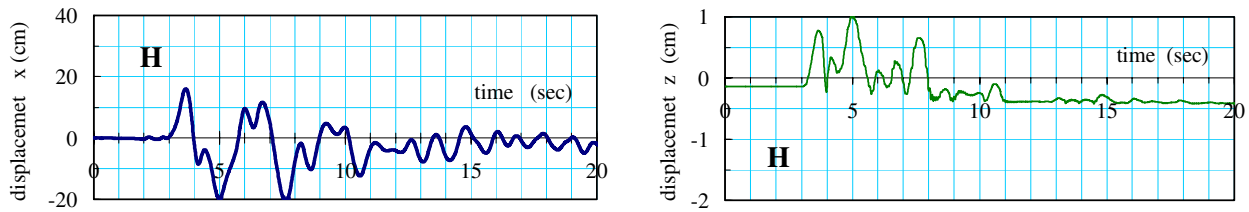
Figure 12. Effect of vertical motion on restoring forces ($n_0 = 0.20$ and $\Omega = 0.85$)

Influence of Vertical Motion on the Time History

Figure 13 shows the time history of horizontal displacement *x* and vertical displacement *z* with or without the input of vertical motion. Without vertical motion, occasionally, the residual horizontal displacement is small at the end of computation. However, the simultaneous input of vertical motion results a *10cm* drift of horizontal displacement after the time of *4sec*, and the final divergence of displacement is due to the loss of horizontal resistance. If vertical motion is also inputted, the vertical displacement drifts to the compressive direction, because the accumulation of compressive strain at the plastic hinge. The low-frequency vibration



(a) Displacements with simultaneous input of vertical motion



(b) Displacements without input of vertical motion but with consideration of vertical vibration

Figure 13. Effect of vertical motion on displacement response ($n_0 = 0.20$ and $\Omega = 0.85$)

of vertical displacement is due to the movement of neutral axis of plastic hinge, while the high-frequency vibration is due to the vertical input motion and it results a large vertical force.

CONCLUSIONS

Dynamic response of reinforced concrete column models until collapse were studied using a simplest lumped mass model that was developed from fiber model of plastic hinge, and shear deformation and shear failure of columns was not considered. Factors that may affect column response such as $P-delta$ effect and deterioration of materials were taken into consideration. Dynamic collapse of RC column is attributed to the divergence of displacements and the loss of resistances at both horizontal and vertical directions. Parametric studies using time history analysis were carried out to find the multiplier Ω for Sylmar-converter station records leading to column collapse. The main findings about the dynamic response of column models may be summarized as follows.

(1) Horizontal strength deterioration of RC columns is mainly due to the stress deterioration of materials at plastic hinge; this is aggravated by vertical ground motion.
(2) The collapse of RC columns may occur suddenly from a relatively stable state if the earthquake input is slightly increased. Resistances at both the horizontal and the vertical directions are lost after a monotonic increase of downward vertical displacement.
(3) The capacity of RC columns for avoiding collapse may be better understood if the gravity load was plotted against earthquake input. If the long-term axial stress ratio n_0 or the capacity ratio n_{ax} is doubled, then the capacity against earthquake input will be reduced to 1/4. The ultimate safety of structures should be discussed considering the vertical resistance of columns.
(4) The simultaneous input of vertical motion results large axial compressive forces, and enable the column collapse at a smaller input level. The omission of vertical motion will over-estimate the earthquake resistant capacity by 20%.
(5) Even the vertical motion is not inputted, vertical vibration occurs and this may result a large vertical compressive force. The restoring force models obtained from static test under constant axial force may also over-estimate the earthquake resistant capacity.

The investigation was made using a lamped mass model; it could not account the effect of distributed mass

and the vertical vibration of beams and floors. Assumptions about the vertical damping and the length of plastic hinge were made, and simplified stress-strain relations of materials were assumed. It is expected that the 3D restoring force characteristics of RC columns will be studied by future 3D shaking table test.

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