EXTREME OF STRUCTURAL CHARACTERISTIC FACTOR OF BUILDINGS FOR DIFFERENT TYPES OF COLLAPSE MECHANISM

Tetsuhiro ASARI¹, Keiichi INOUE² and Yuji ISHIYAMA³

SUMMARY

Many seismic codes in the world adopt a structural characteristic factor that is to reduce design seismic forces considering an energy absorbing capacity of a structure, however its value varies much. In order to investigate the extreme of the structural characteristic factor for different types of collapse mechanism, elasto-plastic earthquake response analyses are carried out for three types of analytical models, i.e. Strong-Column/Weak-Beam (SCWB), Soft-First-Story (SFS), and Multi-Story (MS) models.

Conclusions obtained from the analyses considering P-delta effect are as follows:
1. It can not be expected to reduce the design seismic force by energy absorbing capacity in case of single story collapse.
2. The extreme of the structural characteristic factor of MS model can be approximated by SFS model.
3. The structural characteristic factor should be given as a function of the natural period of the structure.
4. The probabilistic approach is recommended for the determination of the structural characteristic factor.

INTRODUCTION

Observation of the damage induced by earthquakes indicates that buildings suffered severer damage in certain specific parts and collapsed in one story. This damage concentration is not preferable from the view point of seismic safety of buildings. Therefore, the Strong-Column/Weak-Beam (SCWB) concept is introduced to seismic design in order to avoid single story collapse. This design concept is due to the design basis relying on the ductility of the structure against severe earthquake ground motions. Therefore many seismic codes in the world adopt a structural characteristic factor that is to reduce seismic design forces considering an energy absorbing capacity of a structure.

In order to discussed to what extent the design seismic forces can be reduced due to energy absorbing capacity, i.e. extreme of structural characteristic factor of buildings, Ishiyama and Asari¹ have shown the extreme of structural characteristic factor of SCWB buildings by the analytical study for a single-degree-of-freedom model considering P-delta effect.

¹ Instructor, Hokkaido University, JAPAN. Email: asari@eng.hokudai.ac.jp
² Associate Professor, Kushiro National College of Technology, JAPAN.
³ Professor, Hokkaido University, JAPAN. Email: yuji@eng.hokudai.ac.jp
While SCWB buildings have the largest energy absorbing capacity, a story collapse caused by the damage concentration to a specific story is frequently observed in actual seismic damage.

In this paper, the earthquake response analyses have been carried out for a SCWB model, a Soft-First-Story (SFS) model, and a Multi-Story (MS) model in order to investigate the extreme of the structural characteristic factor for different types of collapse mechanism.

ANALYTICAL MODEL AND PROCEDURE

SCWB (Strong-Column/Weak-Beam) model

The analytical model is a finite rotation model as shown in Fig.1 which takes into account P-delta effect. Multi-story buildings in which yield hinges are formed at the end of beams can be treated as equivalent-single-degree-of-freedom systems. The equation of motion is:

$$\ddot{\phi} + \frac{C}{I} \phi + \frac{M(\phi)}{I} = -\frac{\ddot{X}}{R} \cos \phi + \frac{g + \ddot{Y}}{R} \sin \phi$$

(1)

where $\phi$ is the rotation angle, $C$ is the damping coefficient, $M(\phi)$ is the restoring moment, $\ddot{X}$ and $\ddot{Y}$ are the horizontal and vertical accelerations of the ground motion, $g$ is the acceleration due to gravity, $I$ is the moment of inertia of the whole structure about the base, $R$ is the effective height, and they are given as follows:

$$I = \sum_{i=1}^{n} (m_i, R_i)$$

(2)

$$R = \frac{I}{\sum_{i=1}^{n} (m_i, R_i)}$$

(3)

$$R_i = \sum_{i=1}^{n} r_i$$

(4)

where $m_i$ and $r_i$ are the mass and the height of the $i$-th story, respectively.
If the mass and the height of each story are equal, the effective height $R$ converges to $2/3$ of the height of the structure as $n$ increases.

**SFS (Soft-First-Story) model**

![SFS model](image)

SFS model in which all masses concentrate to the first story is shown in Fig. 2. The building in which the strength and stiffness of the first story are significantly smaller than other stories can be simulated with this model. Equation of motion of this model is identical to Eq. (1) substituting $R = r_n$.

**MS (Multi-Story) model**

![MS model](image)

Fig. 3 shows a MS model where any story can collapse in this model. The equation of motion is:

$$
[I_t] \{ \ddot{\phi} \} + [I_n] \{ \ddot{\phi}^\ast \} + \{ C \dot{\phi} \} + \{ M_r \} = -X \{ S_y \} + (g + \ddot{Y}) \{ S_y \}
$$

(5)

where $[I_t]$ and $[I_n]$ are the matrices of moment of inertia for tangent and normal directions, respectively, $\{ \phi \}$ is the rotation angle vector, $\{ C \dot{\phi} \}$ is the damping vector of moment, $\{ M_r \}$ is the restoring moment vector, $\{ S_y \}$ and $\{ S_y \}$ are the vectors of mass-moment for horizontal and vertical components, respectively. They are given as follows:
\[
[I_n]=
\begin{bmatrix}
m_{1}r_{1}^2 & m_{1}r_{1}r_{2}\cos(\phi_1-\phi_2) & \cdots & m_{1}r_{1}r_{n}\cos(\phi_1-\phi_n) \\
m_{1}r_{1}r_{2}\cos(\phi_2-\phi_1) & (m_{1}+m_{2})r_{2}^2 & \cdots & (m_{1}+m_{2})r_{2}r_{n}\cos(\phi_2-\phi_n) \\
\vdots & \vdots & \ddots & \vdots \\
m_{1}r_{1}r_{n}\cos(\phi_n-\phi_1) & (m_{1}+m_{2})r_{2}r_{n}\cos(\phi_n-\phi_2) & \cdots & (\sum_{i=1}^{n}m_{i})r_{n}^2
\end{bmatrix}
\tag{6}
\]

\[
[I_n]=
\begin{bmatrix}
0 & m_{1}r_{1}r_{2}\sin(\phi_1-\phi_2) & \cdots & m_{1}r_{1}r_{n}\sin(\phi_1-\phi_n) \\
m_{1}r_{1}r_{2}\sin(\phi_2-\phi_1) & 0 & \cdots & (m_{1}+m_{2})r_{2}r_{n}\sin(\phi_2-\phi_n) \\
\vdots & \vdots & \ddots & \vdots \\
m_{1}r_{1}r_{n}\sin(\phi_n-\phi_1) & (m_{1}+m_{2})r_{2}r_{n}\sin(\phi_n-\phi_2) & \cdots & 0
\end{bmatrix}
\tag{7}
\]

\[
\{\phi\} = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_n \end{bmatrix}^T
\tag{8}
\]

\[
\{C\phi\} = \begin{bmatrix} C_1\phi_1 & C_2\phi_2 & \cdots & C_n\phi_n \end{bmatrix}^T
\tag{9}
\]

\[
\{M\phi\} = \begin{bmatrix} M_1(\phi_1) & M_2(\phi_2) & \cdots & M_n(\phi_n) \end{bmatrix}^T
\tag{10}
\]

\[
\{S_x\} = \begin{bmatrix} m_{1}r_{1}\cos\phi_1 \\
(m_{1}+m_{2})r_{2}\cos\phi_2 \\
\vdots \\
(\sum_{i=1}^{n}m_{i})r_{n}\cos\phi_n \end{bmatrix}
\tag{11}
\]

\[
\{S_y\} = \begin{bmatrix} m_{1}r_{1}\sin\phi_1 \\
(m_{1}+m_{2})r_{2}\sin\phi_2 \\
\vdots \\
(\sum_{i=1}^{n}m_{i})r_{n}\sin\phi_n \end{bmatrix}
\tag{12}
\]

\[I_n\] of Eq.(7) is asymmetric because of the non-linearity of Eq.(5) in case of the large deformation, and similar to the equation of motion of the multi-pendulum considering large deformation (Clough and Penzien [2]).

**Analytical procedure**

The natural period of the model is taken as \(T = 0.1n\) (s), the mass of each story is equal, and the story height is chosen as 4 meters. The analyzed natural period is \(T = 0.1 \sim 4.0\) (s). The fraction of critical damping is given as 0.05 and the damping is proportional to the instantaneous stiffness. The restoring moment is perfect elasto-plastic.

In the inelastic analysis, the yield level of restoring moment is gradually decreased until the model structure collapses by the input ground motions. Then \(C_y\) is obtained as the maximum yield base shear.
coefficient that leads the structure to collapse. The collapse is assumed to happen when the rotation angle $\phi$ reaches $\pi/2$.

In the analysis for MS model, the story shear distribution factor is $A_i$ that is stipulated in the Building Standard Law of Japan, and the stiffness distribution along the height is chosen so that the first mode becomes an inverse triangle. Preliminary analyses for various distributions of story shear coefficient and stiffness were carried out, and it has been confirmed that the effect of the distributions on the analytical results is small.

The input ground motions are listed in Table 1. All horizontal ground motions are scaled multiplying the factor so that the maximum horizontal velocity becomes 100 kine (cm/s). The vertical component, in case it is available, is considered simultaneously and the same factor as that for the horizontal component is also multiplied to the vertical component.

<table>
<thead>
<tr>
<th>Earthquake Record (Year)</th>
<th>Component</th>
<th>PGA (gal)</th>
<th>PGV (kine)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El Centro (1940)</td>
<td>NS</td>
<td>341.7</td>
<td>33.5</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>210.1</td>
<td>36.9</td>
</tr>
<tr>
<td>Taft (1952)</td>
<td>NS</td>
<td>152.7</td>
<td>15.7</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>175.9</td>
<td>17.7</td>
</tr>
<tr>
<td>Tokyo 101 (1956)</td>
<td>NS</td>
<td>74.0</td>
<td>7.6</td>
</tr>
<tr>
<td>Sendai 501 (1962)</td>
<td>NS</td>
<td>57.5</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>47.5</td>
<td>4.2</td>
</tr>
<tr>
<td>Osaka 205(1963)</td>
<td>EW</td>
<td>25.0</td>
<td>5.1</td>
</tr>
<tr>
<td>Hachinohe (1968)</td>
<td>NS</td>
<td>225.0</td>
<td>34.1</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>182.9</td>
<td>35.8</td>
</tr>
<tr>
<td>Tohoku Univ. (1978)</td>
<td>NS</td>
<td>258.2</td>
<td>36.2</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>202.6</td>
<td>27.6</td>
</tr>
<tr>
<td>Kushiro-BRI (1993)</td>
<td>N063E</td>
<td>711.4</td>
<td>33.5</td>
</tr>
<tr>
<td></td>
<td>N153E</td>
<td>637.2</td>
<td>42.0</td>
</tr>
<tr>
<td>Sylmar (1994)</td>
<td>NS</td>
<td>826.8</td>
<td>128.9</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>592.6</td>
<td>76.9</td>
</tr>
<tr>
<td>Tarzana (1994)</td>
<td>NS</td>
<td>970.7</td>
<td>77.2</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>1744.5</td>
<td>110.2</td>
</tr>
<tr>
<td>Haruka-oki (1994)</td>
<td>N164E</td>
<td>415.9</td>
<td>46.1</td>
</tr>
<tr>
<td></td>
<td>N254E</td>
<td>319.7</td>
<td>32.0</td>
</tr>
<tr>
<td>Kobe-JMA (1995)</td>
<td>NS</td>
<td>818.0</td>
<td>90.2</td>
</tr>
<tr>
<td></td>
<td>EW</td>
<td>617.3</td>
<td>74.2</td>
</tr>
<tr>
<td>Fukiai (1995)</td>
<td>N240E</td>
<td>686.5</td>
<td>57.4</td>
</tr>
<tr>
<td></td>
<td>N330E</td>
<td>802.0</td>
<td>122.8</td>
</tr>
</tbody>
</table>

Since the equation of motion in this study has geometric non-linearity, the response of the model shows non-linearity not only in inelastic range but also in elastic range, i.e. the response of the model is not proportional to input ground motions. Therefore, preliminary analyses for different levels of input ground motions were also carried out for 25 and 50 kine of the maximum horizontal velocity. In this range of ground motion (25~100 kine), both the elastic response ($C_e$) and the inelastic response ($C_{in}$) are almost proportional to the input motions. Therefore analytical results for 100 kine input motions are shown and discussed in the next section.

The numerical integration method is a linear acceleration method, and the time interval for integration is 1/500 seconds.
ANALYTICAL RESULTS

SCWB model
In Fig.4, a thick solid line indicates the average of elastic response of the base shear coefficient \( b C_e \), assuming the logarithmic normal distribution of base shear response. A thin solid curve in the figure shows the approximation of the thick solid curve, given by three curves as follows:

\[
b C e = \begin{cases} 
0.95 + 4.48 \frac{\text{T}}{} & \text{for } T \leq 0.3 \text{ (s)} \\
2.75 - 1.52 \frac{\text{T}}{} & \text{for } 0.3 \text{ (s)} \leq T \leq 1.4 \text{ (s)} \\
1.48 e^{-0.62 T} & \text{for } 1.4 \text{ (s)} \leq T 
\end{cases} \tag{13}
\]

Two dotted curves in Fig.4 indicate the average \( \pm 1 \sigma \) (standard deviation) of \( b C_e \).

A thick solid curve in Fig.5 is the average of the maximum yield base shear coefficient \( b C_y \), assuming the logarithmic normal distribution. A thin solid curve in the figure is the approximation of the thick solid curve, given by two curves as follows:

\[
b C y = \begin{cases} 
\frac{1}{14T + 3} & \text{for } T \leq 1.0 \text{(s)} \\
\frac{1}{17T} & \text{for } 1.0 \text{(s)} \leq T 
\end{cases} \tag{14}
\]

Two dotted curves in Fig.5 indicate the average \( \pm 1 \sigma \) (standard deviation) of \( b C_e \).

Fig.6 shows \( b C_y / b C_e \) which indicates the extreme of structural characteristic factor in case of SCWB model. In the figure, a thick solid curve is the average of \( b C_y / b C_e \), two dotted curves indicate the average \( \pm 1 \sigma \) of \( b C_y / b C_e \), and the thin solid curve is the approximation of the average of \( b C_y / b C_e \), i.e. Eq.(14) or the thin solid curve of Fig.5, divided by Eq.(13) or the thin solid curve of Fig.4.

SFS model
In Fig.7, a thick solid curve indicates the average of elastic response of the base shear coefficient \( p C_e \), two dotted curves indicate the average \( \pm 1 \sigma \) of \( p C_e \), and a thin solid curve is given by Eq.(13). Fig.7 is almost identical to Fig.4 because the P-delta effect is considerably small in elastic range.

A thick solid curve in Fig.8 is the average of the maximum yield base shear coefficient \( p C_y \), two dotted curves indicate the average \( \pm 1 \sigma \) of \( p C_y \), and a thin solid curve in the figure is the approximation of the thick solid curve, given by two curves as follows:

\[
p C y = \begin{cases} 
0.2 + 0.2T - 0.13T^2 & \text{for } T \leq 1.4 \text{(s)} \\
1.48(0.133T + 0.176)e^{-0.62T} & \text{for } 1.4 \text{(s)} \leq T 
\end{cases} \tag{15}
\]

Fig.8 is somewhat similar to the elastic spectrum in Fig.7. It becomes a gentle peak at 0.7~0.8(s), and gradually decreases as the period becomes longer. The divergence of \( p C_y \) is larger than that of \( b C_y \) in Fig.5 for any period \( T \).
Fig. 4: Elastic base shear coefficient $b C_e$ for SCWB model

Fig. 5: Yield base shear coefficient $b C_y$ for SCWB model

Fig. 6: $b C_y / b C_e$; extreme of structural characteristic factor for SCWB model

Fig. 9 shows $p C_y / p C_e$, which indicates the extreme of structural characteristic factor for SFS model. In the figure, a thick solid curve is the average of $p C_y / p C_e$, two dotted curves is the average $\pm 1\sigma$ of $p C_y / p C_e$, and a thin solid curve is the approximation of the average of $p C_y / p C_e$, i.e. Eq.(15) or the thin solid curve of Fig.8, divided by Eq.(13) or the thin solid curve of Fig.7 (or Fig.4). The average of $p C_y / p C_e$ increases linearly as the period becomes longer. The figure indicates that seismic design forces can not be reduced for the SFS structures considering an energy absorption by inelastic deformation, when the natural period of structure becomes longer. The divergence of $p C_y / p C_e$ is large because of the large divergence of $p C_y$. This implies that the extreme of structural characteristic factor is strongly influenced by the characteristics of input ground motions.
Fig. 7: Elastic base shear coefficient $\rho C_e$ for SFS model

Fig. 8: Yield base shear coefficient $\rho C_y$ for SFS model

Fig. 9: $\rho C_y/\rho C_e$; extreme of structural characteristic factor for SFS model
Fig. 10: Elastic base shear coefficient $m_C$ for MS model.

Fig. 11: Yield base shear coefficient $m_C$ for MS model.

Fig. 12: $m_C / m_C$; extreme of structural characteristic factor for MS model.
**MS model**

In Fig.10, a thick solid curve indicates the average of elastic response of the base shear coefficient \( m C_e \), two dotted curves indicate the average \( \pm 1 \sigma \) of \( m C_e \), and a thin solid curve is given by Eq.(13). Since the response is in elastic range, Fig.10 is almost identical to Fig.4 or Fig.7.

A thick solid curve in Fig.11 is the average of the maximum yield base shear coefficient \( m C_y \), two dotted curves indicate the average \( \pm 1 \sigma \) of \( m C_y \), and a thin solid curve is given by Eq.(15). Although any story can collapse in this model, 394 cases of 504 analytical cases (78.2%) collapsed at the first story, 109 cases (21.6%) collapsed at the second story, and only 1 case (0.2%) collapsed at the third story. Therefore, \( m C_y \) is always defined as the base shear coefficient whichever story collapses. Fig.11 is almost identical to Fig.8.

Fig.12 shows \( m C_y / m C_e \) which indicates the extreme of structural characteristic factor for MS model. In the figure, a thick solid curve is the average of \( m C_y / m C_e \), two dotted curves indicate the average \( \pm 1 \sigma \) of \( m C_y / m C_e \), and a thin solid curve is given by Eq.(15) divided by Eq.(13). Since Fig.12 is almost identical to Fig.9, seismic design forces can not be reduced for the MS structures considering an energy absorption by inelastic deformation as the natural period becomes longer just as the SFS structures. The divergence of \( m C_y / m C_e \) is quite large for MS model. This implies that the extreme of structural characteristic factor is strongly influenced by the characteristics of input ground motions.

**DISCUSSION**

In this section, discussion is carried out through the comparison of results for SCWB, SFS, and MS models.

Fig.13 shows the average of elastic response of the base shear coefficient \( C_e \) for three models. \( C_e \) for SFS model is almost identical to that of SCWB model, because both models are single-degree-of-freedom models. Since the MS model is of multi-degree-of-freedom, \( C_e \) for MS model is comparatively smaller than that of SCWB model and SFS model because of the higher mode effect.

Fig.14 shows the average of the maximum yield base shear coefficient \( C_y \) for three models. The spectral shape of \( C_y \) for SCWB model is considerably different from other two models. The \( C_y \) for SCWB model exponentially decreases as the period becomes longer, however \( C_y \) for SFS model and MS model decrease as the period becomes longer after a gentle peak at about 0.7(s). \( C_y \) for SFS model and MS model at 4(s) is 1/2 of the maximum \( C_y \) at 0.7(s). The difference between \( C_y \) for SFS model and MS model is small. \( C_y \) for MS model is comparatively smaller than that of SCWB model.
Fig. 13: Elastic base shear coefficient $C_e$ for three models.

Fig. 14: Yield base shear coefficient $C_y$ for three models.

Fig. 15: $\frac{C_y}{C_e}$; extreme of structural characteristic factor for three models.
Fig. 15 shows the average of the extreme of structural characteristic factor $C_y/C_e$ for three models. Two horizontal lines indicate the extreme of structural characteristic factor $D_s$ in Japanese code and $1/R$ in U.S. code. The figure indicates that it is possible to reduce seismic design forces due to energy absorption by inelastic deformation for SCWB model. Since the maximum of the $C_y/C_e$ is approximately 1/8, seismic design forces can be reduced down to 1/8 for SCWB model. On the other hand, the extreme of structural characteristic factor for a structure, which has a story collapse mechanism as SFS and MS models, increases linearly as the period becomes longer. Therefore, the seismic design force can not be reduced considering the energy absorption by inelastic deformation as the natural period becomes longer. It is concluded that the structural characteristic factor is greatly influenced by the collapse mechanism.

**CONCLUSIONS**

In order to investigate the extreme of the structural characteristic factor for different types of collapse mechanism, non-linear response analyses have been carried out for Strong-Column/Weak-Beam (SCWB), Soft-First-Story (SFS), and Multi-Story (MS) models.

The conclusions of this work are as follows:

1. The extreme of the structural characteristic factor of SFS and MS models are quite larger than that of SCWB model. Therefore, it can not be expected to reduce the design seismic force by energy absorbing capacity in case of single story collapse.
2. The extreme of the structural characteristic factor of MS model can be approximated by SFS model, because the difference between the analytical results of the two models is very small.
3. For three models, the extreme of structural characteristic factor is a function of the natural period of the model. Therefore, the structural characteristic factor should be given as a function of the natural period of the structure.
4. Since the divergence of structural characteristic factor caused by the input ground motions is significantly large, the probabilistic approach is recommended for the determination of the structural characteristic factor.

**REFERENCES**