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TRANSMITTING BOUNDARIES FOR TRANSIENT ELASTIC RESPONSE IN SOIL-STRUCTURE INTERACTION SYSTEM

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SUMMARY

The objective of present work is to investigate the effects of various boundaries in analyzing the problems in dynamic soil-structure interaction (SSI) system. The fixed soil boundaries in the soil structure interaction system under seismic event, reflect the waves which are radiated outward from the excited structure towards infinity. A general and efficient Finite Element method through which an infinite system may be approximated to a finite system for the solution directly in the time domain of transient soil-structure interaction problems is the main concern. A study has been performed here to investigate the effectiveness of various boundaries such as viscous boundaries, transmitting boundary etc.

The dynamic response of an elastic block on a homogeneous and on layered half space is studied. The block and the near field soil region are discretised with plane strain finite elements and far field soil region is modeled by transmitting cone boundary in which dashpots are combined with stiffness at the boundary nodes. The SSI system is subjected to transient load of a rectangular pulse either in vertical or in horizontal direction at top surface of the block. The displacement time history responses at the top of the block are compared for the cases of free, fixed and transmitting boundary in graphical form . The transient response of SSI system is evaluated by varying the stiffness of the homogeneous soil. The parametric studies of the SSI system under transient loads are carried out to investigate the effect of stiffness ratio between the soil layers on transient response of the system in a layered half space. The maximum response and the behavior of present responses are compared with that of other investigators in tabular form. The response of block-soil interaction under horizontal and vertical transient load, indicate that the present method is effective compared to other modeling methods.

INTRODUCTION

The well known numerical problem in dynamic soil-structure interaction analysis is how to simulate far field soil medium, the phenomena of waves that radiate outward from the excited structures towards infinity. This radiation condition leads to boundary value problem for an unbounded domain. The dynamic response of massive structures, such as nuclear power plants, high rise buildings, dams, etc.; may be influenced by the soil-structure interaction as well as the dynamic characteristics of the exciting loads and

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the structures. The effect of soil-structure interaction is noticeable especially for stiff and massive structures resting on relatively soft ground. It may alter the dynamic characteristics of the structural response significantly. Thus the interaction effects have to be considered in the dynamic analysis of the structures in a semi-infinite soil medium. A surface or zone is chosen for analyzing the semi-infinite domain of the soil. The constitutive associated with the nodes on this surface represents the significant features of the far field. The interaction horizon can be situated depending upon the problem. Depending on the modeling method for the soil region, method of SSI analysis can be classified into two groups: the substructure method and the direct method [Wolf, 1].

In substructure method, based on fundamental solution, which satisfy exactly the radiation condition formulated at infinity and on the principal of superposition, applies only to regular linear half space models. Thus, the procedure of the soil-structure interaction analysis becomes very simple, and effort for analysis can be minimized. However, this method is restricted to the cases for simple geometry of foundation and linear behavior of soil medium.

The direct method models numerically the structure and a region of soil in contact up to the artificial boundary. Hence the direct method has the advantages of considering a complex geometry, spatial variations of soil properties, and non-linear behavior of soil medium.

Several kinds of direct methods for soil-structure interaction analysis have been developed to consider the radiational damping of an unbounded soil medium. They are transmitting boundary [Lysmer, 2], boundary elements [Estorff, 3], infinite elements [Medina, 4] and system identification method [Tzong, 5]. The infinite element method, of which concept was originally introduced by Ungeles [6] and Bettess [7] about two decades ago, has one of the popular techniques, since its concept and formulation procedure are similar to those of the finite element method except for the infinite extent of the element region and shape functions. The shape functions of infinite elements are usually formulated depending on the type of the problem in order to describe the behavior of the infinite medium effectively [Yun, 8].

Numerical procedures for the dynamic SSI analysis may be classified according to the nature of the time dependence as either time harmonic or transient. In the linear case, time-harmonic solutions can be used indirectly employing Fourier transform to solve the transient problems. Still a direct time integration approach is necessary whenever nonlinearities occur and may be advantageous for some classes of linear problems. For example, in linear problems exhibiting broadband phenomena, the indirect approach may not be computationally feasible. Also, the measurements of actual performance of SSI problems are usually recorded directly in the time domain, so it may be of interest to use or compare this information with that predicted directly by the mathematical models. In this paper, local transmitting boundaries applicable for transient analysis are considered and a direct method for SSI analysis in two-dimensional (2-d) medium is presented in time domain.

METHODOLOGY

When an impulse is acting on elastic half space medium, the energy is radiated by shear and dilational waves (S and P waves). In order for the waves to transmit energy at infinity, the displacement amplitude must die off at large distance in a special low. A radiation criterion states that radiation of energy occurs when the displacement amplitude decays at infinity in inverse proportion to the square root of the surface area at infinity. For, 2D plane strain analysis, the surface at infinity for body waves can be considered as an infinitely long cylinder with radius tends to infinite. From the radiation criterion, in the plane strain case, body waves can be considered to decrease in inverse proportion to the product of the distance from the input source to boundary node and Poisson's ratio.

For, any point with (x, y) coordinate system in plane strain 2-d analysis(Fig. 1a), the stress component at the boundary location , considering the angle of incidence can be given in general as [Kellezi, 9]:

$$\{\sigma\} = [D_k]\{u\} + [D_c]\{u_{,t}\} \quad \ldots\ldots..(1)$$

where, $\{u\}$ = displacement at the boundary location node,
$\{u_{,t}\}$ = velocity at the same location,
constitutive stiffness matrix $[D_k]$ is given as:

$$[D_k] = (\rho V_p^2/2r)(\mathbf{n.r})[N] + (\rho V_s^2/2r)(\mathbf{n.r})\{[I]-[N]\} \ldots.(2)$$

and the constitutive damping matrix $[D_c]$ is given as :

$$[D_c] = (\rho V_p)(\mathbf{n.r})[N] + (\rho V_s)(\mathbf{n.r})\{[I]-[N]\} \ldots\ldots.(3)$$

In Eq.(2) and Eq.(3),
ρ = mass density of soil,
V_p = velocity of P waves,
V_s =velocity of S waves

[N] = Transformation matrix = $\begin{bmatrix} n_x^2 & n_x n_y \\ n_y n_x & n_y^2 \end{bmatrix}$,

[I] = Identity matrix of order 2,
$\mathbf{n}(\mathbf{n_x}, \mathbf{n_y})$ = outward unit vector normal to the boundary surface,
$\mathbf{r}(\mathbf{r_x}, \mathbf{r_y})$ =unit vector to represent the direction of wave propagation,
r = |**r**| = distance from boundary node to the source of location,
n.r = cosα, α= angle between **n** and **r** .

For plane strain analysis, the coordinates of these unit vectors are **n** (n_x , n_y) and **r** (r_x , r_y) . For axisymmetric case of circular boundary, r is constant for all boundary nodes and **n.r** =1.0 while for rectangular boundary of plane strain analysis , r for each boundary nodes together with **n** and **r** are considered to calculate the constitutive stiffness and damping matrices. So, for the plane strain analysis Eq.(1) can be employed as the transmitting boundary in the area where body waves propagate. The boundary or geometric stiffness and damping matrices for the whole SSI system are obtained by assembling those for the finite element (FE) boundaries. The consistent matrices locally couple the nodes along the boundary giving a more realistic implementation. The equations of motion for the visco-elastic system are:

$$[M]\{u_{,tt}\} + [C]\{u_{,t}\} + [K]\{u\} = \{P\} \ldots\ldots.(4)$$

where, [M] is the mass matrix,
$\{u_{,tt}\}$ is acceleration,
[C] is damping matrix, and
[P] is transient load.

Internal damping is implemented as Rayleigh damping. The direct step by step Newmark's β method is used to solve the equations of motion for evaluating displacement at various locations in SSI system.

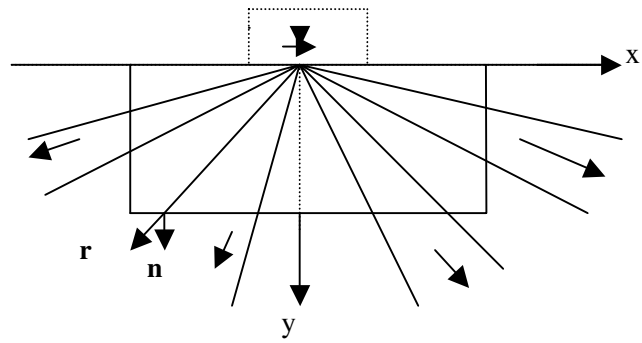

Fig. 1a Cone Transmitting Boundary

BLOCK-SOIL INTERACTION

The dynamic response of an elastic block on a homogeneous and on layered half space, as shown in Fig. 1b, is studied.

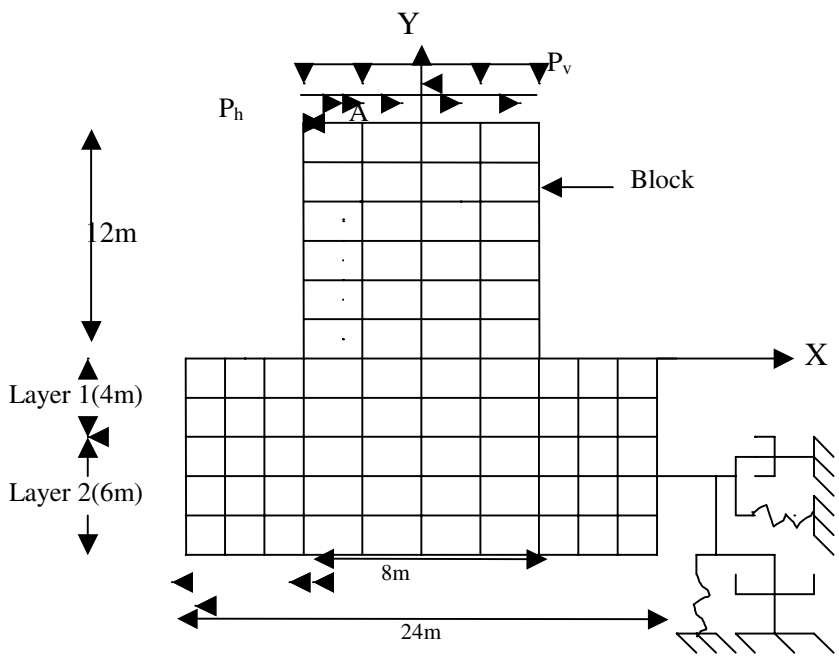

Fig. 1b Block-soil interaction system-analysis model

The elastic block and soil is discretised with 4 noded plane strain finite elements and the remaining far field region is modeled by cone transmitting boundary having both stiffness and dashpots at the boundary nodes in horizontal and vertical direction. The transient load of a rectangular impulse over $5\Delta t$, shown in Fig. 1c, is applied on the top of the block either in vertical or in horizontal direction, in which Δt is 0.0008.

The horizontal and vertical distances from the centre of the foundation to the lateral boundary and vertical boundary are taken as 3b and 2.5b, respectively, where b is half width of the block (=4m).

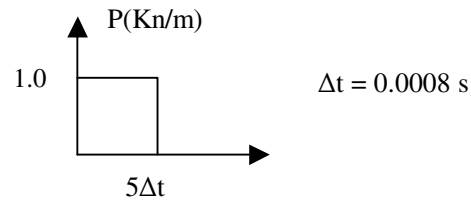

Fig. 1c Loading History for P_v and P_h

The mass density (ρ) and Poisson's ratio (υ) of block are taken $2.0x10^3$ kg/m^3 and 0.25 . The Young's modulus of Block (Eb) is $3.0x10^{10}$Pa. The size of the block is 8mx12m. For the homogenous soil medium , Poisson's ratio (υ1) and mass density (ρ1) are taken to be same as those of the block. On the other hand, two cases of Young' modulus (E1) of homogeneous soil are taken, $1.0x10^{10}$Pa & $3.0x10^{10}$Pa. The material damping in the block and the soil is not included as in the case analysed by Estorff [10].

The vertical displacement at Point A (shown in Fig. 1b) is computed for the vertical load case, while the horizontal displacement at Point A is computed for the horizontal load case.

Parametric studies are carried out to investigate the effect of stiffness ratio (E2/E1) between the soil layers in a layered half-space, where E1 and E2 are Young's moduli of the upper horizontal layer and the underlying half-space. The depth of the upper layer is taken as 4m. The value of (E1) is taken as $1.0x10^{10}$ Pa, while three cases of E2 are considered: ie., $1.0x10^{10}$ Pa , $3.0x10^{10}$ Pa and $10.0x10^{10}$ Pa. The results obtained for a homogeneous half space are compared with those of Estorff [10] and Kim [11].

RESULTS AND DISCUSSION

Comparison of Response for the cases of free, fixed and transmitting boundary

The results are obtained for a homogeneous half space with Young's Modulus (E1)= $3.0x10^{10}$ Pa and compared for three cases of boundaries. The displacement histories at A on the block are plotted against the dimensionless time(t_0= tV_s /b) , b is half width of block (4m), V_s is the shear wave velocity of soil medium with Young's modulus(E) of $1.0x10^{10}$ Pa. The horizontal displacement response at A under horizontal transient load on top of the block is plotted in Fig. 2a for fixed boundary, viscous boundary and transmitting cone boundary cases. Similarly, Fig. 2b shows the vertical displacement time history at A due to vertical transient load at top of the block for fixed boundary, viscous boundary and of transmitting cone boundary cases. It is observed from Fig. 2a that horizontal displacement response is diverging for the fixed boundary case while the response is similar for viscous and cone boundary, that is, slowly converging as time lapses. In case of vertical displacement response under vertical load, the response is diverging for the fixed boundary case; converging very slowly for the case of viscous boundary and fast converging for the case of cone boundary. It is concluded that response in the block is die off in case of transmitting boundary compared to other cases, as shown in Fig. 2a & 2b. So, the transmitting cone boundary represents effectively the semi-infinite behavior of soil medium.

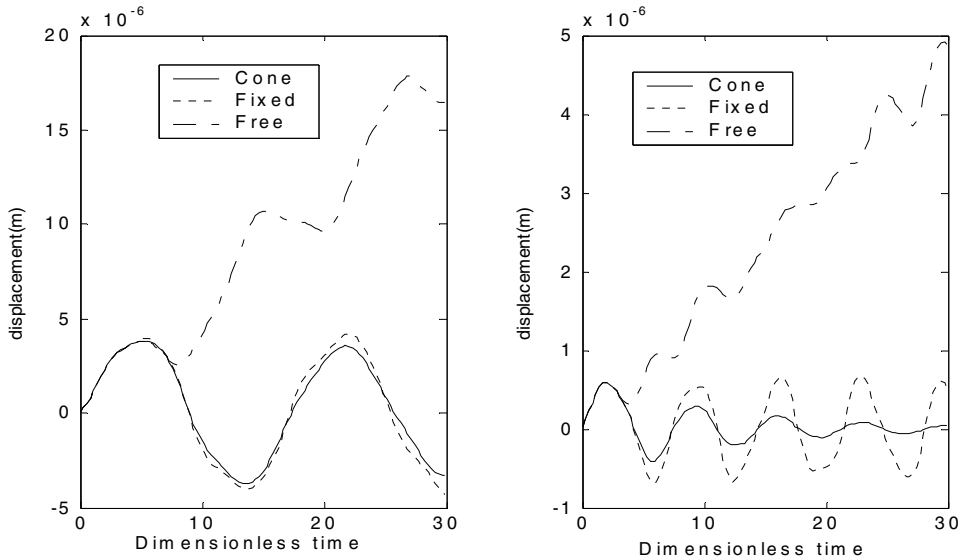

Fig. 2(a) Hor. disp. at A due hor. Load (b) Ver. Disp. At A due to ver. Load for various boundaries

Comparison of Response by varying the stiffness ratio of block and homogenous soil

Fig. 3a & 3b shows the horizontal displacement at A on the block due to horizontal load and vertical displacement at A on the block due to vertical load for two case of stiffness ratio of block and soil. It is observed that maximum displacement at A decreases with increase of stiffness of the soil. Also frequency contents of the motions move into higher frequency range with increase of soil stiffness. Table 1 shows the comparison of maximum horizontal response for present study and that of Estorff [10] and Kim [11] under horizontal transient load for two stiffness ratio of block and homogeneous soil and observed that present modeling is much effective.

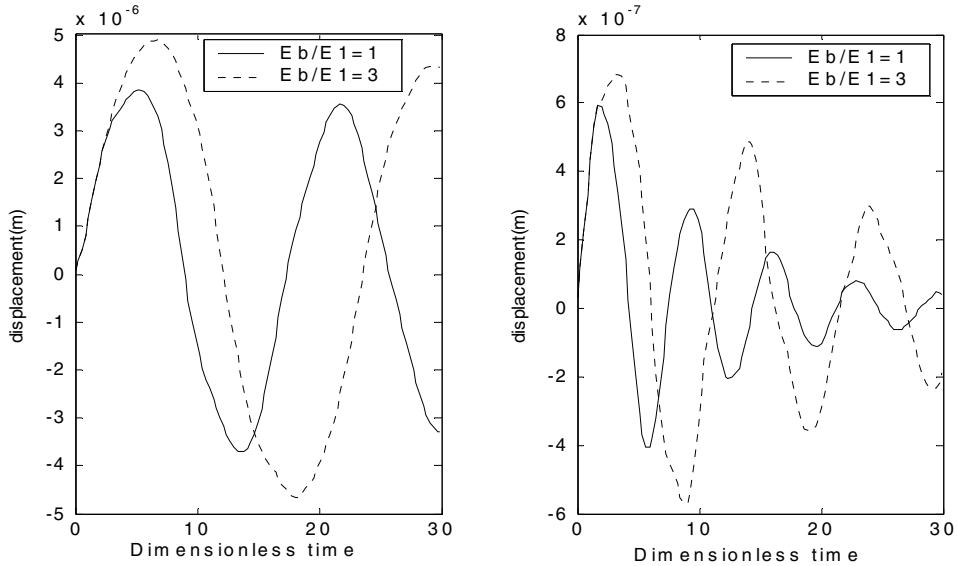

Fig. 3(a) Hor. disp. at A due hor. Load (b) Ver. Disp. At A due to ver. Load for different stiffness ratio of block and homogeneous soil

Table 1 Comparison of Horizontal Displacement at A due to Horizontal load by other Investigators for various stiffness ratios of the block and of the homogeneous soil

Investigators	Maximum Displacement (m) for Eb/E1=1	Maximum Displacement (m) for Eb/E1=3
Present study	0.6 E-6	0.7 E-6
Estorff's [10]	3.8E-6	5.3E-6
Kim's [11]	3.6E-6	4.7E-6

Effect of Stiffness ratio of layered soil on transient response of block

The Parametric studies are carried out to investigate the effect of the stiffness ratio between the soil layers in a layered half space. The horizontal displacement due to horizontal load and vertical displacement due to vertical load at A are plotted for three different stiffness ratio of soil layers and shown in Fig. 4a & 4b. It is found that maximum response of point A on the block increases with increase of stiffness ratio of soil layers from 1 to 10, that is, by increase the stiffness of bottom soil compared to top layer, displacement at A increases, damping decreases and frequency content of motion moves into higher frequency range. It is due to increase of wave reflection at the interface between two layers .Therefore, the radiation damping effect decreases. Table 2 shows the maximum horizontal response due to horizontal load, for the present study and that of Kim [11] for various stiffness ratio of layered soils. It is found that present study shows faster pattern of die off the transient response in the block as compared to that by other investigators. So, the modeling of half space soil medium by cone transmitting boundary, represents truly behavior of the medium.

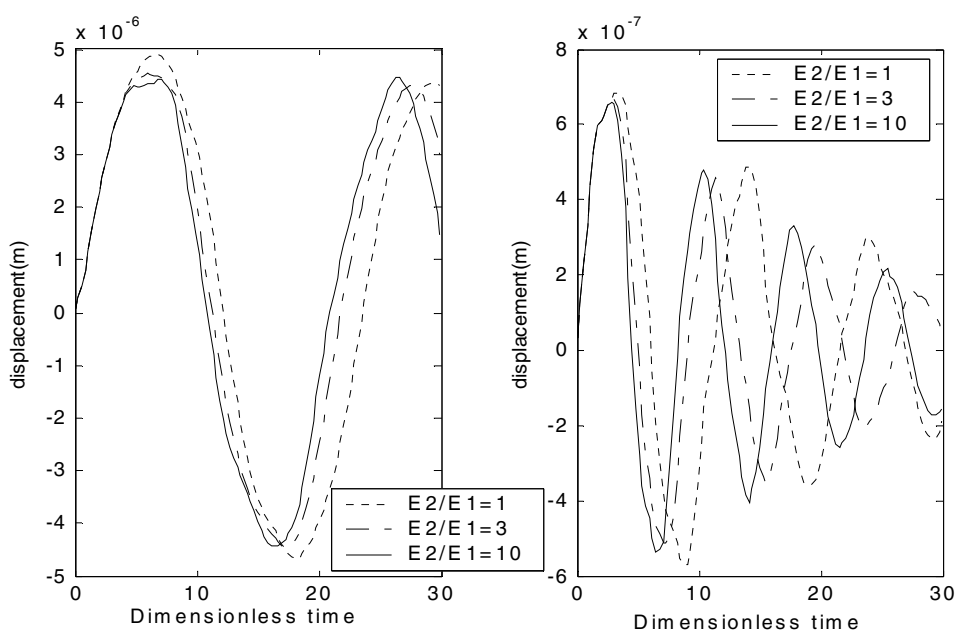

Fig. 4(a) Hor. disp. at A due hor. Load (b) Ver. Disp. At A due to ver. Load for various stiffness ratio of layered soils

Table 2 Comparison of Horizontal Displacement at A due to Horizontal load by other Investigators for different ratios of the soil layers

Investigators	Maximum Displacement (m) for E2/E1=1	Maximum Displacement (m) for E2/E1=3	Maximum Displacement (m) for E2/E1=10
Present study	0.65 E-6	0.60 E-6	0.55 E-6
Kim's [11]	5.00 E-6	4.50 E-6	4.00 E-6

CONCLUSION

In this study, transient response of an elastic block on a homogeneous and on layered half space is investigated. It is observed that radiation and boundary conditions can be interpreted as constitutive equations for the interaction forces between the near and far fields. A tendency for improvement of transient response is reported while using the combination of dashpot and stiffness at the boundary nodes. The displacement response at top of the block decreases with increase of stiffness of homogeneous soil while for the layered soil, the displacement response in the block increases when top soft soil is underlying by hard soil. It is due to reflection of waves at the interface between two layers which leads in decrease of radiation damping effects.

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