CONCEPT AND APPLICATION OF MODE DESIGNATION
FOR SEISMIC DESIGN

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SUMMARY

Rocking Center Designated (RCD) Mechanism is proposed to control the vibration modes of building structures. A RCD mechanism is constructed simply by replacing the first story of a cantilever wall with a pair of inclined RCD columns. If RCD mechanisms are incorporated into a frame structure, vibration modes of the system will be largely rely on the inclination of RCD columns. The concept of designation of vibration modes is discussed using a simplified 2DOF model, and a practical application for systems with a predominant first vibration mode is introduced for seismic design. RCD mechanism is effective for eliminating the deformation concentration of building structures, and may be applied for meeting various performance needs and controlling different kinds of physical quantities in seismic response.

INTRODUCTION

Usually, a frame structure vibrates for a preferably reversed triangular first mode, while an ideal base isolation building vibrates for a uniformly rectangular first mode. The difference in the vibration modes exhibits different structural performances such as response in acceleration, lateral force distribution, overturning moment and displacement distribution, thus requires different needs in resisting seismic forces and in accommodating the displacements for structural members. Therefore, optimal structural design for different performance needs may be investigated from the point of view of vibration modes combination. It is expected that seismic design and seismic control may be studied based on new structural systems with utterly different vibration characteristics without the limitation of current practice.

However, until now, this is difficult for lack of structural model that are able to show arbitrary mode combinations, as the conventional structural systems exhibit only a predominant first vibration mode. Moreover, even in the category of seismic design based on predominant first vibration mode, there exist a problem or strategy of how to utilizing different kinds of structural members to effectively meet the requirements for resisting seismic forces, realizing displacement distribution and accommodating the resulted deformation.

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Rocking Center Designated (RCD) Mechanisms was developed to control the vibration modes of building structures [1]. In this paper, RCD mechanism and its effectiveness for vibration mode control are explained using a Two-Degree-Of-Freedom model, and shaking table test of structural models is described for confirmation of the function of RCD mechanism.

Finally, as a practical example for application of RCD structures, displacement distributed structural system [2] with a somewhat large displacement at the first story and its use in structure design is discussed. As the earthquake response of this type of structures are dominated by the first vibration mode, an equivalent Single-Degree-Of-Freedom model and some points in seismic design is discussed. The displacement distributed structural system is expected to be an alternative for replacing base isolation in structural design.

RCD STRUCTURE AND VIBRATION MODES DESIGNATION

Rocking Center Designated (RCD) Mechanism
For seismic design, unless the displacement or deformation are specified at only one story, as in the case of base isolation, the displacement must be diversified or averaged over every story to prevent the collapse at a single story. Therefore, it is recommended to use weak beams and strong columns. For this yielding mechanism, in order to meet the requirement for strength, we have to use very big columns. However, big columns have small ductility at the base of the first story where axial stress is very high. Compared with the large deformability of slender beam and columns, bracings and structural walls have great strength and rigidity but less deformability. An ideal structural system is one that uses different kinds of structural members for strength, ductility, and displacement distribution. In order to cope with this contradictory dilemma, Rocking Center Designated mechanism was developed to control the vibration mode of buildings, and to distribute the displacement.

As shown by Figure 1, a RCD mechanism consists of a pair of inclined RCD columns and a RCD wall. RCD mechanisms are incorporated into a frame structure to designate the vibration modes, and earthquake energy may be consumed using supplementary dampers. While the RCD wall above the second floor smoothing the upper story drift, the RCD columns pointed to the rocking center (or rotation center) resist the moment resulted from the story force shared by the RCD wall. Because we can use slender columns
even pinned ends columns, it is easy to insure the deformability of RCD columns and rotation of the RCD mechanism. Moreover, RCD wall has large rigidity and is more efficient for displacement distribution than columns in beam yielding mechanism.

If the rotation center is located at the ground level, i.e., \( R = -h \); then, RCD mechanism will function as a pin-ended wall. The pin-ended wall smooths the deformation distribution, and has the same effects as fix-ended wall for control the maximum story drift [3].

As a mechanism, RCD mechanism itself does not exhibits resistance. If the axial rigidities of RCD columns and shear-bending rigidity of RCD wall are infinite (stiff RCD mechanism), RCD mechanism will deform around the rotation center geometrically according to the inclination of the RCD columns as shown in Figure 2. If the rotation center is above the building height, RCD mechanism itself will vibrate as an inverted pendulum, this was firstly proposed by Todorovska [4].

**Mode Designation Using Stiff RCD Mechanism**

For a stiff RCD mechanism with rocking center located under the ground level, Figure 3 illustrates its deformation again in detail. RCD mechanism imposes the same lateral deformation via boundary beams. If the rotation angle of RCD wall is \( \theta \), the upper stories of the main structure will have the same story drift angle. At the first story, story drift \( x_1 \) will be expressed by Equation (1). Here, \( -R \) is the distance between the rotation center and the second floor.

\[
x_1 = -R \theta
\]  

(1)

Shaking table tests are carried out for verifying the vibration mode designation using RCD mechanism. In Figure 4, rigid RCD mechanism is connected to a 2DOF main structure by rigid links. Therefore, the main structure will vibrate according to the movement of RCD mechanism. The relative displacement at the second story will be expressed by Equation (2).

\[
x_2 = s \cdot \theta
\]  

(2)
Actually, the top of model is supposed as the center of upper stories. Let $l_2$ be the distance between the bottom ends of RCD columns, then rotation radius will be $R = 200\sqrt{(220^2 + 200^2)} - l_2$ (unit: mm). From Equations (1) and (2), the displacement ratio of the 2DOF model in Figure 4 will be expressed as follows.

$$\frac{X_2}{X_1} = \frac{-s}{R}.$$  \hspace{1cm} (3)

Results obtained from free vibration tests are shown in Figure 5. Amplitude ratios of relative displacement $X_2 / X_1$ are approximately the same as expressed by Equation (3). For $(l = 50\text{mm}, R = 494\text{mm})$, then $X_2 / X_1 = 2$, and the main structure almost vibrates with a reversed triangle first mode. For $(l = 200\text{mm}, R = \pm\infty)$, then $X_2 / X_1 = 0$, and the main structure vibrates like an ideal base isolation. For $(l = 300\text{mm}, R = 775\text{mm})$, then, $X_2 / X_1 = -1$, and the first story and the top of the model vibrate oppositely.

The function of RCD mechanism for mode designation is also confirmed by input an earthquake wave, here NS component of JMA Kobe, 1995 Hyogo–Ken-Nanbu earthquake was scaled to $PGA = 0.5m/s^2$. In Figure 6, time histories of relative displacements of the main structure are shown for several different rotation radii. The relative displacements of main structure are decided by the first vibration mode, because the very high frequency of the second vibration mode. Even the displacements are very small in the case of $(l = 300\text{mm}, R = 775\text{mm})$, however, base shear force will be decided by the predominant second vibration mode. How to exploitation of the higher vibration modes will be investigated in the future.

**Vibration Modes Based on 2DOF Analytical Model**

In order to investigate the basic characteristics of RCD structures for vibration mode control, a 2DOF model is assumed as shown in Figure 7, where non-stiff RCD mechanism is utilized. The main structure is simplified as a concentrated mass with rotation inertia $I_G$ about the mass center in the vertical direction. As the main structure is forced to deform laterally, the rotation inertia at each story level may ignored. The horizontal stiffness of main structure is represented by a bending spring $k_2$ for upper stories and a shear spring $k_1$ for the first story. The axial deformations of RCD columns are included in the previous study [1], but are not described in this paper for simplicity.
Figure 8 shows the deformations of main structure, and displacement \(x_c\) at mass center and displacement \(x_1\) at the first story are relative to the ground level. The inclination \(\varphi\) of RCD columns is decided by the rotation radius \(R\) and the span \(2L\) of RCD wall as expressed by Equation (4). The displacement \(x_c\) is the result of first story displacement \(x_1\) and the rotation of RCD wall \((\theta + \gamma_w)\) as expressed by Equation (5). Here, \(\theta\) is the rotation of RCD wall for a stiff RCD mechanism, and \(\gamma_w\) is the equivalent shear deformation of RCD wall with a bending-shear stiffness of \(k_w\). From Equation (1), \(\theta\) is related with \(x_1\) by Equation (6).

\[
\tan \varphi = \frac{L}{R} \quad (4)
\]

\[
x_c = x_1 + s(\theta + \gamma_w) \quad (5)
\]

\[
\theta = -\frac{x_1}{R} \quad (6)
\]

Equilibrium of lateral force at the first story leads to Equation (7). Considering moment equilibrium of main structure at the second floor level, it leads to Equation (8). Here, \(\Delta N\) is the axial force variation of RCD columns, and \(F_w\) is the force acted on RCD wall from the mass center of main structure and is expressed by Equation (9).

\[
m(\ddot{x}_c + \ddot{x}_0) + k_1x_1 + 2\Delta N \sin \varphi = 0 \quad (7)
\]

\[
sm(\ddot{x}_c + \ddot{x}_0) + I_G(\ddot{x}_c - \ddot{x}_1) + k_2s^2(x_c - x_1) + s + F_w s = 0 \quad (8)
\]

\[
F_w = k_w \cdot \gamma_w s \quad (9)
\]

However, from the moment equilibrium of RCD mechanism at the end of RCD wall, the axial force variation of RCD columns related with the interaction force \(F_w\) as shown by Equation (10).

\[
F_w \cdot s = \Delta N \cdot 2L \cos \varphi = 0 \quad (10)
\]

If the mass is assumed be uniformly distributed from the second floor to the top of the building, then, rotation inertia of the main structure will be \(I_G = ms^2/3\), noting \(s\) is the distance from the second floor to the mass center. Using Equations (4)-(6) and Equations (9) and (10), the following equations of motion are obtained for the 2DOF system without consideration of viscous damping.
Figure 9 shows the participation vector of the 2DOF model, where $\alpha_i$ and $\beta_i$ are the participation vector of mass center and first story for the $i^{th}$ vibration mode respectively. Three cases for different $k_w/k$ are shown, where $k_w/k$ is the stiffness ratio of RCD wall to main structure. Here, for a 10 story building, $5/2 = \frac{L_s}{s}$, $5/1 = \frac{k_k}{s}$, $0.02/\pi = \frac{EA_h}{k_m}$, $1/2 = \pi$ are assumed.

Participation vector and vibration modes of the 2DOF model vary with rocking radius ratio $s/R$. If rocking center is outside the building, i.e., $R/s < 0$ or $R/s > 2$, then, the first vibration mode will be predominant. However, if rocking center is within the building height, i.e., $0 < R/s < 2$; then, the second vibration modes will be large. Both the first and the second vibration modes vary and depend on $R/s$ and the rigidity $k_w$ of RCD wall. For a very stiff RCD mechanism with $k_w/k = 100$, as shown in Figure 9(a), the system may exhibit predominant second vibration mode. In particularly, the first vibration mode disappears when $R/s = 1$. Participation vector of the second vibration mode decrease with smaller $k_w/k$, as shown in the cases (b) and (c).

Vibration periods also vary with $R/s$ and $k_w/k$. For $0 < R/s < 2$, the first vibration period become smaller, and the second vibration period increases. As a special example, for $R/s = 1.1$ in case (b) of $k_w/k = 20$, the first and the second vibration periods will become very close, and the participation vector will be $\alpha_1 = 2, \alpha = 1/2$ and nearly equal.

RCD structures are able to designate the vibration modes of building structures. How to exploit the mode combination of higher order of vibration mode should be investigated in the future, both theoretically and experimentally.
DISPLACEMENT DISTRIBUTED STRUCTURAL SYSTEM

If the rocking center or the rotation center is designated below the super structure, i.e., \( R < -h_1 \); then the building will vibrate in a trapezoidal shape of predominant first vibration mode. As shown in Figure 9, when \( R/s < 0 \), participation vector of the system are depending on the rocking radius ratio \( R/s \), and are not varied with the stiffness of RCD mechanism. Because the displacement of the first vibration mode is distributed according to the rocking radius, thus, we name the system as Displacement Distributed Structural system. A building plan is shown by Figure 10, a part of earthquake energy will be consumed by dampers placed at the first story. It must be pointed out that RCD mechanisms must have the same rotation radius for each lateral direction, otherwise the deformation will be constrained each others. Further, RCD mechanisms must be separated from each other, although a three-dimensional plan is possible.

Equation of Motion and Restoring Force of Displacement Distributed Structural System

For simplicity, the RCD columns as well as the boundary beams linking RCD mechanism with the main structure are supposed to be axial members and be pin-connected at both ends, and the member deformations of RCD mechanism is neglected. Therefore, these members must have sufficient stiffness or rotation capacity at the ends against stress and deformation response. However, RCD columns are not needed be pin-ended, fix-ended RCD columns such as concrete filled steel tube columns may be more economical for axial stiffness and rotation capacity.

As the main structure follows the lateral movement of RCD mechanism that deform geometrically around the rocking center according to the inclination of the RCD columns as shown in Figure 3, the building will be simplified as an equivalent SDOF analytical model shown in Figure 11 by considering the first vibration mode. Introducing initial viscous damping \( c \), equivalent height \( H \) and equivalent restoring force \( F \) as well as equivalent displacement \( x \), the equation of motion will be expressed by Equation (13).

\[
m\ddot{x} + c\dot{x} + F = -m\ddot{x}_0
\]

Equation (13)
Here, $m$ is the total mass of the building. $\sum Q_i$ is the equivalent restoring force by the main structure obtained from Equation (16), and is calculated from the story force $Q_i$ of upper stories. At the first story, $\sum Q_i$ is the total shear force of hysteric dampers, and $c_1Q$ is the story shear by vertical columns. $e_{fd}$ is a factor decided by Equation (17).

$$
\bar{H} = \frac{[1 + R/h)(R/h - n) + (1 + n)(1 + 2n)/6]}{[(n - 1)/2 - R/h]}
$$

$$
F = \sum_{i=2}^{n} Q_i - c_1 Q \cdot (R/h) - d \bar{Q} \cdot (R/h)
\quad (n - 1)/2 - R/h = \sum Q + e_{fd}(c_1 Q + d \bar{Q})
$$

The factor $e_{fd}$ converts restoring force of the first story to equivalent force of SDOF, and may be explained as an effective factor of damper in the equivalent SDOF model. For base isolation structures with rigid super structures; let $-R/h = \infty$, then $e_{fd} = 1$. For ordinary frame structures; let $-R/h = 1$, then $e_{fd} = 2/(n + 1)$. In the latter case, RCD wall function as a pin-ended structural wall for averaging deformation. The pin-ended wall may has the same effect with increase the strength capacity [3]. Therefore, the Displacement Distributed Structural system is a unified expression of ordinary structure including base isolation structure, with both vibration mode and structural model.

Restoring force of the equivalent SDOF system is illustrated by Figure 12, and the contributions of each part are calculated from Equation (15). Restoring force of dampers is assumed as rigid-plastic, and $\bar{Q}_y$ is the yielding strength of dampers. For $\sum Q$ of the upper stories and $c_1Q$ of the first story, bilinear models are assumed. $c_1x_y$ and $\bar{Q}_y$ are the equivalent displacement corresponding to the yielding of first story column and upper structure, $x_0$ is the equivalent ultimate displacement related to the deformation capacity of structural members, these displacement are transformed from the story drifts $x_1$ or $x_s$ using Equations (18) and (19).

$$
x_1 = -R \theta = \frac{R}{\bar{H}} x
$$

$$
x_s = h \theta = \frac{h}{\bar{H}} x
$$

Seismic Design of Displacement Distributed Structural System

The procedure for seismic design of Displacement Distributed Structural system is based on the response evaluation of the equivalent SDOF model. Simple methods such as the method of Equivalent Linearization or the method of Energy Equilibrium [5] may be applied; this was described in the previous study [2]. As the main structure is librated from displacement distribution, columns may be more slender than that in frame buildings, and a longer vibration period is recommended for reduce the acceleration response. Some points in seismic design are summarized as follows.
For performance design, usually story drifts are limited within an allowable limit. Given the allowable upper story drift $x_s$ and an expected first story drift $x_1$, then from Equations (18) and (19), ratio of rotation radius and story height $-(R/h)$ will be decided by Equation (20). Substituting Equation (20) into Equation (14) and using Equation (19), the equivalent displacement $x$ will be decided from Equation (21).

$$-(R/h) = x_1 / x_s$$  \hspace{1cm} (20)

$$x = \frac{[(x_1 / x_s - 1)(x_1 / x_s + n) + (1 + n)(1 + 2n)/6]}{(n - 1)/2 + x_1 / x_s} x_s$$  \hspace{1cm} (21)

In order to reduce the repair cost of building after an intense earthquake, it is prefer to limit the upper story drift $x_s$. The most of earthquake energy is expected be consumed by dampers, and the required damping force may be estimated from the design velocity spectrum and the expected first story drift $x_1$.

For design of RCD wall and RCD columns, story shear $Q_i$ ($i = 1$ to $n$) of the structural system will be obtained from Equation (22) considering the first vibration mode and using the equivalent response acceleration $S_a$. At the first story, let $i = 1$, then story shear $\Delta_{c-n} Q$ at the first story by RCD columns will be obtained from Equation (23). For upper stories, let $2 \leq i \leq n$, story shear $\Delta_{w} Q_i$ by RCD walls will be obtained from Equation (24) by subtracting the story force of main structure.

$$Q_i = \sum_{j=i}^{n} m_j S_a \frac{[-R + (j - 1)h]}{H} = [(n+i) / 2 - (1 + R / h)] \frac{(n+1-i)}{n} \frac{h}{H} mS_a$$  \hspace{1cm} (22)

$$\Delta_{c-n} Q = Q_i - c\Delta_d Q_s$$  \hspace{1cm} (23)

$$\Delta_{w} Q_i = Q_i - \Delta_{d} Q_i$$  \hspace{1cm} (24)

RCD columns should be designed with sufficient axial strength and rigidity against the axial force variation induced by $\Delta_{c-n} Q$, and if not pin-end fixed, they must be designed to have sufficient deformation capacity under this axial force variation. For RCD wall, it is recommended to limit the stress within the cracking strength, as we assumed the wall be rigid. As shown by Equation (23), at the first story, story shear $\Delta_{c-n} Q$ or the axial force of RCD columns may be adjusted by changing the story shear $Q_i$ shared by vertical columns, or changing the damping force. 

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**Figure 12. Restoring force of equivalent SDOF**

![Figure 12. Restoring force of equivalent SDOF](image)

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CONCLUSIONS

Rocking Center Designated (RCD) Mechanism was proposed as a tool to control the vibration modes of building structures. If the RCD mechanisms are incorporated into a frame structure, the vibration modes of the structure can be designated according to the rocking center of RCD Mechanism, and even a predominant second vibration mode may be realized. The effectiveness of RCD mechanism in vibration mode control was described by shaking table tests and mode analysis of a 2DOF model. Future researches are needed for investigating how to exploit the advantages of different mode combinations.

As a practical application of RCD structures, Displacement Distributed Structural system with a predominant first vibration mode is introduced for seismic design. In this new structural system, columns including the 1st story columns are librated from displacement distribution as is required for weak-beams and strong-columns yielding mechanism in current seismic design practice. Seismic forces are resisted by the main structure and additional dampers, and are distributed by RCD walls and RCD columns. Earthquake response of this new structure may be simplified as an equivalent SDOF model. The benefits of the structural system are a more easily increase of the predominated period, a rational displacement distribution satisfying the performance needs, and a more economical energy absorption scheme at the first story. It gives a unified expression of ordinary structure including base isolation structure respecting to vibration mode, and is expected for providing more choice for seismic design.

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