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A REFINED SUB-REGIONAL RECIPROCAL METHOD IN TIME DOMAIN AND ITS APPLICATION IN DYNAMIC INTERACTION ANALYSIS

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SUMMARY

The structure-soil dynamic interaction is mainly confined in frequency domain because evaluating the dynamic effects of infinite medium on structural response is difficult. In this paper, an adaptive damping-extraction procedure, which can simulate the dynamic properties of non-homogeneous infinite medium by applying artificial damping and then extracting the effects, is proposed for the dynamic interaction analysis of structure-unbounded rock system. Based on the proposed procedure, it is convenient to get dynamic interaction force in time domain acted on interface between unbounded rock and structure. The corresponding FE numerical implementation of the procedure in time domain is given by precise step-by-step time integration scheme. Some key factors in implementation are discussed and the numerical tests demonstrate satisfactory accuracy and excellent application prospect.

INTRODUCTION

The structure and the infinite medium are two main distinct parts with different properties in the dynamic soil-structure interaction analysis. It has been demonstrated that the radiation damping of infinite medium plays an important role in the seismic responses of large hydraulic structures, tunnels and long spanned bridges etc. In unbounded medium, the motion from wave source can only propagate outwardly and can be reflected back by the outer boundary if the unbounded region is modeled by finite region. Thus the problem of simulating infinite medium attracts intense attention of researchers and many approaches are suggested. Some methods model the infinite medium by study the dynamic characteristics at structures-infinite medium interface, such as boundary element method, trial function method and dynamic infinite element etc; Other methods simulate the infinite medium by applying local artificial boundary to make the outwardly propagating wave transmit the outer boundary of finite region, such as transmitting boundary method and visco-elastic boundary method. All these are convenient to be applied to the dynamic soil-structures interaction analysis in frequency and difficult in time domain because of the problem of convolution integral.

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As shown in Fig 1, if introducing seismic excitations via the elastic unbounded soil, the discrete equation of motion of the structure in time domain can be expressed as

$$\begin{bmatrix} [M_{ss}] & [M_{sb}] \\ [M_{bs}] & [M_{bb}] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}_s^t(t)\} \\ \{\ddot{u}_b^t(t)\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -\{R_b(t)\} \end{Bmatrix} \tag{1}$$

$$\{R_b(t)\} = \int [S_{bb}^g(t-\tau)](\{u_b^t(\tau)\} - \{u_b^g(\tau)\})d\tau \tag{2}$$

The right-hand side of Eq.(1) represents that, only the structural nodes in contact with the soil are loaded. For the total time history, the ground's interaction forces $\{R_b(t)\}$ in theory are equal to the convolution

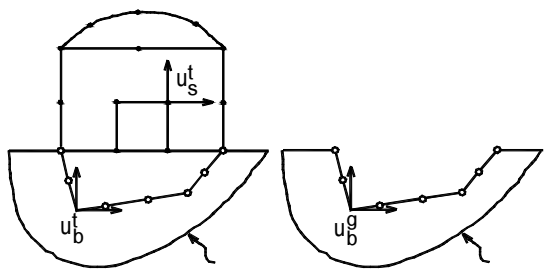


Fig.1 Dynamic soil-structure interaction and the wave scattering motions

integrals of the soil's dynamic stiffness $[S_{bb}^g(t)]$ and the relative motion ($\{u_b^t(t) - u_b^g(t)\}$), in which $\{u_b^g(t)\}$ is the excavated ground scattering motions. Analogous to the familiar Duhamel integral, if the $\{R_b(t)\}$ in Eq.(2) are directly numerical evaluated in time domain, the current interaction forces are related to all of the ground motions heretofore, and thus unavoidably leads to a huge computational effort.

Also in other important aspects, many investigations have been done on the overcoming of convolution integrals existing in the time-domain dynamic analysis. Wolf [2] proposed a simple recursive algorithm, with considering the effects of only m time segments before the current time t_n on the current ground interaction forces ($m < n$). Zhang [3] suggested fitting the dynamic stiffness matrix of unbounded soil in time domain by a few piece-wise linear segments. And then based on an implicit integral scheme, the convolution integral can be evaluated only by using the response matrices $[S_{bb}^g(t)]$ at these few coupling time steps. To some extent, the above-simplified numerical algorithms result in an improvement over current methods and obviously reduce the computational efforts of convolution integrals in time domain, however, unavoidably inducing some algorithm errors. A real need exists to develop a new procedure to calculate the interaction forces of unbounded medium in time domain without convolution integrals.

Damping-Solvent Extraction Method (DSE method) is a simple, accurate and generally applicable method [4], it provide a new approach in the substructure method to approximately simulate the radiation conditions of unbounded medium by applying artificial material damping to attenuate both outgoing and reflected waves at the outer boundaries of bounded computational soil region, and then 'extracting' the artificial damping to remove its undesirable effects. Although the convolution integral do not appeal in the formula of DSE method, the time parameter t is retained because of the incomplete decoupling in the process of time-frequency transforming, which in fact is an alternative form of convolution integral.

The objective of this paper is to propose a complete numerical step-by-step integral scheme for time-domain implementation of DSE method, which can avoid the convolution integral and greatly reduce the computational efforts by dividing the total displacements into regular part and an additional modified part. Some discussions are presented and the unconditional convergence and good accuracy of the new developed algorithm are demonstrated.

Ⅰ.REVIEW OF THE DAMPING SOLVENT EXTRACTION METHOD

The damping extraction method can succeeds to simulate the radiation condition of infinite medium and efficiently evaluate the desirable dynamic response matrix of unbounded soil, owing to the following three main steps in general.

Step 1: A bounded soil domain adjacent to the generalized structure is selected. Artificial material damping is first introduced into the computational soil region, depending on the approximate equivalency between radiant damping and material damping in reducing the amplitudes of outwardly propagation waves. For a sufficiently large artificial damping ratio, the reflected waves g at the outer boundary are greatly attenuated by the introduced material damping and fail in influencing the motion of structure-soil interface, therefore the desired performances for the radiation condition of unbounded region is perfectly simulated in a bounded domain. The outer boundary may be also defined as an absorbing boundary to decay the waves further.

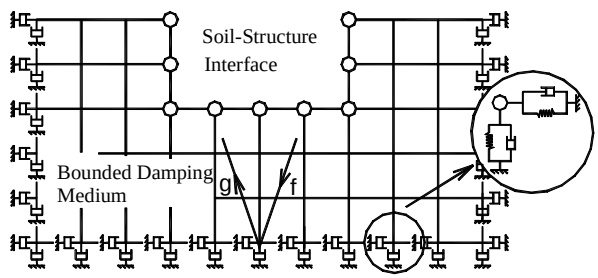


Fig.2 FE numerical model for the time-domain implementation of DSEM

Step 2: With one certain frequency ω, the dynamic stiffness $[S_\zeta(\omega)]$ of the bounded damped domain on the soil-structure interface is then computed, which and whose first-order frequency derivative are assumed to be equal to the corresponding values of the artificially damped unbounded medium.

Step 3: Influences of the introduced artificial damping on the equivalent dynamic stiffness $[S_\zeta(\omega)]$ of damped unbounded domain is extracted, by means of Taylor expansions at the dimensionless frequency a_0^* with respect to ω, and finally the dynamic stiffness of unbounded medium can be obtained

In attempting to simplify the complex expressions of $[S_\zeta(\omega)]$ in the time domain analysis, a equivalently artificial mass-proportional nodal damping matrix $2\zeta[M]$ and additional nodal stiffness matrix $\zeta^2[M]$ are introduced into the same bounded domain, instead of the previous hysteretic material damping in the frequency domain.

The equation of motion of the artificially damped bounded domain in time domain is given by [4]

$$[M]\{\ddot{u}\}+2\zeta[M]\{\dot{u}\}+([K]+\zeta^2[M])\{u\}=\{P\} \tag{3}$$

Eq.(3) results in a desirably simple expression of the dynamic stiffness matrix of the damped bounded soil in the frequency domain, as

$$[S_\zeta(\omega)]=[K]-(\omega-i\zeta)^2[M] \tag{4}$$

and in dimensionless form as

$$[S_\zeta(\omega)]=Gr_0^{s-2}([K]-a_0^{*2}[M])=Gr_0^{s-2}\left[\bar{S}(a_0^*)\right] \tag{5}$$

Where $a_0^*=(\omega-i\xi)r_0/C_s$; G is the shear elastic modulus and the dimensionless frequencies a_0 and a_0^* correspond to the natural material and artificially damped material respectively,

To calculate $\left[\bar{S}(a_0)\right]$ from $\left[\bar{S}(a_0^*)\right]$, the first two terms of a Tailor expansion of $\left[\bar{S}(a_0^*)\right]$ at frequency a_0 are formulated

$$[\bar{S}^{\infty}(a_0)]=[\bar{S}(a_0^*)]+[\bar{S}(a_0^*)]_{,a_0^*}(a_0-a_0^*) \tag{6}$$

and the same relation applies as in Eq(5)

$$[S^{\infty}(\omega)]=Gr_0^{s-2}[\bar{S}^{\infty}(a_0)] \tag{7}$$

Substituting Eqs.(5) and (7) into Eq.(6), the dynamic stiffness in frequency domain of un-damped unbounded medium can be obtained

$$[S^{\infty}(\omega)]=[S_{\zeta}(\omega)]+i\zeta[S_{\zeta}(\omega)]_{,\omega} \tag{8}$$

By applying inverse Fourier transformations to Eq.(8), the impulse displacement response matrix (dynamic stiffness matrix in time domain) of natural unbounded soil will be finally expressed in time domain as

$$[S^{\infty}(t)]=(1+\zeta t)[S_{\zeta}(t)] \tag{9}$$

Defining $\left\{R^{\infty}(t)\right\}$ and $\left\{u(t)\right\}$ as the force and displacement vector at the interface of infinite medium and structure, then the interaction force at the interface can be obtained by Duhamel integral as

$$\{R^{\infty}(t)\}=\int_0^t[S^{\infty}(t-\tau)]\{u(\tau)\}d\tau \tag{10}$$

Substituting Eq.(9) into Eq.(10), a simple time-domain formulation with two-terms superposition will be gotten as the follows

$$\{R^{\infty}(t)\}=(1+\zeta t)\{R_{\zeta}(t)\}-\zeta\{R_{\zeta r}(t)\} \tag{11}$$

with

$$\{R_{\zeta}(t)\}=\int_0^t[S_{\zeta}(t-\tau)]\{u(\tau)\}d\tau \tag{12}$$

and

$$\{R_{\zeta r}(t)\}=\int_0^t[S_{\zeta}(t-\tau)]\tau\{u(\tau)\}d\tau \tag{13}$$

It's worth noting that all of the new presented algorithms in the next sections come from Eqs.(11)-(13). However, the above time-domain equations with convolution integrals are only exact in theory, and its computational efforts of solution are too substantial to afford in the practical applications. From an engineering point of view, $\{u(t)\}$ and $t\{u(t)\}$ can be considered as the displacement vectors at the surface of damped bounded soil region, under prescribed loadings $\{R_{\zeta}(\tau)\}$ and $\{R_{\zeta r}(t)\}$ at the same places, respectively. And obviously, with the known enforced loadings, both of them can be easily solved from the dynamic equilibrium equation of damped bounded soil region (see Eq.3).

II. NUMERICAL IMPLEMENTATION FOR REFINED DSE METHOD IN TIME DOMAIN

Though the convolution integral is avoided, the time t is appear in Eq.(11) and it is inconvenient for the application. A updated method is proposed here and the compete numerical step-by-step integral procedure is derived to overcome the shortage of the DSE method.

The equation (3) of motion of damped bounded soil is firstly represented in partition matrices form as

$$\{\begin{matrix}0\\R_{\zeta b}\end{matrix}\}=[\begin{matrix}M_{mm} & 0\\0 & M_{bb}\end{matrix}]\{\begin{matrix}\ddot{u}_m\\\ddot{u}_b\end{matrix}\}+[\begin{matrix}2\zeta M_{mm} & 0\\0 & 2\zeta M_{bb}\end{matrix}]\{\begin{matrix}\dot{u}_m\\\dot{u}_b\end{matrix}\}+[\begin{matrix}K_{mm}+\zeta^2M_{mm} & K_{mb}\\K_{bm} & K_{bb}+\zeta^2M_{bb}\end{matrix}]\{\begin{matrix}u_m\\u_b\end{matrix}\} \tag{14}$$

in which the subscripts b and m denote the soil-structure interface nodes and inside nodes of the bounded soil (see Figure 2), respectively.

Defining $[\overline{M}]=[M],[\overline{C}]=2\zeta[M],[\overline{K}]=[K]+\zeta^2[M]$ (15)

Then Eq.(14) can be decomposed into two sub-formulations as

$$-[\overline{K}_{mb}]\{u_b\}=[\overline{M}_{mm}]\{\ddot{u}_m\}+[\overline{C}_{mm}]\{\dot{u}_m\}+[\overline{K}_{mm}]\{u_m\} \tag{16a}$$

$$\{R_{\zeta b}\}=[\overline{M}_{bb}]\{\ddot{u}_b\}+[\overline{C}_{bb}]\{\dot{u}_b\}+[\overline{K}_{bb}]\{u_b\}+[\overline{K}_{bm}]\{u_m\} \tag{16b}$$

Similarly, according to Eq.(13), the equation for modifying motions of the same damped region subjected to the prescribed force $\{R_{r\zeta b}(t)\}$ is given by

$$\begin{Bmatrix} 0 \\ R_{r\zeta b} \end{Bmatrix}=\begin{bmatrix} \overline{K}_{mm} & \overline{K}_{mb} \\ \overline{K}_{bm} & \overline{K}_{bb} \end{bmatrix}\begin{Bmatrix} u_{rm} \\ u_{rb} \end{Bmatrix}+\begin{bmatrix} \overline{C}_{mm} & 0 \\ 0 & \overline{C}_{bb} \end{bmatrix}\begin{Bmatrix} \dot{u}_{rm} \\ \dot{u}_{rb} \end{Bmatrix}+\begin{bmatrix} \overline{M}_{mm} & 0 \\ 0 & \overline{M}_{bb} \end{bmatrix}\begin{Bmatrix} \ddot{u}_{rm} \\ \ddot{u}_{rb} \end{Bmatrix} \tag{17}$$

where $\{u_{rb}\}$、$\{\dot{u}_{rb}\}$、$\{\ddot{u}_{rb}\}$ are the vectors of modified motions of the nodes at the soil-structure interface, with $\{u_{rm}\}$、$\{\dot{u}_{rm}\}$、$\{\ddot{u}_{rm}\}$ denoting the additional motions of inside nodes.

In order to determine the modified displacements inside of the soil region, it's necessary to follow the numerical relations of Eq.(18a) for the interface nodes.

Defining $\{u_{rb}\}=t\{u_b\}$ (18a)

Then the derivation of Eq.(18a) with respect to time will result in

$$\{\dot{u}_{rb}\}=t\{\dot{u}_b\}+\{u_b\}$$

$$\{\ddot{u}_{rb}\}=2\{\dot{u}_b\}+t\{\ddot{u}_b\} \tag{18b}$$

Obviously in Eqs.(11) and (18), containing the absolute time parameter t will brings some inconveniences for the practical adoptions by the dynamic interaction analysis in time domain. And thus also perhaps causes unsteady numerical results, owing to the parameter t continuously enlarging the diversity of quantity among the displacement, velocity and acceleration till the end of seismic excitations. To a certain extent, Eq.(11) is a representation of the incomplete space and time decoupling of convolution integral in the time domain. Therefore, to remove the absolute time in the final numerical formulations, a valid assumption is introduced in this section for the expressions of modified motions of the inside nodes in bounded soil region, as the following

$$\{u_{rm}\}=t\{u_m\}-\{v_m\} \tag{19a}$$

By applying operation of derivates in Eq.(19a), the modified velocity and acceleration can be also obtained as

$$\{\dot{u}_{rm}\}=t\{\dot{u}_m\}+\{u_m\}-\{\dot{v}_m\}$$

$$\{\ddot{u}_{rm}\}=2\{\dot{u}_m\}+t\{\ddot{u}_m\}-\{\ddot{v}_m\} \tag{19b}$$

For the Eq.(16) considered, substituting Eqs.(18) and (19) into the modified equation of (17), will leads to the equivalently decomposed expressions of Eq.(17) as the follows

$$[\overline{M}_{mm}]\{\ddot{v}_m\}+[\overline{C}_{mm}]\{\dot{v}_m\}+[\overline{K}_{mm}]\{v_m\}=2[\overline{M}_{mm}]\{\dot{u}_m\}+[\overline{C}_{mm}]\{u_m\} \tag{20a}$$

$$\{R_{r\zeta b}\}=t\{R_{\zeta b}\}+2[\overline{M}_{bb}]\{\dot{u}_b\}+[\overline{C}_{bb}]\{u_b\}-[\overline{K}_{bm}]\{v_m\} \tag{20b}$$

And then, by using Eqs.(16) and (20) in Eq.(11), one can achieves the final time-domain formulation of interaction forces.

$$\{R^{\infty}(t)\}=[\overline{M}_{bb}]\{\ddot{u}_b\}+([\overline{C}_{bb}]-2\zeta[\overline{M}_{bb}])\{\dot{u}_b\}+([\overline{K}_{bb}]-\zeta[\overline{C}_{bb}])\{u_b\}+[\overline{K}_{bm}]\{u_m\}+\zeta[\overline{K}_{bm}]\{v_m\} \tag{21}$$

in which, $\{u_m\}$ and $\{v_m\}$ can be solved form the Eqs.(16a) and (20a) by usual finite element methods. Since the influence of absolute time on the numerical calculation of Eq.(21) has been removed, the interaction forces $\{R^{\infty}(t)\}$ in Eq.(21) can be expediently evaluated by directly applying some various step-by-step integral schemes. A computational program is developed to verify the new proposed algorithms.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, the unbounded soil's interaction forces $\{R^{\infty}(t)\}$ is evaluated and compared to demonstrate the validity and accuracy of the proposed procedure.

Computational model

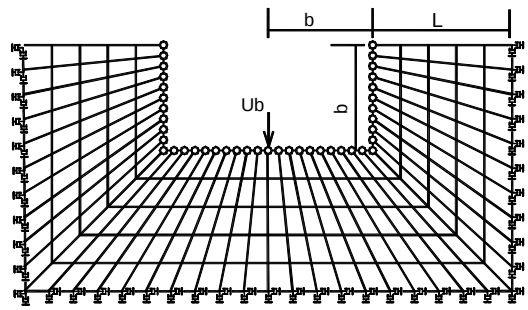


Fig.3 FE discretization of finite region adjacent to structure-medium interface

A rectangular rigid foundation embedded in homogenous half-plane with length of bounded region l equal to b =20m as shown in Fig.3 is selected as practical example, assumed to be subjected to transient excitation of prescribed vertical displacement at the center of the rigid base.

The material parameters of soil are as the follows: $G = 3.2e8Pa$ (Shear elastic modulus) $\mu = 0.25$ (Poisson ratio) and $\rho = 2.0e3kg/m^3$, with the according speeds of acoustic waves $c_s = 400m/s$ and $c_P = 693m/s$.

To easily compare the numerical results, transient excitations are taken as the simple harmonic motions, with the period $T = 8b/c_s = 0.4s$ and the amplitude $u_0 = 0.20m$, expressed as the following

$$u_b(t) = \begin{cases} \frac{u_0}{2}(1-\cos(\frac{2\pi t}{T})) & t \le 2T = 0.8s \\ 0 & t > 2T = 0.8s \end{cases} \tag{22}$$

The dimensionless nodal damping coefficient is $d = \zeta b / c_s$. For different radiation radium l , $d = 0.5b/l$, b/l and $2b/l$ are selected for computation.

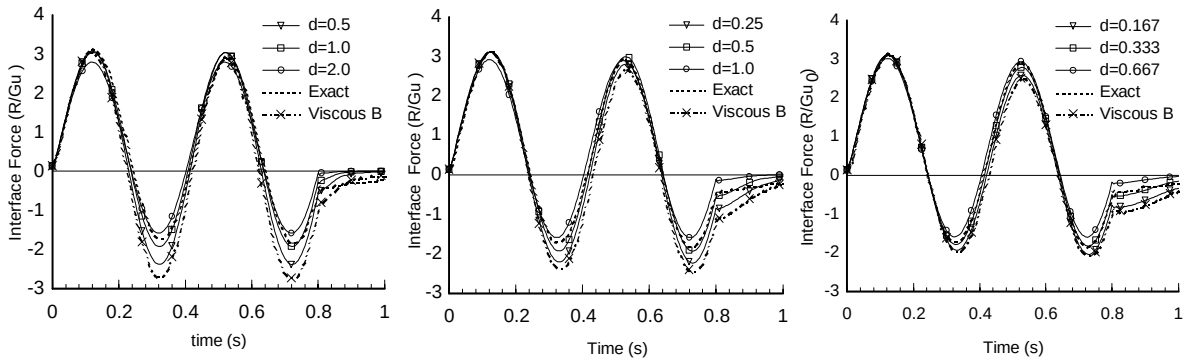
Results and analysis

Based on the proposed algorithm of DSE method, Fig.4 and 5 show the numerical results for three values (0.5,1 and 2) of the dimensionless artificial damping factor d , in which FB stands for Fixed Boundary and VB for Viscous Boundary. Accordingly, other related parameters in the evaluation include: viscous dashpots at outer boundaries, the damping ratio (an approximate value of 0.2) for viscous boundary model, and the damping ratio of lower modes (selected in the range of 16%~20%) for damped bounded soil model. In addition, extended mesh model are also applied in this paper to get exact results, with a large computational soil region of length $l \ge c_P t_a$, in which t_a denotes the effective total time of seismic excitation.

It is evident from Figure 4 that, when compared with viscous boundary model, the agreement with exact results for DSE time-domain method is commonly improved in the case of any damping factor, only causing a little increase of computational effort. In particular, the best precision exists for the case of $d = 1.0$, which is consistent with the conclusion of Reference [7]. Additionally, the step-by-step integral scheme for the DSE time-domain method, proposed in this paper, is highly advantageous to the adoption in computer programming for dynamic interaction analysis.

The vibration phase difference between outer boundary and interface is $e^{-i\omega l/c_s}$ for un-damped finite region with radiation radium l. In terms of Eq.(4), the corresponding phase difference become $e^{-\zeta l/c_s}e^{-i\omega l/c_s}$ ($e^{-dl/b}e^{-i\omega l/c_s}$ if expressed in dimensionless form) if ω is replaced by $\omega - i\zeta$. That is to say, after introducing artificial damping, the wave attenuation ratio propagate from interface to outer boundary is

$$\beta^* = 1 - e^{-dl/b} \tag{23}$$



(a) L=1b (b) L=2b (c) L=3b

Fig.4 dynamic interaction force of unbounded rock for various region length L

Thus the wave attenuation ratio at interface is $1 - e^{-2dl/b}$。

The attenuation ratios for different value of $d \cdot l/b$ are listed in Tab.1 and can be referenced for the selection of artificial damping coefficient d。

Tab.1 attenuation of vibration amplitude on the interaction face

$d \cdot l/b$	0.25	0.5	1	2
Attenuation ratio	0.394	0.632	0.865	0.982

It is also indicate that the proposed procedure can improve the accuracy of the simulation of unbounded medium while the computational effort increase a little compared to classical visco-elastic local artificial boundary

Figure 6 shows that it is relatively worse and of insufficient accuracy for the results for the high damping bounded soil models, instead of the naturally infinite medium. So, the advantages in precision for DSE time-domain method are more prominent for the smaller computational soil region, also greatly reducing the evaluation efforts as other numerical algorithms usually requiring a relatively large soil region.

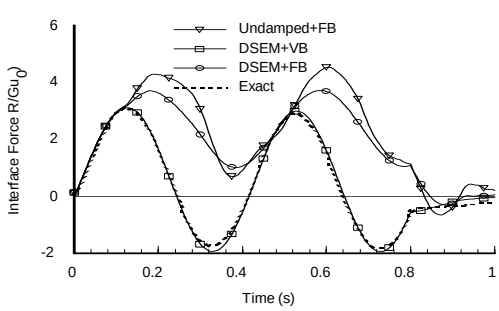


Fig.5 comparison of numerical results as l=2b and d=0.5

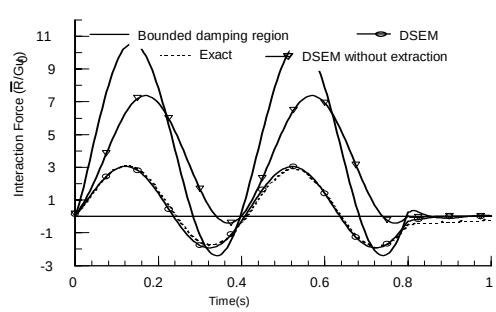


Fig.6 comparison of numerical results at various step of DSEM

To some extent, the results in Figure 5 for bounded damped medium model (DSE method without extraction step) are similar to the dynamic responses of high damping bounded soil model, besides differences only existing in the amplitude and phases at various time steps. Therefore, in order to ensure

the precision of results, it has been exhibited as a necessary step for extracting the influences of artificial damping further in the implementation of DSE method.

CONCLUSION

Dynamic stiffness with a certain frequency (See Fig.1) cannot sum up all of the dynamic properties of natural soil region in time domain, especially for seismic excitations with a frequency range, so seeking after completely time-domain numerical method is important. In this paper, a refined damping-solvent extraction method is proposed, which completely avoid the convolution integrals, as required in other time domain algorithms. The new time-domain method ensures the seismic excitations introduced at the structure-soil interface, avoiding the inverse computations from the recorded background seismic motion to base-rock motions. Finally, multiple support excitations and traveling waves can also be considered through the different initial oscillatory time at various excitation points of the interface nodes in time domain analysis.

ACKNOWLEDGEMENT

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REFERENCES

1. Wolf J P. Soil-Structure-Interaction Analysis in Time Domain[M]. N.J.: Prentice-Hall Englewood Cliffs, 1988.
2. Wolf J P, Motosaka M. Recurive Evaluation of Interaction Forces of Unbounded Soil in the Time Domain[J]. Earthq. engng struct. dyn. 1989,18(3):345-363.
3. Xiong Zhang, Wegner J L. Three-Dimensional dynamic soil-structure interaction analysis in the time domain[J], Earthq. engng struct. dyn. 1999,28(12):1501-1524.
4. Chongming Song, Wolf J P. Dynamic Stiffness of Unbounded Medium based on Damping Solvent Exrtraction[J]. Earthquake engng struct. dyn. 1994,23(2): 169-181.
5. Wanxie Zhong. On Precise time-integration method for structure dynamics[J]. Journal of Dalian University of Technology, 1994,34(2):189-192. (in Chinese)
6. Wolf J P, Chongmin Song. Finite-Element Modelling of Unbounded Media[M]. N.Y.: John Wiley&Sons Inc,1997.
7. Ushnish Basu, Anil K. Chopra. Numerical Evaluation of the Damping-Solvent Extraction Method in the Frequency Domain[J]. Earthquake engng struct. dyn. 2002,31(11): 1231-1250.
8. Bahaa E.A., John F.Hall. Non-linear Earthquake Response of Concrete Gravity Dams Part 1: Modelling. Earthq. engng struct. dyn. 1989,18:837-851.