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Study on Control of Reducing Seismic Response for Cable-stayed Bridge Based on Mode Analysis

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SUMMARY

In this paper, the algorithm of reducing seismic response control for cable-stayed bridges, which is sliding mode control based on mode analysis, is put forward. The method is used for reducing seismic response control of cable-stayed bridge, and it is shown that the methods are feasible and effective by the study of simulating control for long-span bridge, and the sliding mode control based on mode analysis achieved ideal control result by comparatively less external energy.

INTRODUCTION

The control of flexible bridge structure especially for long-span cable-stayed bridge is new, difficult and unique problem, with many ramifications, both in modeling and control design. The control of very flexible bridge structures has not studied to the extent that buildings have. As a result, little expertise has been accumulated. Since the active control of medium- and long-span bridges has little history associate with it, it is recommended that the study be as open ended as possible.

In the year of 1979, J. N. Yang et al performed some of the earliest work in this area, when they studied the application of active control to cable-stayed bridges under the excitation of strong wind gusts. In the year of 1993, Warnitchai et al experimentally and analytically studied control of cable-stayed bridges subject to vertical sinusoidal force utilizing a simple cable-supported contilever as model. In the year of

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1993, Hutton used active control to control vertical deflections of a cable-stayed guideway under a moving constant load. In the year of 1994, Volz et al using a simply supported, lumped mass beam supported by two cable, employed a decentralized active controller to reduce the bridge's vertical deck deflection and Magana et al examined robust non-linear active control to attenuate displacements under adverse condition. In the year of 1996, Achkire et al experimentally and analytically examined active vibration control of cables and cable system using a single active tendon. To a certain degree, active control technology for cable-stayed bridges overlaps with the active control technology developed for large space structures. In the year of 1993 Hyland et al presents a wealth of information on the state of the art of active control technology for large space structures.

The main difficulty of active control for long-span cable-stayed bridges is that the freedoms of the structures are very excessive. Though the analysis in the mode space should make reduction the scale, the basic period is long and the vibration modes are close. As a result, the truncation number of the modes is also too excessive by traditional method of truncation number of the modes, for the simulation analysis is very difficult. The accumulative total contributing ratio of modes is 0.90 in the U. S. UBS code. To reach the standard, it should be calculate 130 modes for WUHU Cable-Stayed bridge. In the paper, it is put forth control technology by the sliding mode control based on modes space. In the analysis choosing the main modes according to the contribution ratio in the modes space to make control for reduction seismic response by above control technology, and the feasibility of the technology is discussed.

THE CONTROL ALGORITHM OF REDUCTION SEISMIC RESPONSE BASED ON SLIDING MODE IN MODES SPACE

The Equation of Motion of Structure Controlled in Physical Coordinate

$$M\ddot{X}+C\dot{X}+KX=HU+\eta\ddot{X}_0 \qquad (1)$$

in which X denoting the shift displacement of the structure;

M、C、K denoting the mass, damping and stiffness matrices, respectively;

U denoting control forces;

H denoting the location of controllers.

η denoting the influence of the earthquake excitation.

The Equation of Motion of Structure Controlled in modal Coordinate

$$\ddot{\overline{X}}+\Lambda\dot{\overline{X}}+\Omega\overline{X}=\Theta U+\Phi^T\eta\ddot{X}_0 \qquad (2)$$

in which $\Lambda=\begin{bmatrix} 2\zeta_1\omega_1 & 0 & \cdots & 0 \\ 0 & 2\zeta_2\omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\zeta_m\omega_m \end{bmatrix}$; $\Omega=\begin{bmatrix} \omega_1^2 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_m^2 \end{bmatrix}$; $\Theta=\Phi^T H$.

The State Equation

$$\dot{Z}(t)=AZ(t)+BU(t)+E(t) \tag{3}$$

in which $Z(t)=\begin{Bmatrix} X \\ \dot{X} \end{Bmatrix}$; $A=\begin{bmatrix} 0 & I \\ \Omega & -\Lambda \end{bmatrix}$; $B=\begin{bmatrix} 0 \\ \Theta \end{bmatrix}$; $E(t)=\begin{bmatrix} 0 \\ \Phi^T\eta \end{bmatrix}\ddot{X}_0(t)$.

The Design of Sliding Surface

Sliding surface is a linear combination of the state variables $S=PZ$, to simplify, let S＝0， i. e .

$$S=PZ=0 \tag{4}$$

in which S is r sliding variables, S_1，S_2，⋯，S_r， r is the total number of controller, i. e.

$S=[S_1, S_2, \cdots, S_r]^T$

The key to design sliding mode is to be determined P matrix.

1) To educe D, $\overline{A}$, $\overline{B}$ and T matrixes by A，B matrixes and weighting matrix Q

$$D=\begin{bmatrix} I_{2n-r} & -B_1B_2^{-1} \\ 0 & I_r \end{bmatrix};\quad D^{-1}=\begin{bmatrix} I_{2n-r} & B_1B_2^{-1} \\ 0 & I_r \end{bmatrix}; \tag{5}$$

$\overline{A}=DAD^{-1}$, partitioning as $\overline{A}=\begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{bmatrix}$; (6)

$$\overline{B}=\begin{bmatrix} 0 \\ B_2 \end{bmatrix}\quad; \tag{7}$$

$$T=(D^{-1})^T QD^{-1};\quad T=\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{8}$$

in which I_{2n-r} and I_r are (2n-r)x(2n-r) and (rxr) identity matrices respectively, B_1 and B_2 are submatrices of B.

T_{11} and T_{22} are (2n-r) ×(2n-r) and (r×r) matrices, respectively. Q is $2n\times 2n$ positive definite weighting matrix.

2) To solve following Riccati equation:

$$\hat{A}^T\hat{P}+\hat{P}\hat{A}-\frac{1}{2}\hat{P}\hat{A}_{12}T_{22}^{-1}\overline{A}_{12}^T\hat{P}=-2(T_{11}-T_{12}T_{22}^{-1}T_{12}^T) \tag{9}$$

in which $\hat{P}$ is a (2n-r) ×(2n-r) Riccati matrix， $\hat{A}=\overline{A}_{11}-\overline{A}_{12}T_{22}^{-1}T_{21}$

（3）、$\hat{P}\Rightarrow\overline{P}_1\Rightarrow\overline{P}\Rightarrow$P，i. e.

$$\overline{P}_1=0.5T_{22}^{-1}(\overline{A}_{12}^T\hat{P}+2T_{21}) \tag{10}$$

$$P = \overline{P}D = \left[\overline{P}_1 \quad I_r\right]D \tag{11}$$

The Design of Controller

The continuous controller can be designed as following

$$U = G - \overline{\delta}\lambda^T \tag{12}$$

in which λ^T and G are r vectors, respectively, $\lambda = S^T PB$; $G = -(PB)^{-1} P(AZ + E)$

$\overline{\delta}$ is (rxr) diagonal matrix, the elements are δ_1, δ_2, $\cdots$, δ_r .

Simulation Analysis by MATLAB

SIMULATION ANALYSIS OF WUHU CABLE-STAYED BRIDGE

Summarize of the Bridge

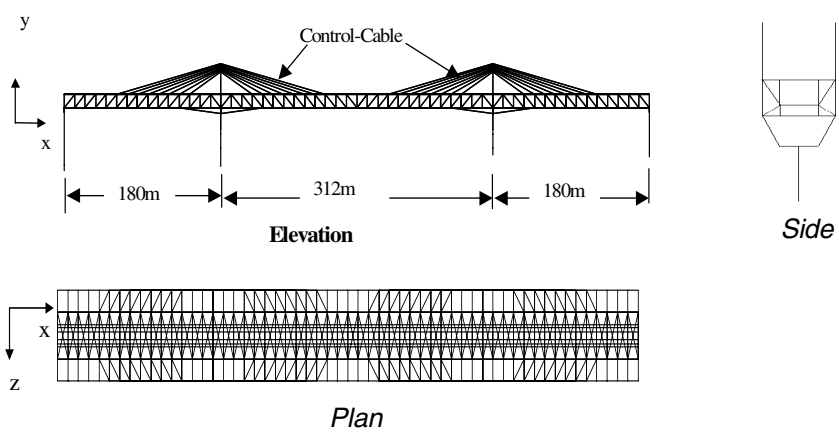

Fig. 1 The Structure Analysis Model

In this paper, the WUHU Cable-stayed Bridge (Fig. 1) located in the main channel of Yangtse River is as the researching example of simulation control. The members of main tower, the main truss and the lateral bracing etc. are taken as space beam elements in the finite element model. The cables are taken as space cable elements , and using Ernst equation to take account of the effect of cable sag. There are 740 nodes and 1820elements in the whole bridge. The periods and contributing ratio of the modes are listed in Table1 and shown in Fig. 2

TABLE 1

No.	Period (s)	Mode Contribute Ratio in X	Mode Contribute Ratio in Y	Mode Contribute Ratio in Z	General Mode Contribute Ratio	No.	Period (s)	Mode Contribute Ratio in X	Mode Contribute Ratio in Y	Mode Contribute Ratio in Z	General Mode Contribute Ratio
1	2.8137	.0000	.0000	.2455	.0818	66	.1955	.0000	.0000	.0000	.0000
2	2.5352	.5870	.0003	.0000	.1956	67	.1929	.0000	.0000	.0001	.0000
3	2.2592	.0135	.0352	.0000	.0141	68	.1921	.0000	.0001	.0000	.0000

4	1.9318	.0000	.0000	.0448	.0149	69	.1912	.0000	.0000	.0001	.0000
5	1.8105	.0000	.0000	.3440	.0740	70	.1901	.0000	.0000	.0001	.0000
6	1.5146	.0000	.0000	.0415	.0063	71	.1827	.0000	.0000	.0002	.0001
7	1.4575	.0000	.0000	.0000	.0075	72	.1801	.0000	.0008	.0000	.0003
8	1.3979	.0000	.0000	.0000	.0000	73	.1758	.0000	.0000	.0000	.0000
9	1.3978	.0000	.0001	.0000	.0000	74	.1717	.0000	.0139	.0026	.0009
10	1.3103	.0040	.0002	.0000	.0014	75	.1702	.0000	.0001	.0000	.0108
11	1.1872	.0000	.0000	.0061	.0020	76	.1688	.0000	.0000	.0000	.0000
12	1.0928	.0001	.2382	.0000	.0794	77	.1679	.0000	.0000	.0017	.0006
13	1.0646	.0000	.0000	.0000	.0000	78	.1625	.0000	.0000	.0009	.0003
14	.8818	.0004	.0000	.0000	.0001	79	.1598	.0001	.0003	.0055	.0020
15	.8433	.0000	.0000	.0000	.0000	80	.1598	.0023	.0000	.0001	.0055
16	.8197	.0000	.0000	.0000	.0047	81	.1594	.0000	.0015	.0000	.0080
17	.6895	.0000	.0000	.0053	.0018	82	.1579	.0000	.0000	.0000	.0103
18	.6230	.0000	.0000	.0012	.0004	83	.1562	.0000	.0000	.0000	.0000
19	.6035	.0000	.0000	.0055	.0018	84	.1532	.0000	.0000	.0000	.0174
20	.5774	.0071	.0137	.0000	.0091	85	.1517	.0000	.0000	.0014	.0005
21	.5290	.0000	.0265	.0000	.0150	86	.1488	.0000	.0000	.0002	.0001
22	.5196	.0000	.0000	.0142	.0351	87	.1469	.0000	.0325	.0000	.0392
23	.4960	.0000	.0000	.0000	.0056	88	.1466	.0000	.0000	.0000	.0000
24	.4829	.0010	.0001	.0000	.0120	89	.1442	.0000	.0000	.0019	.0006
25	.4762	.0000	.0068	.0000	.0023	90	.1407	.0000	.0000	.0000	.0326
26	.4605	.0000	.0018	.0000	.0331	91	.1400	.0000	.0000	.0001	.0000
27	.4275	.0000	.0000	.0001	.0000	92	.1373	.0000	.0000	.0000	.0000
28	.4097	.0000	.0000	.0522	.0174	93	.1360	.0000	.0011	.0000	.0004
29	.4012	.0002	.0000	.0000	.0001	94	.1351	.0000	.0005	.0000	.0002
30	.3813	.0183	.0000	.0007	.0002	95	.1344	.0000	.1178	.0000	.0090
31	.3744	.0000	.0000	.0340	.0029	96	.1333	.0657	.0000	.0000	.0000
32	.3550	.0000	.0000	.0088	.0113	97	.1313	.0000	.0000	.0001	.0000
33	.3467	.0464	.0000	.0000	.0155	98	.1303	.0000	.0000	.0000	.0000
34	.3321	.0000	.0000	.0010	.0003	99	.1288	.0001	.0000	.0308	.0152
35	.3296	.0000	.0000	.0010	.0003	100	.1285	.0000	.0271	.0000	.0144
36	.3284	.0012	.0000	.0307	.0004	101	.1268	.0000	.0000	.0187	.0000
37	.3202	.0008	.0002	.0000	.0003	102	.1263	.0000	.0000	.0523	.0000
38	.3182	.0000	.0000	.0002	.0001	103	.1261	.0000	.0000	.0000	.0000
39	.3148	.0045	.0002	.0000	.0016	104	.1252	.0000	.0000	.0000	.0344
40	.3106	.0013	.0143	.0000	.0035	105	.1250	.0000	.2010	.0000	.0000
41	.3104	.0018	.0070	.0000	.0029	106	.1236	.0000	.0000	.0000	.0000
42	.3073	.0000	.0000	.0000	.0000	107	.1234	.0000	.0000	.0000	.0000
43	.3039	.0359	.0075	.0000	.0028	108	.1223	.0000	.0000	.0002	.0001
44	.2959	.0976	.0000	.0006	.0002	109	.1206	.0000	.0000	.0000	.0000
45	.2954	.0001	.0001	.0000	.0001	110	.1183	.0000	.0000	.0001	.0000
46	.2943	.0000	.0093	.0000	.0048	111	.1164	.0000	.0000	.0000	.0000
47	.2901	.0001	.0003	.0000	.0142	112	.1160	.0000	.0000	.0000	.0000
48	.2651	.0000	.0000	.0031	.0010	113	.1158	.0000	.0000	.0000	.0000
49	.2631	.0000	.0000	.0002	.0001	114	.1156	.0000	.0012	.0000	.0004
50	.2594	.0057	.0002	.0000	.0065	115	.1150	.0000	.0454	.0000	.0000
51	.2554	.0000	.0000	.0002	.0001	116	.1149	.0000	.0000	.0000	.0000
52	.2534	.0000	.0000	.0008	.0003	117	.1141	.0000	.0031	.0000	.0010
53	.2522	.0425	.0000	.0000	.0000	118	.1140	.0000	.0000	.0000	.0000
54	.2476	.0193	.0009	.0000	.0022	119	.1133	.0000	.0000	.0000	.0000
55	.2307	.0000	.0000	.0000	.0000	120	.1131	.0000	.0000	.0000	.0000
56	.2252	.0000	.0000	.0000	.0102	121	.1131	.0007	.0000	.0000	.0002
57	.2191	.0000	.0000	.0000	.0062	122	.1127	.0000	.0000	.0000	.0000
58	.2175	.0000	.0049	.0000	.0016	123	.1127	.0000	.0000	.0000	.0000
59	.2162	.0000	.0000	.0000	.0000	124	.1123	.0000	.0000	.0001	.0000
60	.2110	.0000	.0046	.0000	.0015	125	.1117	.0000	.0000	.0000	.0000
61	.2077	.0000	.0000	.0002	.0001	126	.1116	.0000	.0000	.0000	.0000
62	.2057	.0000	.0000	.0001	.0000	127	.1108	.0000	.0000	.0000	.0000
63	.2036	.0000	.0000	.0003	.0001	128	.1105	.0000	.0000	.0000	.0000
64	.2025	.0000	.0000	.0000	.0000	129	.1089	.0000	.0000	.0002	.0001
65	.1990	.0000	.0000	.0000	.0000	130	.1076	.0000	.0000	.0000	.0000

note：the initial cable force in 10# pier is 1550KN; and 3100KN in 11# pier.

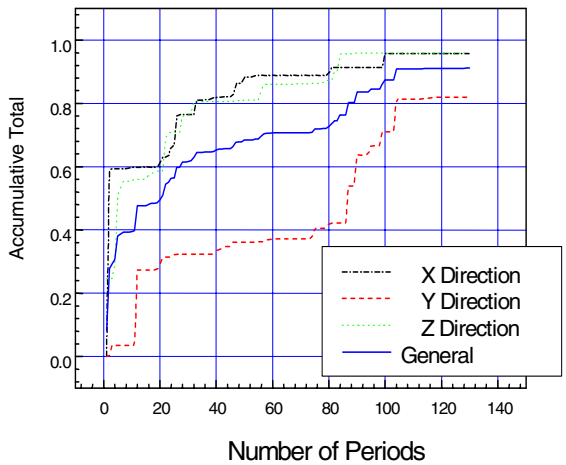

Fig. 2 130 modes contributing ratio

Four cables in the mid-span, which are near main-span middle of the bridge, are taken as control cables, and set up controller at the ends of these cables(Shown in Fig. 1)。The Modified Modal Superposition Method is taken in the analysis. This means to choose modes from Table 1 according to the standard of modal contributing ratio(more than 0.01 in the X or Y direction) , by this standard the chosen modes are: the 2, 3, 12, 20, 21, 30, 33, 40, 43, 44, 53, 54, 74, 87, 95, 96, 100, 105, 115 th, the total is 19 modes， and the simulation control research are made in according to these modes, while other modes are ignored. The artificial seismic waves of X and Y direction in 10# pier are shown in Fig. 3 and Fig. 4.

The control force by sliding mode control based on Modified Modal Analysis is shown in Fig.5，the value of maximum control force is 1062KN. The longitudinal displacement history of 10# tower top is shown in Fig. 6. The displacement history of 11# tower top is shown in Fig. 7；the displacement history of main span middle is shown in Fig. 8. The longitudinal bending moment history of 10# tower base is shown in Fig. 9. The longitudinal bending moment history of 11# tower base is shown in Fig. 10. The maximum values are compared between with-control and without control.

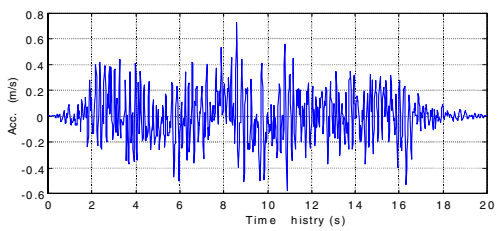

Fig. 3 longitudinal artificial seismic wave of 10# pier

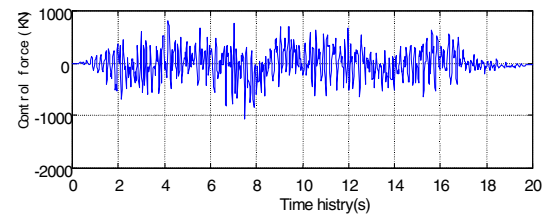

Fig. 5 the control force history

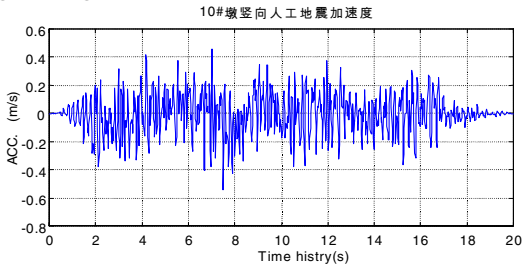

Fig. 4 vertical artificial seismic wave of 10# pier

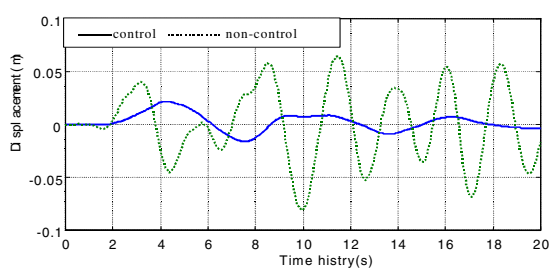

Fig.6 Long. dis. history of 10# tower top

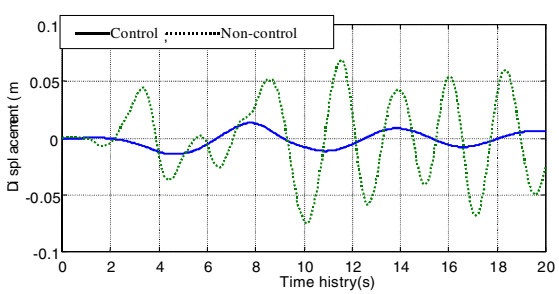

Fig.7 Long. dis. history of 10# tower top

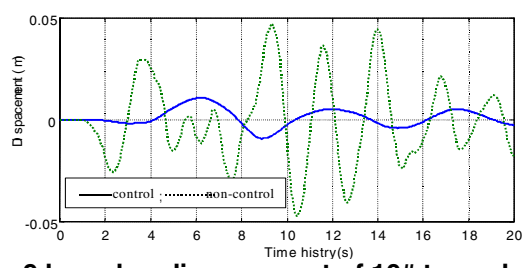

Fig.9 long. bending moment of 10# tower base

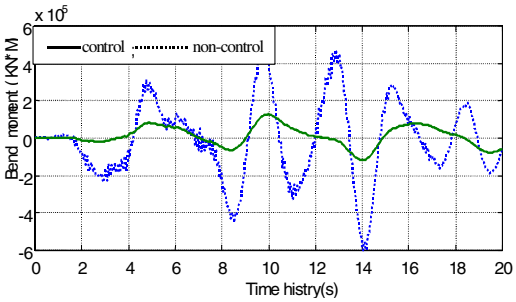

Fig.8 Vertical. dis. history of span mid.

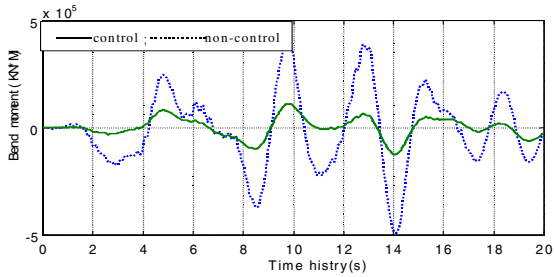

Fig.10 long. bending moment of 10# tower base

TABLE 2 Maximum Values Comparison between With-Control and Without Control

Location	The Maximum of Modified Modal Superposition (without-control)	The Maximum of Sliding Modal Control (with-control)
Long. Displacement of 10# tower top (m)	0.08143	0.0218
Long. Displacement of 11# tower top (m)	0.07528	0.0139
Vertical Displacement of span mid. (m)	0.04621	0.0108
Long. Bending Moment of 10# tower base（KN*M）	559180.0	127413
Long. Bending Moment of 11# tower base（KN*M）	491900.0	122681

CONCLUSION

In this paper, the algorithm of reducing seismic control based on modal analysis by sliding mode is used for reducing the seismic response of cable-stayed bridge. In the light of the characteristic of long-span cable-stayed, such as the seismic responses of the vertical displacement of the span-middle and the bending moment of the tower base are higher, so the longitudinal and vertical control should be analyzed in vertical plan. In the simulation control research the Modified Mode Superposition is taken. The seismic responses of vertical displacement of the span middle and the longitudinal displacement of the tower top should be reduced by the modified controlled-cables forces. As an example, through analysis the WUHU Yangtse River Cable-stayed Bridge by this technology, it is shown the control is effective for reducing the seismic responses of the main parts of the structure.

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