



MULTI-ACTUATOR SUBSTRUCTURE TESTING WITH APPLICATIONS TO EARTHQUAKE ENGINEERING: HOW DO WE ASSESS ACCURACY?

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SUMMARY

Real time dynamic substructuring is a novel experimental testing technique used to test the dynamic behaviour of large structures subject to loading. The technique involves creating a hybrid model of the entire structure by combining an experimental test piece, the substructure, with a set of numerical models. Such testing will allow the behaviour of critical elements to be viewed under extreme dynamic loading at full scale.

We consider the problem of formulating a multi-actuator substructuring model and determine the characteristics of assessing accuracy in both pre-testing estimation and post-process measures using synchronization theory.

Keywords: structural response, numerical-experimental testing, substructuring, synchronization.

INTRODUCTION

In this paper we consider the hybrid experimental-numerical testing technique known as real-time dynamic substructuring. The technique involves creating a hybrid model of the whole structure by combining an experimental test piece - the substructure - with one or more numerical models. This allows design engineers to view the behaviour of critical structural elements under extreme dynamic loading at full scale. So far the technique has been developed successfully using delayed time scales - known as pseudo-dynamic testing, Shing [1] and Donea [2] - with the limitation that dynamic and hysteresis forces must be estimated. Implementing the substructuring process in real-time eliminates the need for these estimations and has been the subject of much recent research in this area; Nakashima [3], Horiuchi [4], Nakashima [5], Blakeborough [6], Darby [7], Darby [8] and Wagg [9].

To couple the experimental and numerical parts of the model, transfer systems are used to transfer the appropriate force and displacement signals between the two parts of the model. Transfer systems are typically single actuators (electric or hydraulic), but could also be in the form of a shaking table. Single actuator substructuring has been developed beyond the 'proof of concept' stage, and experiments with simple substructures have been carried out; Nakashima [3], Horiuchi [4], Darby [8] and Wagg [9]. Multi-

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actuator substructuring presents a significant engineering challenge in terms of real-time implementation but also in terms of measuring the overall accuracy of the substructuring process; Blakeborough [6], Darby [7], Darby [8] and Wagg [10].

In this paper we will consider the problem of formulating a substructuring model with multiple transfer systems. We show that the dynamic characteristics of the transfer systems (actuators) are crucial in determining the likely accuracy of the testing. The control required to carry out successful substructuring has two main sources of error - lag in the transfer system response and the measurement of the feedback force. We demonstrate how coupling between these sources of error can destabilize the substructuring system. Following this we describe how techniques from synchronization theory can be used to give an online accuracy measure based on the delay between the signals passing through the transfer system. Finally we comment on techniques to minimize error and thus significantly increase accuracy in the substructuring control implementation.

THEORETICAL BACKGROUND

The entire structure, which we refer to as the emulated system, is represented by hybrid numerical-experimental substructuring model where the dynamics of the numerical model are combined with the dynamics of the substructure, recorded in time series data format. The general principle of substructuring remains constant regardless of the number of transfer systems present in the system, however the problem from a control point of view becomes more complicated due to the introduction of cross-coupling between the control signals.

In this paper, we will consider the example of a three mass oscillator system with two diametrically opposing excitation walls as shown in Figure 1. This will allow us to demonstrate the problems of achieving accurate control for multiple transfer substructuring using a simple example.

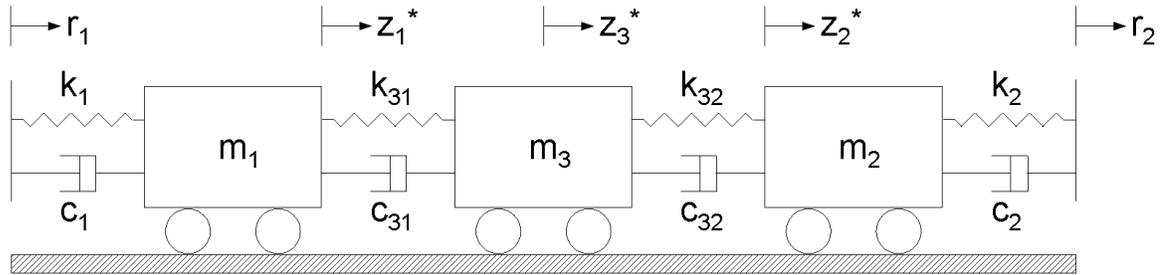


Figure 1: Schematic representation of the three mass system (numbering system chosen as mass 3 will be later removed as the substructure thus simplifying synchronisation enumeration).

In order to simplify coherence testing, the masses are coupled by four identical linear springs, k_i , and damped by coupled viscous dampers, c_i , where $i = 1, 2, 31, 32$. Spring and damper constants 31 and 32 are used to indicate firstly that these operate on mass m_3 and secondly which mass they influence respectively. The system is excited via two moving supports, r_j , where $j = 1, 2$.

The general equation of motion for the system shown in Figure 1 can be written as

$$M\ddot{\xi} + D\dot{\xi} + K\xi = Sr(t), \quad (1)$$

where, M , D and K are the mass, damping and stiffness matrices respectively and $Sr(t)$ is the support excitation. ξ is a vector and represents the states of the system, such

$$\xi = [z_1^*, z_2^*, z_3^*, \dot{z}_1^*, \dot{z}_2^*, \dot{z}_3^*]^T. \quad (2)$$

In order to create a substructured model of the system shown in Figure 1, the middle mass, m_3 , and accompanying springs, k_{31} and k_{32} , are taken to be the substructure. This leaves both excitation walls and respective nearest masses to be used to create two independent numerical models whose influence is imposed on the substructure by two separate transfer systems (actuators) via two independent control signals, u_1 and u_2 . The influence is represented by two autonomous forces, F_1 and F_2 , acting on each side of the substructure which are measured by a set of time series measurements. This new substructured model is shown schematically in Figure 2.

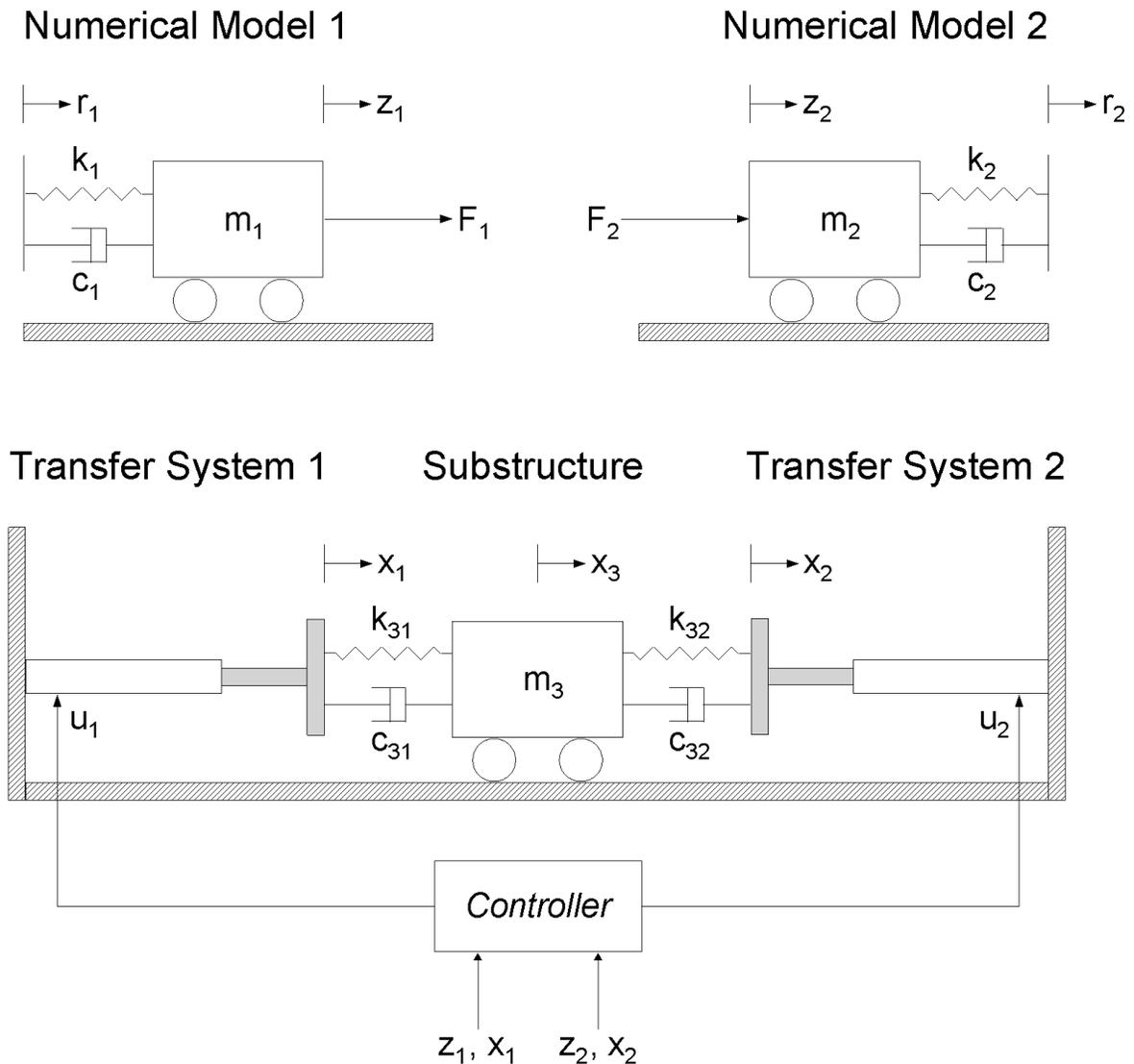


Figure 2: Schematic representation of a substructured three mass system with two transfer systems

The aim of the control algorithm is to achieve synchronization between the output of each numerical model, z_i , and the actual position of the respective transfer system, x_i , where $i = 1, 2$. Mechanical cross-coupling occurs between the transfer systems due to the coupled nature of masses, thus the motion of one can have a significant effect on the motion of the other introducing an added complexity into the formulation of the control algorithm. Therefore, decoupling the dynamics such that this cross-coupling experienced by each transfer system can be dealt with as a disturbance is a crucial step in the substructured model formulation.

Therefore, we can write the equations of motion for the emulated system in full,

$$\begin{aligned} m_1 \ddot{z}_1^* + c_1(\dot{z}_1^* - \dot{r}_1) - c_{31}(\dot{z}_3^* - \dot{z}_1^*) + k_1(z_1^* - r_1) - k_{31}(z_3^* - z_1^*) &= 0, \\ m_2 \ddot{z}_2^* + c_2(\dot{r}_2 - \dot{z}_2^*) - c_{32}(\dot{z}_2^* - \dot{z}_3^*) + k_2(r_2 - z_2^*) - k_{32}(z_2^* - z_3^*) &= 0, \\ m_3 \ddot{z}_3^* + c_{31}(\dot{z}_3^* - \dot{z}_1^*) - c_{32}(\dot{z}_2^* - \dot{z}_3^*) + k_{31}(z_3^* - z_1^*) - k_{32}(z_2^* - z_3^*) &= 0, \end{aligned} \quad (3)$$

and the dynamics of numerical model,

$$\begin{aligned} m_1 \ddot{z}_1 + c_1(\dot{z}_1 - \dot{r}_1) + k_1(z_1 - r_1) &= F_1, \\ m_2 \ddot{z}_2 + c_2(\dot{r}_2 - \dot{z}_2) + k_2(r_2 - z_2) &= F_2, \end{aligned} \quad (4)$$

where,

$$\begin{aligned} F_1 &= c_{31}(\dot{x}_3 - \dot{x}_1) + k_{31}(x_3 - x_1), \\ F_2 &= c_{32}(\dot{x}_2 - \dot{x}_3) + k_{32}(x_2 - x_3). \end{aligned} \quad (5)$$

Due to the linear nature of the emulated system we know explicitly the force at each time interval allowing us to assess the accuracy of an individual test. However, in standard substructuring, these forces would not be known and could only be measured experimentally.

In general, we consider the dynamics of the transfer systems to be linear and in the form

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t), \quad (6)$$

where, x is the state vector of the transfer system (experimental measurement rather than a numerical estimation), A and B are constant matrices which represent the dynamic parameters of the transfer system and $u(t)$ is the control signal.

In our three mass system example (assuming that an actuator can be modeled as a 2nd order system) the equation of motion for the transfer systems are

$$\begin{aligned} m_{t1} \ddot{x}_1 + c_{t1} \dot{x}_1 + k_{t1} x_1 &= u_1 + F_1, \\ m_{t2} \ddot{x}_2 + c_{t2} \dot{x}_2 + k_{t2} x_2 &= u_2 + F_2. \end{aligned} \quad (7)$$

Therefore rewriting Equation 7 in the form of Equations 6, we obtain

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_{r1}} & -\frac{c_1}{m_{r1}} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_2}{m_{r2}} & -\frac{c_2}{m_{r2}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_{r1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_{r2}} \end{bmatrix}, \quad (8)$$

$$x = [x_1 \quad \dot{x}_1 \quad x_2 \quad \dot{x}_2]^T, \quad (9)$$

$$f(t) = [0 \quad F_1 \quad 0 \quad F_2]^T. \quad (10)$$

Mechanical cross-coupling only occurs via the $f(t)$ vector due to the absence of diagonal terms in A and B matrices and thus can be treated solely as a ‘disturbance’. This now allows us to design two individual SISO controllers, one for each transfer system, which can work in parallel and allows us to observe the effect of this cross-coupling.

The aim of the control algorithm is to achieve synchronization between the output of the numerical model, z , and the actual position of the transfer systems, x . However, the physical transfer systems will always be subject to a lag, Δt . The exact nature of the error in the substructuring model has not been fully characterized but from current testing it is thought that there are two coupled components which we can write as,

$$e = g_1(z^*, z, t) + g_2(z, x, t + \Delta t), \quad (11)$$

where, g_1 is a function which describes the accuracy of numerical model compared to the respective part of the emulated system each time step, such that $g_1 = [z_1^* - z_1 \quad z_2^* - z_2]^T$. Note that g_1 also includes a measure of the accuracy of the force measurement of $f(t)$ fed back from the substructure. g_2 is a function which indicates the synchronization of the plant to its respective numerical model, such that $g_2 = [z_1 - x_1 \quad z_2 - x_2]^T$ which is effected by the lag in the system, Δt .

The coupled nature of these errors means that if we achieve perfect synchronization by removing the lag from the plant, $g_2 = 0$, then correspondingly $g_1 = 0$ as the correct force will be added into the numerical model at the correct time, thus achieving the dynamics of the emulated system. However, any error in g_2 and this will result in a corresponding error in g_1 and thus propagate each time step leading to instability in the substructured model.

EXPERIMENTAL SETUP

To implement real-time substructuring we are using a dSpace DS1104 R&D Controller Board running on hardware architecture of MPC8240 (PowerPC 603e core) at 250 MHz with 32 MB synchronous DRAM (SDRAM). This DSP type board offers 4 A/D channels at 16 bit, 4 A/D channels at 12 bit with 8 D/A channels at 16 bit, of which 5 and 4 are required respectively for this substructuring example. This is fully integrated into the block diagram-based modeling tool MATLAB®/Simulink® which is used to build the substructuring model. The dSpace companion software ControlDesk is used for online analysis and control, providing soft real-time access to the hard real-time application. Figure 3 shows the substructure model setup (a) and an enlarged view of the substructure with the load cells measurement locations (b).

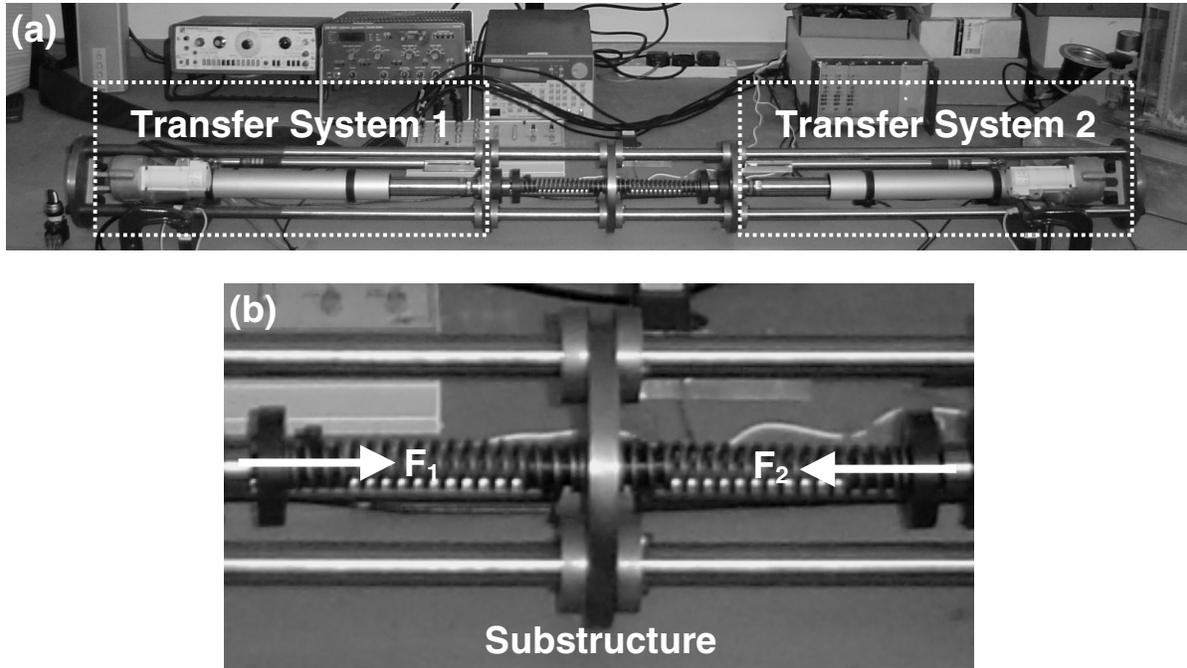


Figure 3: a) Experimental rig set-up of substructured model, b) Substructure enlarged view

Two UBA (timing belt and ball screw configuration) linear Servomech actuators are used as the transfer systems, with maximum force capacity of 500N and maximum linear speed of 640mm/s. These are driven independently by two Panasonic Minas Series AC servo motors which are configured as analogue amplifiers to remove any internal closed loop control functions. Three RDP Electronics DCT captive guided DC LVDT displacement transducers are used to measure the displacement of the two transfer systems and the substructure which have a $\pm 0.11\%$ linearity error on full scale deflection of 50mm. Each unit has an internal bearing that guides the armature built-in dc to dc signal conditioning to help remove noise. Two RDP Electronics model 31 precision miniature tension/compression load cells are used for the force measurements either side of the substructure. The unit is applicable both in tension and compression with linearity $\pm 0.15\%$, hysteresis $\pm 0.15\%$ and non-repeatability $\pm 0.1\%$ of full scale deflection. Each mass is a constant 2.2kg and connected to the rig via three parallel shafts constraining their motion to one degree of freedom with an axial alignment accuracy of $\pm 0.1\text{mm}$. Each mass has three LBBP linear ball bearings with double lip seals and raceway plates to reduce friction. Through system identification the spring constants were found constant for all and equal to $k = 4750\text{N/m}$ and damping ratio of $c = 6\text{Ns/m}$.

THE EFFECT OF DELAY ON THE SUBSTRUCTURED MODEL

There is an important difference between a standard control problem and that of a substructured system. For substructuring, the reference signal for each transfer system is not known at the start of the time interval as in a normal control problem, but must be created in its respective numerical model at the start of each time step. We require two parts to create this signal, firstly the known part made from the dynamics of the continuous equations and secondly the unknown part which is the relevant force measurement from the substructure itself. This is where the first dichotomy arises; we require the dynamics of the substructure at the end of the time step in order to calculate the force on the numerical model for the start of the time step as can be seen from Figure 4. In a purely numerical simulation we can get around this problem due to Simulink scheduling, as the execution order of the model is automatically altered during the initialization phase. However, algebraic loops are strictly prohibited in any real-time

programming as any alterations in the way the model executes during the build phase would effect the timeliness of its operation. The only way round this problem is to hold the feedback signal until the next step, therefore by altering the structure in this way we have effectively introduced a one time step delay into one of the signals which used to create the numerical model.

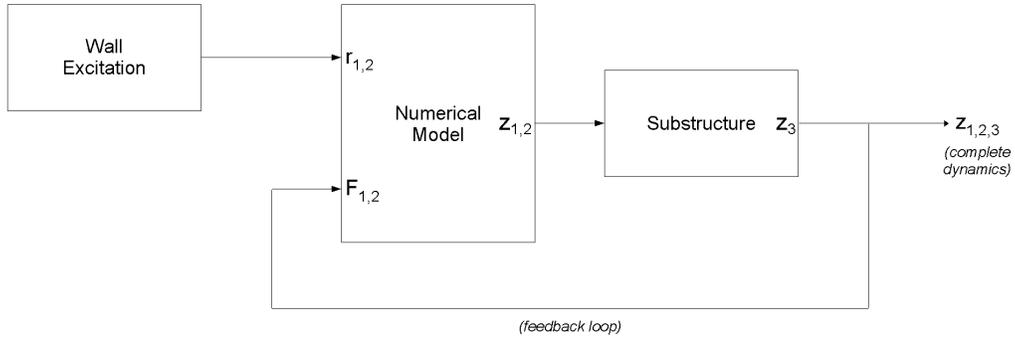


Figure 4: Basic substructuring model structure

The effect of this one time step delay can be seen in Figure 5 which shows the time domain response for one of the transfer systems (with a simulated substructure). It is clear that as resonance is approached, the magnitude of the force signal rapidly increases and this time step delay in feeding the force signal back into the numerical model has a significant effect on the creation of its dynamics z_1 and therefore a reduction in synchronization to the perfect dynamics of the emulated system shown by z_1^* .

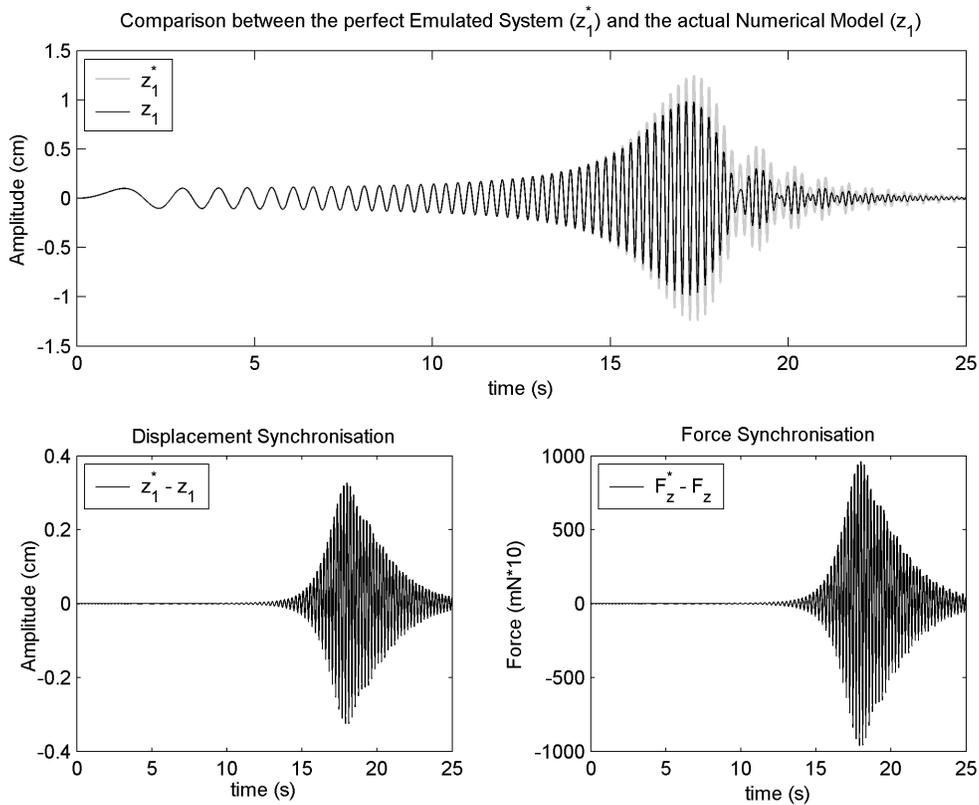


Figure 5: Effect of a single time step delay on the force signal for its respective numerical model

It is important to note that we are assuming no plant dynamics and perfect synchronization of the transfer system, x_I , so the divergence of the numerical model from the emulated system is purely down to the force signal being fed into the numerical model one time step late. However, although there is this discrepancy in synchronization the substructuring algorithm is stable (we are using a time step of 1ms in this case). If we increase the delay of the feedback loop we can see that we can see the effect on the stability, as can be seen Figure 6. Increasing the delay to 2ms, it can be seen that the substructuring algorithm remains stable for around 40 seconds before going unstable during the second mode of resonance, whereas when the delay is further increased to 3ms, the model becomes unstable almost immediately even before any resonance peak is reached. It is clear that as the frequency increases this constant delay error has an increasingly significant effect as a larger percentage of the period is missed. This instability shown by the substructuring algorithm can be characterized by a function of exponential growth, where the feedback delay can be thought of as adding negative damping to the system with instability occurring at the point of sign change, Horiuchi [4].

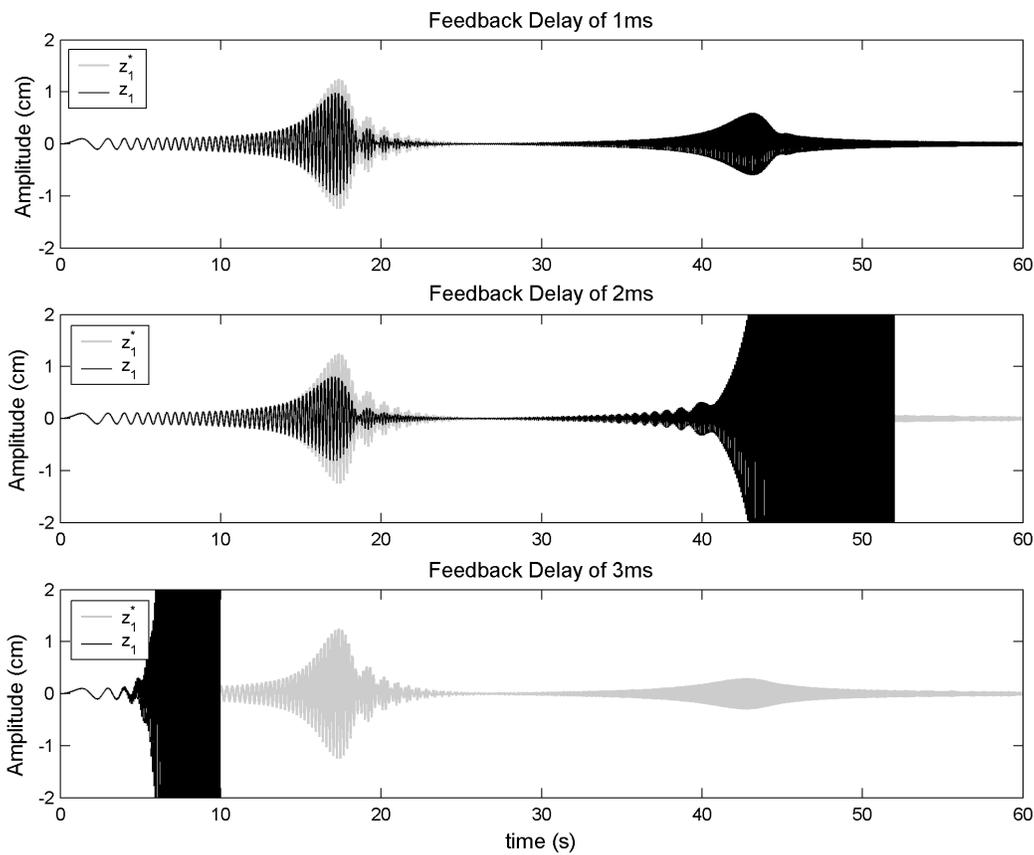


Figure 6: Progression to instability as the feedback delay on the force is increased (numerical substructure simulation)

To complete the substructure model we must include the plant dynamics, which include the characteristics of the transfer systems and control algorithm used, as can be seen in Figure 7. Consequently, the addition of these dynamics increases the phase delay of the feedback force from the substructure subject frequency of excitation. We see the effect on the stability of the numerical model in Figure 8. The close up shown in Figure 8 (b) highlights the coupling between the errors in the substructure model and demonstrates this positive exponential growth of the numerical model z_I compared to the emulated system z_I^* .

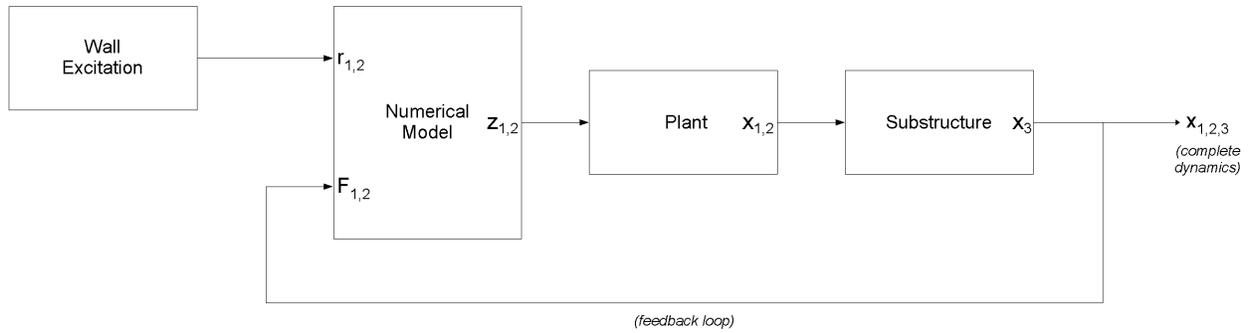


Figure 7: Completed substructure model structure

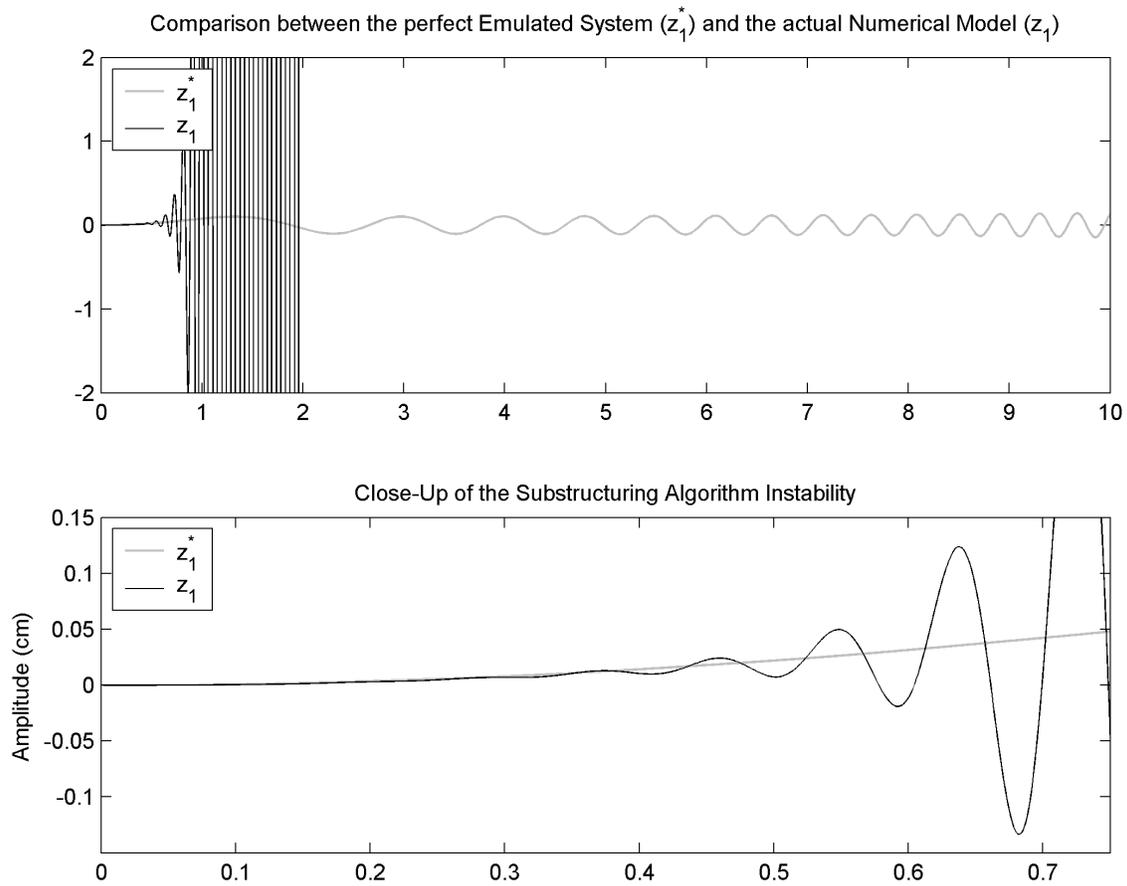


Figure 8: Instability of the substructure algorithm due to phase delay of the plant response (numerical substructure simulation)

MEASURING SYNCHRONIZATION ACCURACY

In order to keep the substructuring algorithm stable when we include the plant dynamics, we must use the force signal from the emulated system as this removes the phase delay on the force signal fed back to the numerical model, thus ensuring that $z_I^* = z_I$. Figure 9 (a) shows a small section of the plant (the transfer system and controller) response x_I in relation to its numerical model z_I using the emulated force. This is a

typical response you would expect from a tuned linear controller, quite high accuracy apart from a phase lag. A 'synchronization subspace' plot is used to show the effectiveness of the controller by plotting the desired versus actual responses, Ashwin [11], as shown in Figure 9 (b). Subspace plots are important as they allow the effectiveness of the controller to be characterized in an online procedure, aiding in tuning and in evaluating the success of the substructure test in a post process procedure. A subspace plot shows the amplitude accuracy and the magnitude of delay coupled together at any one time interval. Perfect synchronization is represented by a straight line at an angle of 45° to the horizontal with maxima and minima of the reference signal. Any reduction in synchronization can be seen as a deviation from this idealized line. For constant wall excitation conditions these plots build up into a repeating periodic pattern, which can appear complex, however, the individual components of amplitude and delay produce their own specific and identifiable patterns if evaluated separately. The result of varying the amplitude accuracy is to change the angular orientation of the subspace plot compared to the idealized 45° line and the consequence of introducing a constant delay between the reference signal and the response is to transform the idealized straight line into an ellipse.

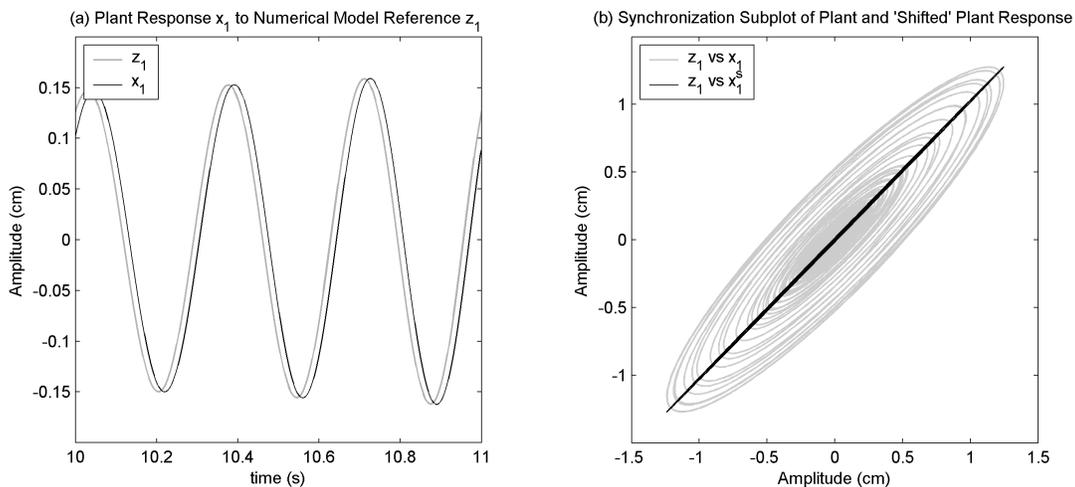


Figure 9: a) Typical experimental plant (transfer system) response to reference signal from the numerical model, b) Comparison in synchronisation subspace of actual plant response x_1 and a forward time shifted (15ms) plant response x_1^s .

We see that the subspace, Figure 9 (b), plot for z_1 against x_1 shows a high amplitude accuracy due to its orientation and the appearance of a constant delay. The plot shows the first 25s of a sine sweep test which is why the maxima and minima of the plot change. If we shift the plant results in a post process procedure by 15ms forward to produce x_1^s and re-plot in the synchronization subspace we see a result far closer to the idealized 45° line of perfect synchronization. This does not hold for the entire test as the plant introduces a phase lag rather than a constant delay, however it does suggest that if we can overcome this problem we can achieve excellent synchronization.

NUMERICAL-EXPERIMENTAL RESULTS

Figure 10 shows the experimental test results for a constant sinusoid input of 3Hz to Excitation Wall 1, r_1 , and 5Hz to Excitation Wall 2, r_2 . Figure 11 shows the experimental test results for a constant sinusoid input which is equal and opposite for both Excitation Walls, r_1 and $r_2 = 6$ Hz. In order to keep the substructuring algorithm stable, we are using the force from the emulated system to create the numerical model.

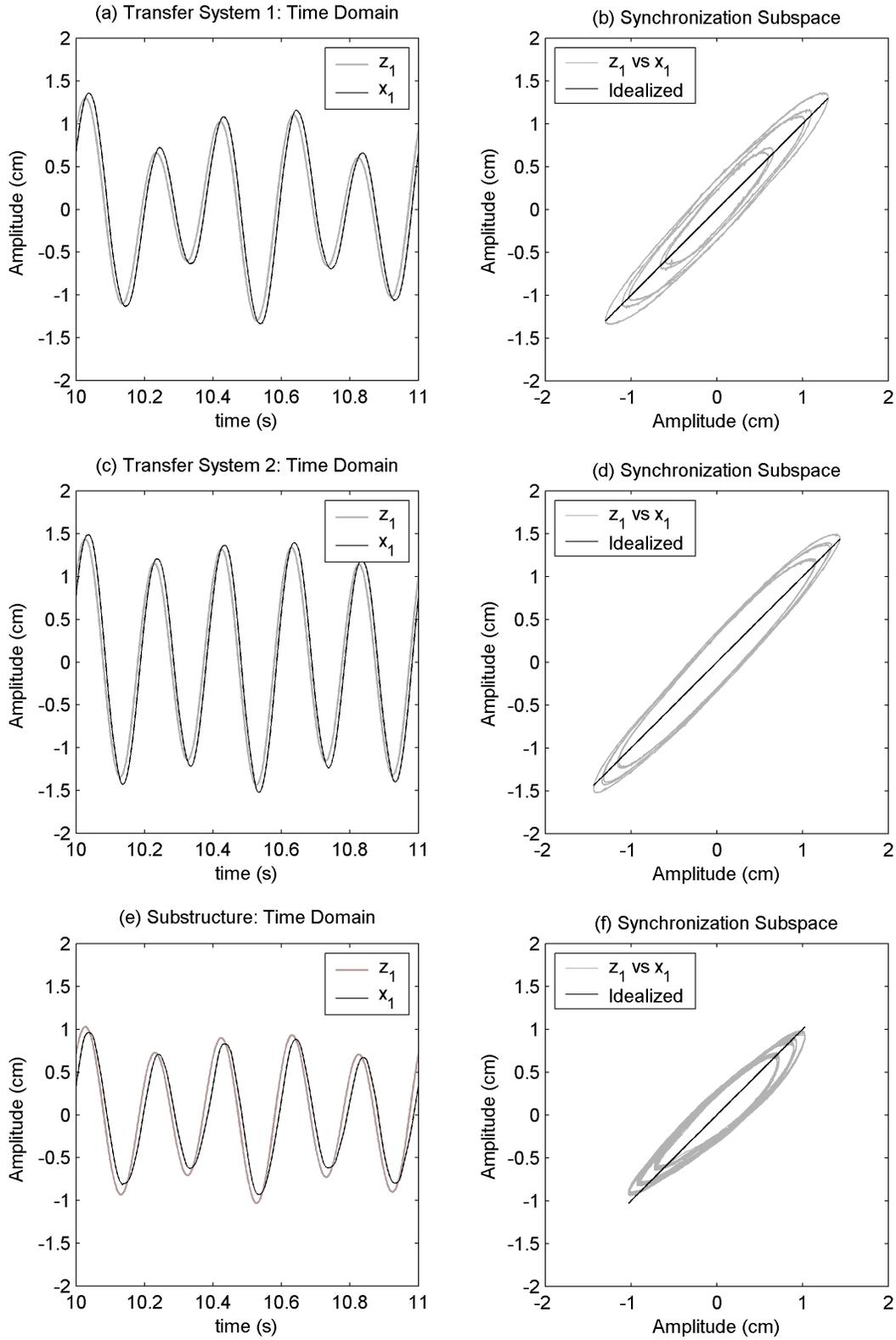


Figure 10: Experimental test results for sinusoid input of $r_1 = 3\text{Hz}$ and $r_2 = 5\text{Hz}$; (a) and (b) for transfer system 1, (c) and (d) for transfer system 2, (e) and (f) for the substructure.

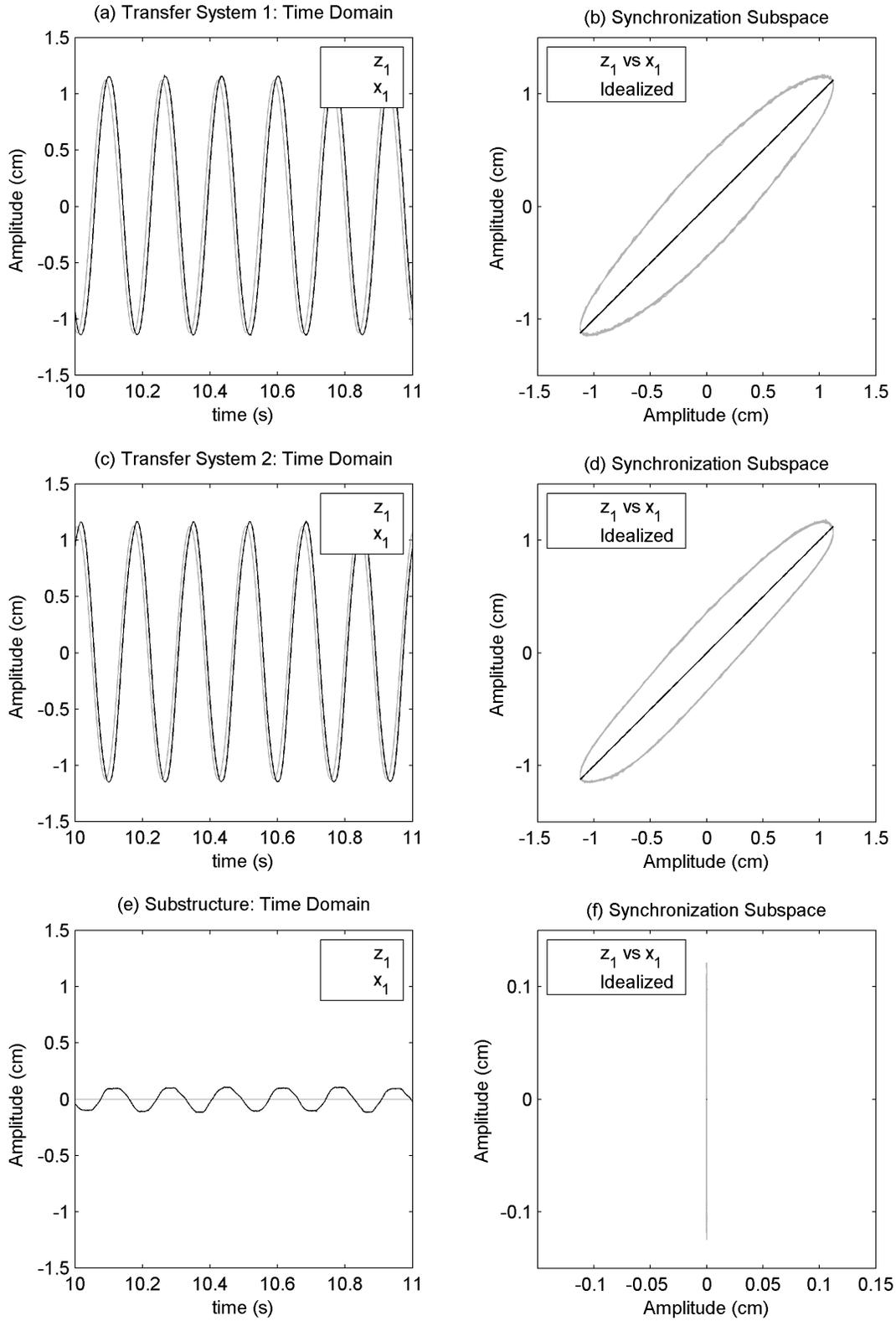


Figure 11: Experimental test results for sinusoid input of $r_1 = 6\text{Hz}$ and $r_2 = 6\text{Hz}$; (a) and (b) for transfer system 1, (c) and (d) for transfer system 2, (e) and (f) for the substructure.

Figure 10 shows the experimental test results for a constant sinusoid input of 3Hz to Excitation Wall 1, r_1 , and 5Hz to Excitation Wall 2, r_2 . Observation of the synchronization subplots for the two transfer systems, Figures 10 (b) and (d), we see a high degree of amplitude accuracy but a phase lag between the numerical models and their respective transfer systems. This translates a phase lag though to the substructure but retains a high degree of amplitude accuracy. This is an important result as it suggests that if the phase error can be removed from the control of each transfer system then we can ensure synchronization of the substructure.

Figure 11 shows the experimental test results for a constant sinusoid input which is equal and opposite for both Excitation Walls, r_1 and $r_2 = 6\text{Hz}$. As the motion of the Excitation Walls are equal and opposite, the output from each numerical model will also be equal and opposite, which in turn should mean that the substructure is stationary. It is clear from Figure 11 (e) that the substructure is not stationary but in fact shows a periodic pattern. Figures 11 (b) and (d) show a different level of synchronization to each other even though the demands on each plant and the transfer systems themselves are identical. Transfer System 1 shows a larger but more constant delay to Transfer System 2 as a constant delay is represented by a perfect ellipse in a subspace plot. This clearly indicates that each plant has different dynamics due to the frictional component associated with each transfer system, which transfers through to the dynamics of the substructure.

The differing transfer system dynamics would mean that different phase delays would be produced in the feedback of the force readings. Although in this case we are using the emulated force, Figure 12 shows the effect of differing levels of synchronization of the two transfer system as we sweep on excitation wall from 1 to 15Hz whilst keeping the other constant. Unidentified resonance peaks in the substructure are produced as can be seen from Figure 12 (d).

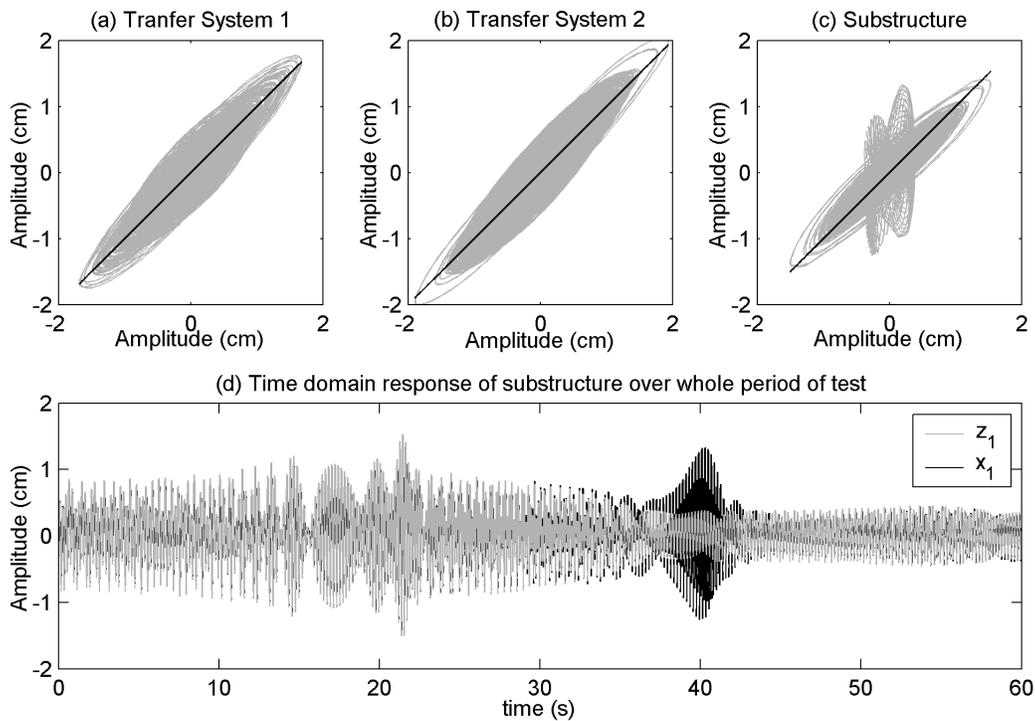


Figure 12: Experimental test results for sinusoid input of $r_1 = 1$ to 15Hz (in 60s) and $r_2 = 5\text{Hz}$

CONCLUSIONS

There are two components to the accuracy in a substructuring test, the accuracy of the numerical model(s) created compared to the emulated system and the level of synchronization the transfer system(s) achieve. These components are coupled such that if perfect synchronization is realized then there will be no error in the numerical model(s), however, any error in synchronization will cause an incorrect reference to be created such that the synchronization of next time step will further diverge from the emulated system. This is due to the hybrid nature of the numerical model(s) in substructuring. The phase lag in the discrete force signal(s) fed back each time step has a significant effect on the stability of the model. If this delay is greater than a nominal value then instability will occur. We can infer from Figure 10 that if we can achieve perfect synchronization of the transfer system(s) then we can achieve synchronization of the substructure.

Multi-transfer system substructuring introduces new problems into dynamical process. We can decouple the transfer system dynamics for simple control however the mechanical cross-coupling puts much higher demands on the system. Additionally, Figure 11 and 12 indicate that it is not only the phase lag which is important between transfer systems but also their relative phase difference as this introduces unidentified resonance dynamics of the substructure.

Future development in substructuring must evolve around overcoming the phase lag of the transfer systems. Two areas under development at the moment are an open-loop phase inversion of the modeled plant using bond graph modeling techniques and a closed-loop adaptive feedback forward prediction algorithm using polynomial statistical analysis.

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