13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 323

STUDY ON A ESTIMATION METHOD FOR BUCKLING CAPACITIES OF BRIDGES DURING EARTHQUAKES

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SUMMARY

Buckling of large bridges during earthquakes is often concerned on design stages. However, quantitative estimations for their buckling capacities have rarely been performed due to the rack of suitable analytical methods. Some conventional methods can provide the judgment whether a structure becomes unstable during an earthquake. But they can't indicate the margin to buckling quantitatively when an analysis is completed stably.

After reviewing existing numerical analysis methods for buckling, we present a new method to get seismic-buckling capacity indicators by means of repeated eigen-analyses in a material and geometrical non-linear time history analysis.

Furthermore three different large arch bridges are analyzed by this method with three-dimensional overall frame models. Results of them suggest that all of three bridges don't fall into buckling under the Great Hanshin Earthquake. New method's effectiveness and reasonability are demonstrated by showing the buckling capacity and the buckling mode of each bridge.

The new method and the buckling capacity indicator can help to rationalize seismic-buckling investigation of bridges. But there are several remained subjects to be considered such as establishment of criteria for design.

INTRODUCTION

Although buckling of large bridges is often concerned on design stages, quantitative estimations for their buckling capacities haven't been performed necessarily and uniformly. This situation can be seen, for example, in a state of the arts report on large concrete arch bridges in Japan [1]. This report shows the information about design and construction of 35 concrete arch bridges that have a center span with a length of over 100m. According to this report, buckling investigations by means of experiments or numerical analyses were carried out for only 6 bridges with different conditions and methods respectively. Therefore their buckling capacities aren't comparable on the same index and can't be stimulated as general information. Little information leads to the next ambiguous concern. In this way, not only concrete arch bridges, it has been repeated while it has been unsolved.

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We thought the fundamental cause of this situation was the rack of general estimation methods and performance indicators for buckling capacity of structural systems. So we started the study for the following points: (1) Review of the past analysis techniques for buckling under static loads or dynamic oscillations; (2) Development of a new analytical investigation method; (3) Proposal of performance indicators for buckling capacity.
In this paper the research results about these points by present are reported.

THE PAST EVALUATION TECHNIQUES AND THEIR PROBLEMS

First of all, we reviewed the past analytical techniques for buckling and considered their problems. As buckling analysis techniques, eigen analysis and mixed (material and geometrical) non-linear static analysis are often adopted. Both of them are essentially the methods to search a critical (bifurcation or limit) point of a tangential stiffness matrix. Mixed non-linear dynamic analyses are applied, too. Meanwhile, the supposed situations where buckling is concerned in designs are mainly under dead load and during severe earthquake. Techniques should be properly used according to the situations.

Eigen Analysis

Linear buckling eigen analysis by equation (1) offers a buckling mode vector and the buckling load-factor (multiple factor for reference loads) with the smallest eigen value. It is a simple method but it can be strictly used only for a buckling investigation under dead load (gravity). And it is available only in elastic and small displacement range. It's a technique with much restriction, too.

$$([K] + \lambda [Kg]) \{u\} = \{0\} \quad (1)$$

$[K]$: Material stiffness Matrix, $[Kg]$: Geometrical stiffness matrix, $\{u\}$: Buckling mode vector, λ : Buckling-eigen value

Static Analysis

Mixed non-linear static analysis controlled by incremental arc-lengths or displacements is often applied to the buckling investigation for the state accompanying plasticity of structure's members. In this method, another buckling-eigen analysis by equation (2) is infixed in an incremental analysis step where the sign (positive/negative) of the tangential stiffness matrix's determinant value is reversed (Figure 1).

$$([Kt] + \lambda [\Delta Kg]) \{u\} = \{0\} \quad (2)$$

$[Kt]$: Tangential stiffness matrix in the previous step, $[\Delta Kg]$: Incremental geometrical stiffness by the initial stress in the current step, $\{u\}$: Buckling mode vector, λ : Buckling-eigen value

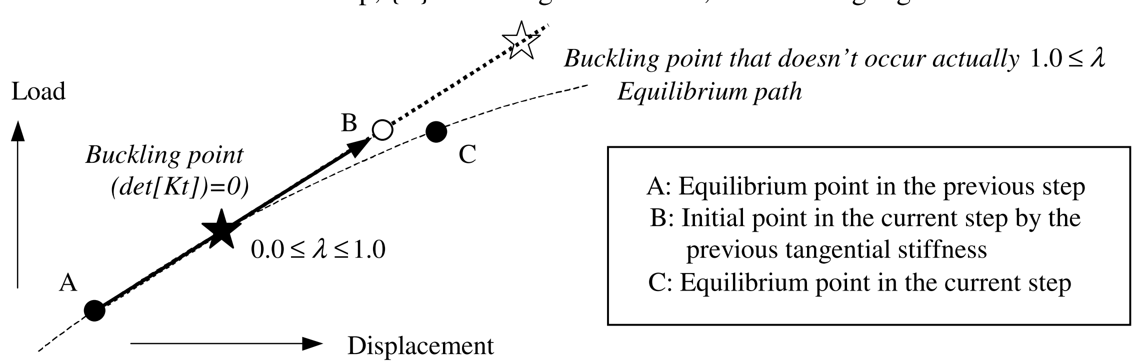


Figure 1: Buckling Point in a Step of Incremental-Static-Analysis

$0.0 \leq \lambda_1 \leq 1.0$ (λ_1: the first eigen value) also suggests that there is a buckling (bifurcation or limit) point between the two continuous steps. It can provide an accurate buckling point and a buckling mode vector. This technique is often used for magnified dead loads and it is expected as a method for the buckling during earthquake because it's available in plastic range or large displacement range. But there is a problem. Many bridges that are concerned about buckling, such as large arch bridges and cable stayed bridges, are generally anticipated to perform complicated actions during earthquakes. Therefore they became to be designed with inelastic dynamic analyses in Japan after the Great Hanshin Earthquake (1995) [2]. When the attention is paid to buckling, it should be thought the same way. In fact, seismic-buckling estimation with incremental static analysis may not be rational due to the difficulty of a setup of seismic loads for the bridges of that type. From this discussion, the necessity for a buckling investigation method based on a non-linear time history analysis has been recognized.

Dynamic Analysis

Buckling investigations with mixed non-linear time history analyses have rarely been carried out on design stages in the past. In many of these rare cases, the stabilities of bridges have been evidenced by the stabilities of the time history analyses' processes. It means that those bridges were regarded to be stable against the supposed earthquake when the mixed non-linear time history analyses had been ended stably without any rapid serious responses or un-converging. It can be said that they were qualitative judgment. Meanwhile Himeno et al. infixed the eigen analysis by equation (3) in each time step of a mixed non-linear time history analysis to account for the negative eigen values of tangential stiffness matrices of a system [3]. The importance of these eigen values is in whether positive or negative. Their absolute values don't indicate the degree of the dangerousness of buckling. It was an attempt for quantitative judgment.

$$([K] + [Kg])\{u\} = \lambda \{u\} \qquad \text{(3)}$$

$[K] + [Kg]$: Tangential stiffness matrix in the current step, $\{u\}$: Mode vector, λ: Eigen value

Anyway, these dynamic methods can provide the judgment whether a structural system becomes unstable during a supposed earthquake. But when a dynamic analysis is completed stably (without any negative eigen values), there is no telling whether the stableness is enough or not. These methods can provide limited information about buckling when buckling doesn't occur in an analysis.

From the above, following recognitions about the past buckling analysis methods were renewed: (1) Eigen analysis and mixed non-linear static analysis are available when a set up of reference loads is explicit enough such as dead loads; (2) In these cases a buckling load factor can be a performance indicator; (3) Some methods can provide the judgment whether a structure becomes unstable during an earthquake; (4) But any method can't provide the degree of the dangerousness of buckling during an earthquake quantitatively.

We especially focused on the last point and tried to solve.

PROPOSAL OF A NEW METHOD

'Dynamic-instability' (instability in a dynamic action) is a complicated issue that includes some kinds of phenomenon [4]. As the first step, we focused on a phenomenon just like buckling under static loads, namely 'Static-instability in dynamic action', because it is the phenomenon of bringing about serious damage to a structure and it may be overlooked only by a mixed non-linear time history analysis. A certain additional processing is needed.

A hint of a quantitative estimation for this kind of instability was suggested by the method of Himeno et al. and a new method is expected to provide an indicator that shows the degree of dangerousness quantitatively. Taking into account them, we tried to apply the eigen-analysis by equation (2) in each time step of a dynamic analysis, instead of equation (3). As a result of equation, eigen values and their paired buckling mode vectors can be obtained. The first (minimum) eigen value indicates the multiple factor for the stepwise incremental stresses of each member of the system to the nearest buckling point (static-unstable point in dynamic action).

We knew the application of equation (2) is originally limited to the static analysis for conservative forces. But we considered it was available in each step of a mixed non-linear dynamic analysis approximately. Although dynamic oscillations such as earthquake motions are not totally conservative of course, we regarded the incremental dynamic force of each time step as conservative because the stepwise incremental responses are accumulated (conserved) as well as in an incremental static analysis.

When the 'Inverse iteration method' is adopted as a buckling eigen analysis method, the first buckling eigen value λ_1 is minimum in absolute. So it can be negative. As has been suggested in the previous chapter, $0.0 \leq \lambda_1 \leq 1.0$ indicates that there is a buckling point within the time step interval. In the state of $\lambda_1 > 1.0$, it is getting near to the buckling point but the larger λ_1 is, the farther to the nearest buckling point. $\lambda_1 < 0.0$ means that it is getting far from the nearest buckling point.

In an incremental static analysis, equation (2) is applied only when the buckling point between continuous two steps is found out by checking determinant value of the tangential stiffness. But in the proposed dynamic method it should be carried out repeatedly at a fixed step interval because the significance to use this method is not only for the judgment of unstableness but also for the suggestion of the degree of the dangerousness for buckling with eigen values even while the system is stable. Time history of λ_1 indicates the dangerous time zone. And the minimum λ_1 in duration can be a performance indicator for buckling during supposed earthquake. It must be noted that when the minimum λ_1s of some cases are compared, integration-time-intervals (Δt) of dynamic analyses must be unified.

We enabled the execution of the analysis mentioned the above on a frame analysis program as shown in Table 1. The application results will be exemplified hereafter.

Table 1: Specification of the Platform Program

Category	Three dimensional non-linear frame analysis program
In-elastic	Setting of the moment-curvature relationships in beam elements' bending
Geometrical stiffness matrix	Estimating from axial forces of beam or truss elements
Formulation of Geometrical non-linear	Updated Lagrange formulation
Time history analysis	Direct integration method (Newmark-β method)
Vibration eigen analysis	Sub-space method
Buckling eigen analysis	Inverse iteration method

EXAMPLE ANALYSIS

Analysis model and conditions

Mixed non-linear time history analyses with eigen analyses by equation (2) at a fixed step interval were performed for an analysis model to verify the new method suggested in the previous chapter. For the verification, a three-dimensional analysis model called 'CU600' shown in Figure 3 were arranged from a trial-designed concrete arch bridge [1] that have a center span with a length of 600m shown in Figure 2.

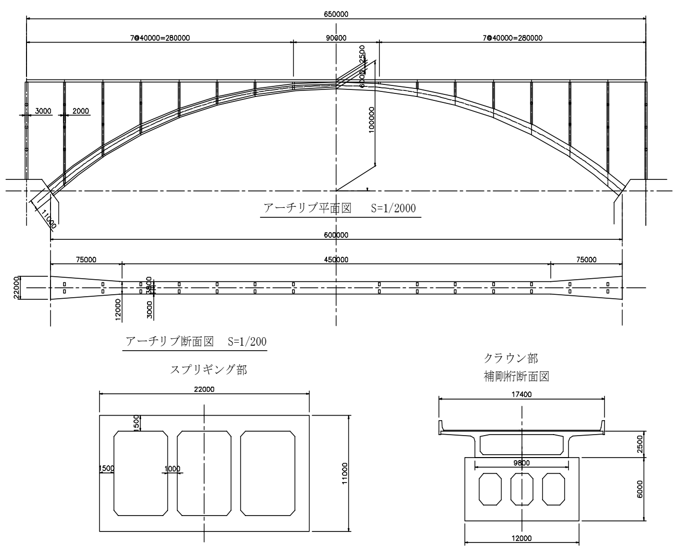


Figure 2: Sample Concrete Arch Bridge (600m Center span)

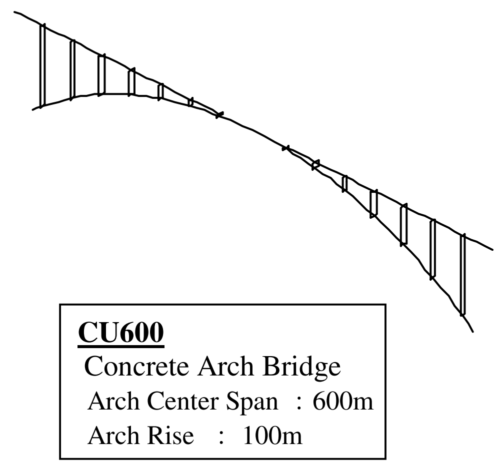


Figure 3 Sample Analysis Model

Analysis conditions were given as below: (1) After dead loading, mixed non-linear time history analyses based on the updated Lagrange formulation were carried out on a three dimensional frame model; (2) Newmark- β method was used to integrate the equation of dynamic motion with $\beta = 0.25$ and Δt=0.001seconds; (3) Buckling eigen analyses were infixed in an interval of 0.01 seconds; (4) Damping ratio of concrete members was given as 3% and the Rayleigh damping with the damping ratios of the appropriate vibration modes was used. Damping matrix was constant in duration; (5) Degrading tri-linear hysteretic model was used on the moment-curvature relationships of concrete arch-lib members. The other members were modeled with elastic beams; (6) NS component of JMA Kobe in the Great Hanshin Earthquake (shown in Figure 9, whose maximum acceleration is $812\, cm/sec^2$) that is one of the standard base waves for design of highway bridges in Japan was used as a horizontal input base wave. The acceleration wave was input in the longitudinal direction in 'CASE1'. 'CAEE2' was an analysis case of the transverse oscillation.

Application results

Time history of eigen values at intervals of 0.01 seconds in all duration of CASE 1 (longitudinal oscillation case) was shown in Figure 4. Dangerous time zone is judged from Figure 4 to be 7-9 seconds. The analysis process ended stably and the minimum λ_1 (generated in 8.178 seconds) was 2629.7. It meant that buckling didn't occur in this analysis. Figure 5 shows a buckling mode that was coming up (although didn't come up) in the most dangerous time. It was a local mode in the stiffening girder mainly.

What should be focused on is the relationship among some dynamic responses including λ_1s shown in Figure 7. All of three diagrams in Figure 7 are only for the dangerous time zone at intervals of 0.001 seconds. Figure 7 shows the time histories of λ_1s (a), vertical displacements of a point 'A' located on the stiffening girder (b), and axial section forces of a girder's member 'B' (c). Near the time T1, while both of vertical displacement and compression were increasing, the positive λ_1 was reducing rapidly (it means buckling was coming up). And then after compression began to reduce, the reducing of λ_1 began to blunt and the minimum λ_1 was generated at T2. After that axial force turned tension λ_1 turned negative (it was getting far from buckling). Then when tension began to reduce, λ_1 turned positive again after the time T3. Vertical displacement was continuing increasing in the meantime. It can be said that their relationships were rational and accountable.

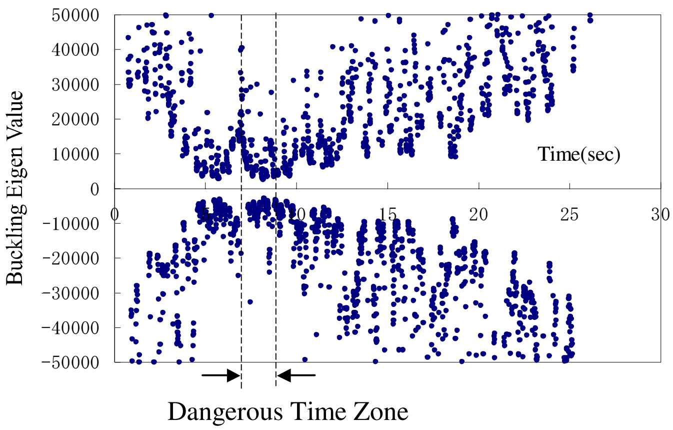


Figure 4: Time History of Eigen-Values (Longitudinal Oscillation Case)

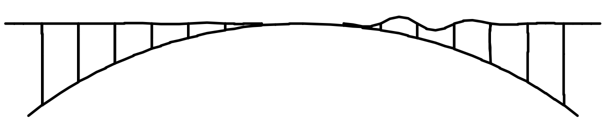


Figure 5: Buckling Mode (Longitudinal Oscillation Case)

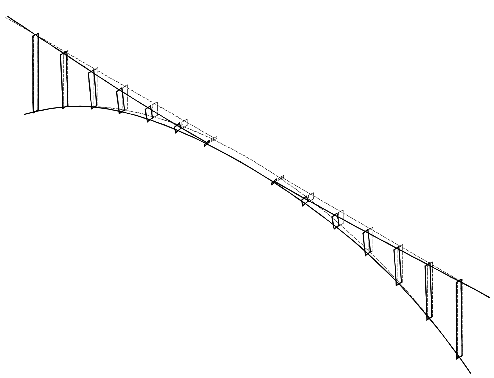


Figure 6: Buckling Mode (Transverse Oscillation Case)

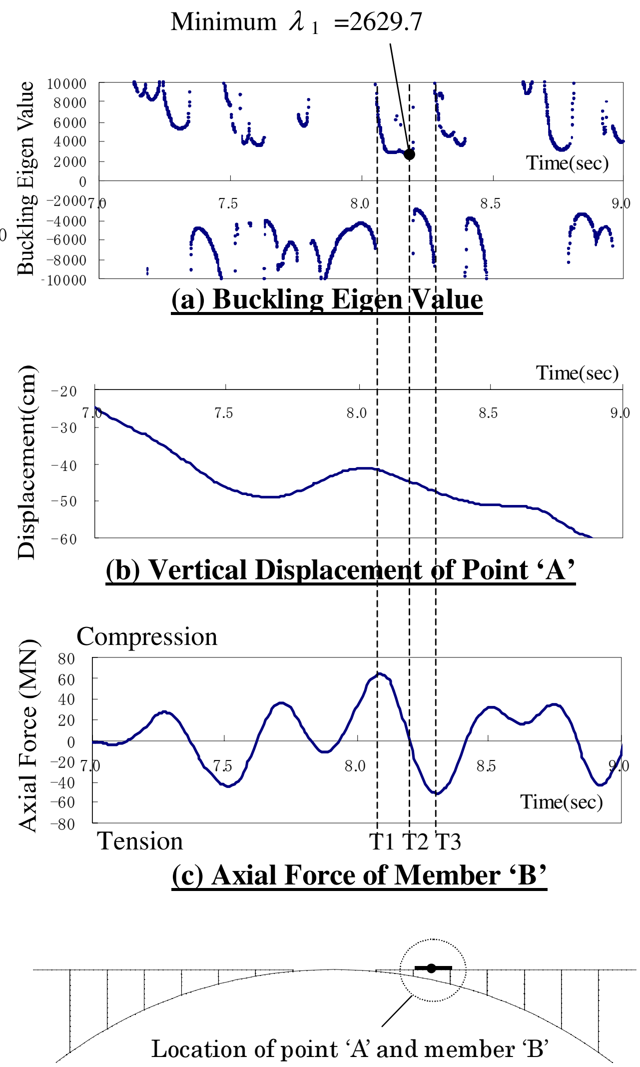


Figure 7: Responses Time History (Dangerous time zone of Longitudinal Oscillation Case)

Figure 6 shows the buckling mode in the most dangerous time of CASE 2 (transverse oscillation case). It was an overall mode in which all vertical members were inclined to the transverse direction. The minimum λ_1 was 221.6. It meant that buckling didn't come up in all duration. But it should be noted that It was about 10% compared with CASE 1's minimum λ_1. It suggested that this bridge's seismic buckling capacities were largely different depending on directions. The past methods would have told only that buckling wasn't seen in both cases. Hereafter, negative eigen-values won't be discussed in this paper because they originally suggest the situations getting far from buckling point and it was illustrated that positive/negative values' time histories were approximately symmetrical in Figure 4. So if only care of positive values is taken, it can be evaluated enough.

A parameter study

Furthermore, Table 2 shows the change of minimum λ_1s when changing the rise of CU600 from 60m to 100m. It also shows buckling eigen values under dead load by equation (1). All dynamic analysis cases suggest that there was a directional variation in CU600's seismic-buckling capacities again.

Table 2: Comparison of the Minimum Buckling Eigen Values

CU600's Rise	Longitudinal Oscillation Cases	Transverse Oscillation Cases	Dead load cases by Equation (1)
100n	2627.7 (100%)	221.6 (100%)	2.885 (100%)
80m	2291.3 (87%)	294.8 (133%)	2.296 (80%)
60m	2420.5 (94%)	372.6 (168%)	1.615 (56%)

In longitudinal oscillation cases, the minimum λ_1s didn't change according to the change of the rise because their paired buckling modes were commonly local modes on the stiffening girder as shown in Figure 5 that had little relation with their rises. Meanwhile the minimum λ_1 in transverse oscillation cases changed according to the change of the rise. It also could be seen that the lower rise became, the safer in the viewpoint of buckling during the earthquake (although the tendency is reverse under dead load). In these cases the paired buckling modes were commonly the states in which all vertical members were inclined to the transverse direction as shown in Figure 6 that become more disadvantageous when the vertical member is longer. The changing status of λ_1s suited the buckling modes' character.

PERFORMANCE COMPARISON

Analysis model and conditions

In the next place, two additional overall arch bridge models, namely 'CM100' and 'SM160' shown in Figure 8 were arranged. And mixed non-linear dynamic analyses using three-dimensional earthquake motions (Figure 9) were carried out for the three models including CU600. It was an attempt to illustrate a quantitative buckling-capacity comparison of the different bridges on the same condition. The analysis conditions were same as described in the previous chapter except for the following points: (1) Damping ratio of steel members was given as 2%; (2) Normal bi-linear hysteretic model was used on the moment-curvature relationships of steel members; (3) NS, EW and vertical acceleration components of JMA Kobe in the Great Hanshin Earthquake were used at the same time as input base wave.

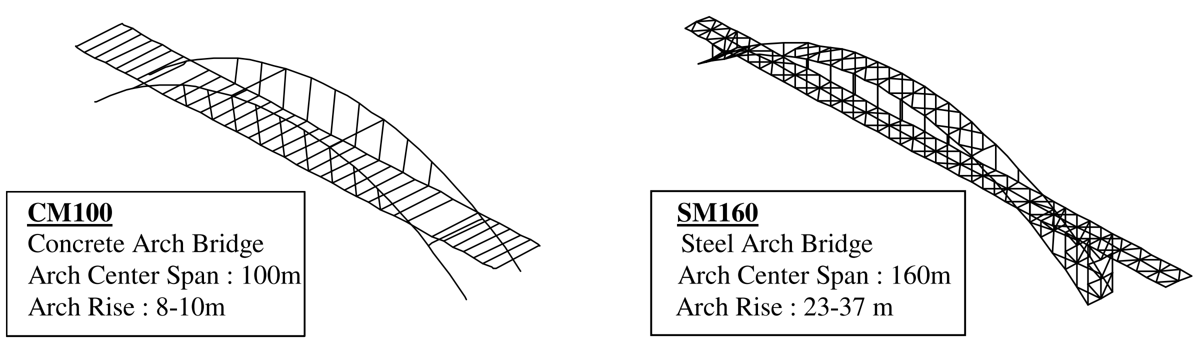
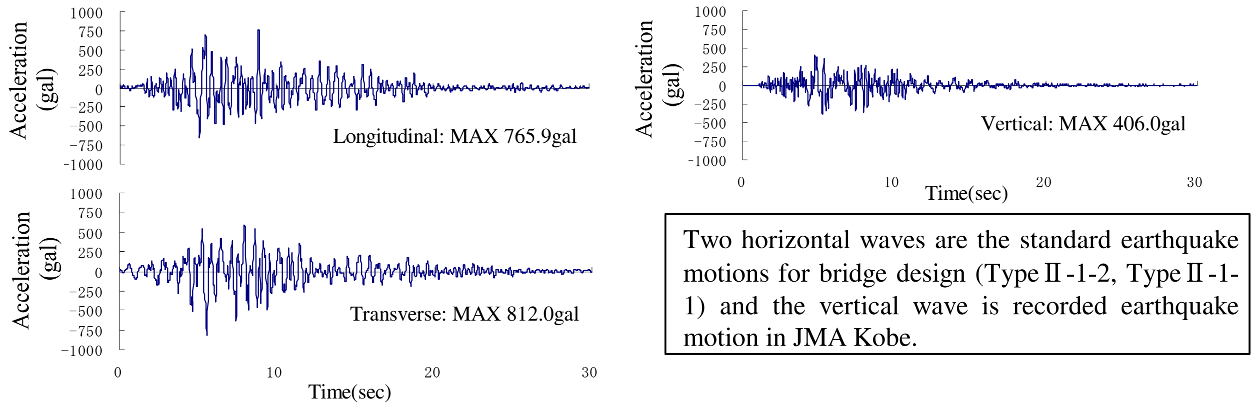


Figure 8: Additional Arch-Bridge Models



Two horizontal waves are the standard earthquake motions for bridge design (Type Ⅱ-1-2, Type Ⅱ-1-1) and the vertical wave is recorded earthquake motion in JMA Kobe.

Figure 9: Input Earthquake Motions (Acceleration Waves)

Performance comparison

Figure 10 shows the time history of CU600's λ_1s in intervals of 0.01 seconds. The minimum λ_1 was 233.9 (in 5.87 seconds). The paired buckling mode with the minimum λ_1 is shown in Figure 11. As it was already seen in the previous chapter, the time history of λ_1s drew the row of irregular V shapes. We focused on the minimum eigen-value that is one of the lowest points of V shapes. So the envelope of the lowest points of V shapes (the dashed line in Figure 10) helps well to check the changes of the dangerousness. The downside envelopes of three models' time history of λ_1s were shown in Figure 12 and their minimum λ_1s were shown in Table 3. All of them didn't fall into buckling in duration but the difference in their anti-buckling-performances could be seen quantitatively with the minimum λ_1s. The capacity is larger in order of CM100, CU600 and SM160. Especially, it attracts attention that SM160's λ_1s bellowed the others' from beginning to last.

The difference in performance seemed to mainly originate in the difference in buckling modes that were coming up in the most dangerous times. CM100's mode (Figure 13) was an overall mode that had no directional variation. CU600's mode (Figure 11) was also an overall mode but it inclined only in the transverse direction. Meanwhile, SM160 showed a local buckling mode in the longest piers (Figure 14). SM160's minimum eigen-value paired with an overall buckling mode shown in Figure 15 was 552.6. It suggested that if some reinforcement were given to lower the possibility of the local buckling, the overall mode would become the first buckling-mode and the performance would be largely improved. In this way, the new method can help to find the reinforcing point of existing structures, too.

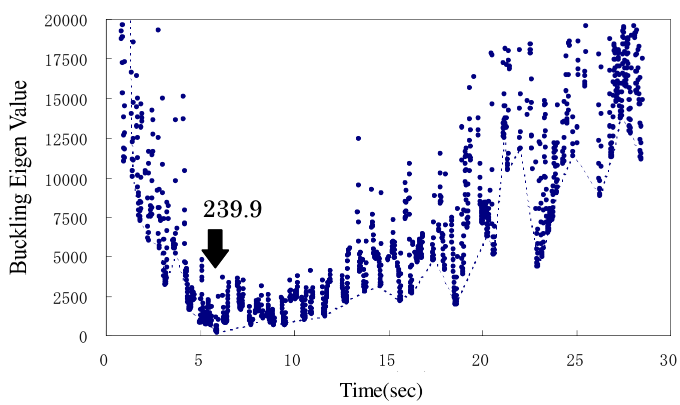


Figure 10: Time History of Eigen Values (CU600)

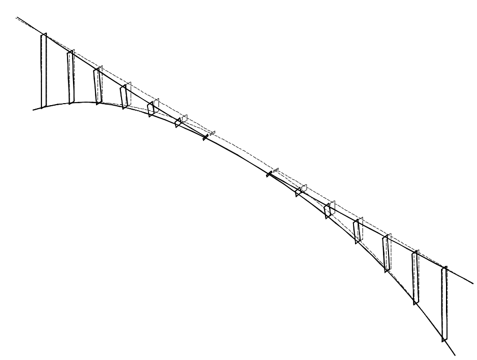


Figure 11: Buckling Mode of CU600 Paired with Minimum Eigen Value

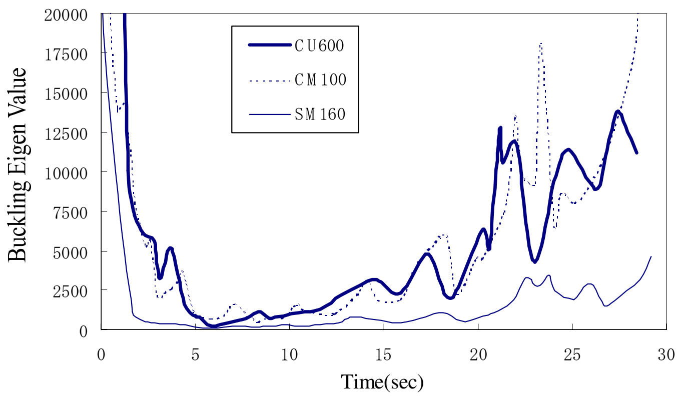


Figure 12: Time History Envelopes of Eigen Values

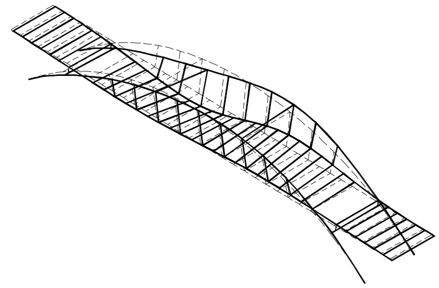


Figure 13: Buckling Mode of CM100 Paired with the Minimum Eigen Value

Table 3: Comparison of Performance Indicators

Model's name	Minimum λ_1 (time)
CU600	239.9 (5.87 seconds)
CM100	493.5 (8.29 seconds)
SM160	75.6 (6.16 seconds)

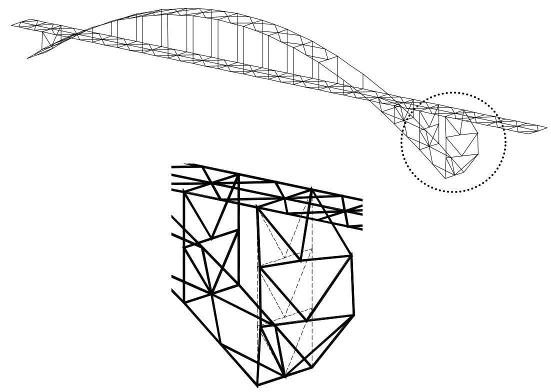


Figure 14: Buckling Mode of SM160 Paired with the Minimum Eigen Value

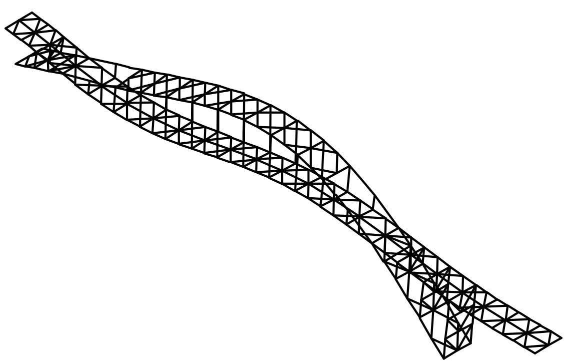


Figure 15: SM160's Overall Mode

CONCLUSIONS AND REMAINED SUBJECTS

1. Existing analytical techniques for buckling under static loads are applicable when a setup of reference loads is explicit. In this case buckling load-factor can be a performance indicator. But these techniques often can't estimate buckling during an earthquake rationally.
2. A new analytical estimation method based on a time history analysis was proposed. The minimum eigen value in all duration obtained from a mixed non-linear time history analysis including repeated buckling-eigen analyses can indicate the buckling (static instability) capacity under an earthquake quantitatively.
3. By some analysis results potential of the new method has been demonstrated. This method can provide the information about not only whether buckling will come up or not in duration, but also whether enough or not to buckling while a structural system is stable. Some rational relations between the seismic-buckling capacities indicated by eigen-values and the characters of buckling modes could be seen in the analysis results.

4. Example analyses showed that buckling in dynamic action becomes to be estimated quantitatively by using eigen-values as indicators. However this index is only for one earthquake condition. More general index based on these eigen values should be considered. And it is expected that accumulated information based on a same index lead to the rational judgment whether a detailed analytical investigation is needed or not.
5. We should show the validity of the proposed method by collation with experimental results later on in the near future.
6. In the viewpoint of the practicality in design, not only analytical techniques, establishment of criteria is needed.
7. 'Dynamic-instability' includes some kinds of phenomenon. We focused on only one of them in this paper. We will study on analytical estimation methods of the other phenomenon, too.

ACKNOWLEDGEMENTS

Our study started from the research activities of the 'Task Committee on Design and Construction of Concrete Arch Bridges with Exceedingly Long Spans' organized in Japan Society of Civil Engineering. Finally, we'd like to express deepest thanks to Professor Tada-aki Tanabe (Nagoya University), the chairman of the committee and the members of the committee for a lot of precious advice.

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