ENERGY BALANCE BASED SEISMIC DESIGN METHOD OF RC STRUCTURES CONSIDERING CYCLIC BEHAVIOR UNDER EARTHQUAKE

Tomohisa MUKAI¹, Hideyuki KINUGASA², Setsurou NOMURA³

SUMMARY

The primary objective of this study is to decide the Required Strength (Py) to control maximum deformation by expressing the intensity of earthquake (seismic demand) with both total input energy (ED) and equivalent number of inelastic cycles (ND), and by applying energy balance method for equivalent Single-Degree-Of-Freedom (SDOF) system. Firstly, dynamic inelastic response is substituted to the stationary response and ND is defined as the amount which total input energy is divided by maximum momentary input energy. Secondly, energy dissipation model of structure is shown in the stationary response and then Py to control the average of maximum deformation is calculated by solving the energy balance. To predict transient response, Maximum Deformation Ratio (MDR) is estimated using both the maximum ductility factor and the hysteretic loop area. As a conclusion of these investigations, it is shown that the assumptions on the model of energy dissipation and the definition of ND (that is, EIV concept) are generally valid. And the accuracy on calculation method of Py is almost high. For further improvement of the accuracy, however, it is important to estimate MDR accurately, particularly in the case of JMAKOBE which is known to have occurred by the fracture of active fault.

INTRODUCTION

In Performance Based Seismic Design, it is important to estimate responses of buildings under earthquake. The maximum response deformation should be the index which represents adequate damages of R/C buildings. Many estimating methods of maximum response deformation were proposed so far, where 'Equivalent Linear Method' is very familiar and widely used. But this method is not able to express the effect of cyclic response and transient response in earthquake. The purpose of this paper is to propose the estimation method on maximum deformation of RC structure modeled by SDOF by applying energy balance method considering cyclic response and transient response. The accuracy of the proposed method is also investigated by performing dynamic analysis.

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CONCEPT OF ENERGY INPUT VELOCITY (EIV)

A structure shows different responses by the type of input earthquake motions. It can be understood if we compare the response of building subjected to strong earthquake occurred in a long-distance seismic center with that subjected to medium earthquake occurred in a near-distance seismic center. In order to consider the type of input motion, we define Energy Input Velocity (EIV [1]) as the index on intensity of energy input shown in Fig.1. The left part of Fig.1 shows time-history response displacement and input energy under earthquake. The random response is replaced to a stationary response with constant displacement amplitude as shown in right part of Fig.1. In actual response, $\Delta \text{Emax}/\Delta T$ represents the intensity of energy input and the value of $\Delta E$ changes in every moment. $\Delta \text{Emax}$ is the maximum value of all $\Delta E$ for the duration. In inelastic response, $\Delta T$ is equivalent period $T_e$ in one cycle corresponding to secant stiffness $K_e$ (Fig.2) given from maximum response points. In the replaced stationary response with constant displacement amplitude, EIV represents intensity of energy input and the value is determined to only one, corresponding to both input earthquake motion and period of structure. EIV is given as eq. (1).

$$\text{EIV} = \frac{\text{ED}}{(\text{ND} \times \Delta T)}$$

(1)

Where: ED is total input energy, ND is equivalent number of inelastic cycles
Assuming that $\Delta \text{Emax}/\Delta T$ is equal to EIV, ND is rewritten by following equation:

$$\text{ND} = \frac{\text{ED}}{\Delta \text{Emax}}$$

(2)

Thus, ND is an index that represents not only the characteristics of input motion (seismic demand), but also the equivalent number of inelastic cycles of structure subjected to earthquake.

Figure 1: Definition on EIV
ESTIMATION METHOD OF REQUIRED STRENGTH

Model of energy dissipation and energy balance:
Energy dissipation model for elasto-plastic R/C structure is shown in Fig. 3 by assuming that time-history response is replaced by stationary response with constant displacement amplitude as mentioned above. ES (Total Dissipation Energy) is comprised of four dissipation energies (Ey, Eds, Ec, Eh) as shown in eq. (3). Ey is elastic strain energy, Eds is plastic one in first one cycle, Ec is plastic one except Eds, and Eh is viscous one. Eds and Ec are calculated using plastic displacement \( \delta_p \) is given by subtracting yield displacement from maximum displacement. In eq. (3), \( \xi \) is energy dissipation capacity coefficient (Fig. 4). It means that when a value of \( \xi \) is large (for example \( \xi = 0.5 \)), the capacity on cyclic hysteretic dissipation energy of structure is high.

![Figure 3: Model of energy dissipation for R/C structure](image)

![Figure 4: Energy dissipation capacity coefficient](image)
ES = Ey + Eds + Ec + Eh

Where:

\( h \) is viscous damping factor

If \( ED \) (Total Input Energy) is equal to \( ES \) as shown in eq. (4), \( Py \) is just the required strength.

\[ ED = ES \] (4)

Eq. (5) is given by solving eq. (4) about \( Py \).

\[ Py = \sqrt{\frac{ED \cdot \text{Key}}{0.5 + 2(\lambda_{ave} - 1) + 4h(ND - 1)(\lambda_{ave} - 1) + 2 \pi h_{ave} \lambda_{ave} ND}} \] (5)

Where: \( \mu_{ave} \) is average ductility factor (average value of positive and negative maximum ductility factor)

Eq. (5) implies that \( Py \) which controls the average ductility factor under input motion can be given. In design, however, it is desirable to calculate \( Py \) which controls the maximum ductility factor (\( \mu_{max} \)) when we consider transient response. Then Maximum Deformation Ratio (MDR) is defined as follows.

\[ MDR = \frac{\mu_{max}}{\mu_{ave}} \] (6)

From the above investigation, it is necessary to estimate \( ED \), \( ND \) and MDR for calculating \( Py \) to control \( \mu_{max} \) under earthquake.

**Estimation methods of ED (Total Input Energy) and ND (Number of Inelastic Cycles), MDR (Maximum Deformation Ratio):**

ED and ND are estimated by using elastic spectra.

ED can be estimated by referring Akiyama’s method [2]. In Fig.5, spectrum of VE which is energy equivalent velocity of ED is shown for each earthquake (EQi is simulated motions). Yield period \( T_{ey} \) is used in elastic response and equivalent period \( T_{e} \) in inelastic response. JMAKOBE has large values in short period (around 1 second), and El Centro has smaller values than JMAKOBE and other motions. A solid line is elastic response value and circle marks are inelastic response values for each viscous damping factor. The marks are almost on the solid line with little influence of viscous damping factor. Thus, ED can be estimated from elastic spectrum.

○ damping 0%(inelastic) —— damping 5%(elastic)

● damping 5%(inelastic)

![Figure 5: VE (equivalent velocity of Total Input Energy) spectrum in 5% damping](image-url)
Fig. 6 indicates ND given by eq. (2). The notes are same to those of Fig. 5. From Fig. 6, value of ND is large in short period range and small in long period one. Especially, in case of JMAKOBE, it shows small values, which indicates the intensity of energy input is severer than other input motions. The circle marks are almost on the solid line with little influence of damping factor. Thus, ND can be also estimated from elastic spectrum by taking account of the fact that period become longer with structure’s plasticity.

![Figure 6: ND (Number of Inelastic Cycles) spectrum in 5% damping](image)

To estimate $\mu_{\text{max}}$ from $\mu_{\text{ave}}$, MDR in eq.(6) that is the difference between maximum response displacement in positive side and in negative side under earthquake, must be determined by taking account of transient response under earthquake. Although the value of MDR is influenced by various factors, it seems that MDR has some relation with $\mu_{\text{max}}$ and $\xi$. Then, to investigate the relationships between MDR and those two factors, residual ductility factor $\mu^*$ (see Fig. 7 (a)) is defined as function of $\mu_{\text{max}}$ and $\xi$ as follows:

$$\mu^* = (\mu_{\text{max}} - 1) / \xi$$  \hspace{1cm} (7)

In Fig. 7 (b), the relation between MDR and $\mu^*$ are shown for each $T_y$ (yield period), $\xi$, input motion. In case of JMAKOBE, MDR has more large value than the others. As a general tendency, in case of $\xi = 0.5$, MDR increases up to the $\mu^* = 1.0$ and keeps at a constant value in a range beyond the value except for a part of JMAKOBE and EQ2. In case of $\xi = 0.125$, MDR is small value in wide range. Focusing the tendency of $\xi = 0.5$, MDR is simplified by eq. (8) and represented by the solid line in Fig. 7 (b).

$$\text{MDR} = 1 + 0.25\mu^*$$ \hspace{1cm} ($\mu^* \leq 1.0$)

$$\text{MDR} = 1.25$$ \hspace{1cm} ($\mu^* > 1.0$)  \hspace{1cm} (8)
**ACCURACY VERIFICATION OF PROPOSED METHOD**

**Procedure to calculate required strength:**
The procedure to calculate required strength (Py) is shown in Fig. 8. Firstly, Tey (yield period) and $\xi$ (energy dissipation capacity coefficient) and h (viscous damping factor) are required as basic structural characteristics of building. When allowable $\mu_{max}$ (maximum ductility factor) is determined, $\mu_{ave}$ (average ductility factor) is given by using eq. (6) and (8). Secondary, it is necessary to calculate $T_{eq}$ (equivalent period) so as to read ED (total input energy) and ND (number of inelastic cycles) in inelastic response from elastic spectrum. The eq. (9) can be applied to $T_{eq}$ when skeleton curve of analysis model is bi-linear and post-yielding stiffness is almost 0.

$$T_{eq} = \text{Tey} \cdot (\mu_{ave})^{0.5}$$  \hspace{1cm} (9)
When $T_e$ was determined from eq. (9), $ED$ and $ND$ are estimated by elastic spectra with viscous damping $h = 0.05$ (Fig.5 and 6). At the same time, total dissipation energy $ES$ of structure is determined. Finally, $Py$ is given by the following energy balance equation ($ED = ES$).

In this chapter, the two verifications on estimation of $Py$ are investigated. The one is verification on validity of energy dissipation (Fig.3) and EIV concept model. The other is verification on accuracy of estimation method (Fig.8). The dynamic response analysis for SDOF systems is carried out for verification and analysis parameters are listed in Table-1. Generally, for RC structure, tri-linear model is frequently used such as Takeda hysteretic model. But it is said that the difference of maximum response displacement is not large between bi-linear and tri-linear model when structure is subjected to strong earthquake. Thus, degrading bilinear model in Fig.2 is used on these investigations. The type of viscous damping is assumed to be proportional to the instantaneous tangential stiffness.

<table>
<thead>
<tr>
<th>Table-1: Analytical parameters</th>
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<tbody>
<tr>
<td>Input Motions</td>
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<tr>
<td>$T_e$ (yield period) (s)</td>
</tr>
<tr>
<td>$\xi$ (energy dissipation capacity)</td>
</tr>
<tr>
<td>$Py$ (yield strength)</td>
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<tr>
<td>$h$ (damping factor) (%)</td>
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Figure 8: Estimation procedure of required strength $Py$. 

<table>
<thead>
<tr>
<th>$T_e$ (yield period), $\xi$ (energy dissipation capacity coefficient), $h$ (damping factor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>determine $\mu_{\max}$ (maximum ductility factor)</td>
</tr>
<tr>
<td>calculate $\mu_{\text{ave}}$ (average ductility factor)</td>
</tr>
<tr>
<td>calculate $T_e$ (equivalent period)</td>
</tr>
<tr>
<td>$ND$ (Number of inelastic cycles)</td>
</tr>
<tr>
<td>$ED$ (Total Input Energy)</td>
</tr>
<tr>
<td>using elastic spectra with 5% damping</td>
</tr>
<tr>
<td>$ES$ (Total Dissipation Energy)</td>
</tr>
<tr>
<td>$ED = ES$</td>
</tr>
<tr>
<td>use eq(5)</td>
</tr>
<tr>
<td>estimation of $Py$ (required strength)</td>
</tr>
</tbody>
</table>

Input Motions: JMAKOBE, El Centro EQ1, EQ2, EQ3 (simulated motions)

$Py_i (i = 1-8)$ is determined so that $\mu_{\max}$ is from 1 to 6.
Accuracy on validity of energy dissipation and EIV concept model:
The validity on model of energy dissipation and EIV concept is verified by using the values, which are MDR and ED and ND, given from dynamic response analysis. The Fig.9 is a part of verification result and relationship between Py/mg (m is mass and g is gravity acceleration) and µmax in case of JMAKOBE in each viscous damping (0 and 5%), Tey, ξ. The estimation values (white circle marks) have suitable relation to the response values (black circle marks) and the tendency of this result was almost same in case of other input motions. Consequently, it is said that the proposed assumption on model of energy dissipation and EIV concept are valid.

![Figure 9: Accuracy on validity of energy dissipation and EIV concept model](image)

Accuracy on estimation method:
According to Fig.8, Py can be calculated by eq. (5). In the calculation, ED and ND are estimated by elastic spectra with viscous damping h = 5% and MDR is given by using eq. (8) and (6). The Fig.10 illustrates the verification result for accuracy on estimation method of ED and ND and MDR. The both axes and all notes are same as those of Fig.10, and the left part of Fig.10 is the case of JMAKOBE (h = 0.05) and the right part is that of El Centro (h = 0.05). The figures show high estimation accuracy except a part of JMAKOBE (Tey = 0.5s), which is due to the not good estimation accuracy for MDR (Fig.7 (b)). For further improvement of the accuracy, it is important to estimate MDR accurately, particularly in the case of JMAKOBE which is known to have occurred by the fracture of active fault.
CONCLUSIONS

This paper proposed the calculation method of Py (required strength) to control a maximum ductility factor for RC structure modeled SDOF. This method is based on energy-balance, where both the cyclic behavior of structure subjected to earthquake and the intensity of energy input to structure are considered. In this method, it is necessary for calculation of Py to estimate ED (Total Input Energy), ND (Number of inelastic cycles) and MDR (Maximum Deformation Ratio) in inelastic response. The estimation procedure of the proposed method was given and the accuracy of the method was verified. The conclusions are as follows.

(1) ED and ND can be estimated by using elastic spectra with viscous damping taking the fact that period become longer with structure's plasticity into consideration. MDR is estimated from maximum ductility factor and $\xi$ (energy dissipation capacity coefficient).

(2) The assumption on model of energy dissipation and EIV concept is valid (Fig.10).

(3) This method has generally enough accuracy except a part of JMAKOBE (Tey = 0.5s) (Fig.11), which means that the accuracy of estimated ED, ND and MDR almost is high. For further improvement of the accuracy, it is important to estimate MDR accurately, particularly in the case of JMAKOBE which is known to have occurred by the fracture of active fault.

REFERENCES