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SHAKING-TABLE TEST ON SEMI-ACTIVE CONTROL FOR SEISMIC RESPONSE OF TALL BUILDINGS USING MRF-04K DAMPER

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SUMMARY

In this paper, a double-ended, shear mode combined with valve mode damper, MRF-04K damper with the maximum force of 20kN, had been designed and manufactured recently. To demonstrate the effectiveness of seismic response control for buildings using MRF-04K damper, a series of shaking-table tests on a 3-story frame model under different seismic input were conducted on the shaking table. And the validity of two passive control situations and semi-active control systems based on three different control algorithms was verified. It can be concluded that the MRF-04K damper is very effective, either two passive control situations or the semi-active control systems are all able to significantly reduce the seismic responses of the model structure.

INTRODUCTION

Magnetorheological (MR) fluid damper, due to its mechanical simplicity, high dynamic range, low power requirement, large and adjustable damping force capacity and robustness, has become one of the most potential vibration control devices for civil structures. The research on its behavior and application to civil structures has been attracted lots of scholastic attention (Spencer [1], Dyke [2], Carlson [3], Hiemenz [4], Gordaninejad [5] and Xu [6]).

In this paper, a double-ended, shear mode combined with valve mode damper, named as MRF-04K damper, had been designed and manufactured in Tianjin University of China, and the completed damper is approximately 0.5m long and with a mass of 50kg and the MR fluid used in this damper is the type of MRF-04K provided by Chongqing Instrument Materials Research Institute of China. The maximum force at a full magnetic field strength is about 20kN while the maximum power required is less than 50w.

To demonstrate the effectiveness of seismic response control for buildings using MRF-04K damper, a series of shaking-table tests on a 3-story frame model under different seismic input were carried out. The validity of two passive control situations, in which the applied current was fixed at the minimum value of 0A and the maximum value of 2.0A respectively, and semi-active control systems based on three different

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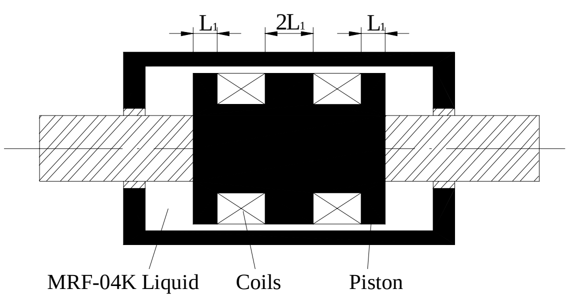
control algorithms, IOC (Instantaneous Optimal Control) algorithm, COC (Classical Optimal Control) algorithm and LQG (Linear Quadratic Gaussian) control problem, was verified.

SHEAR-VALVE MODE MRF-04K DAMPER

To take full use of the advantages of MR fluid to devices, a double-ended, shear mode combined with valve mode MRF-04K damper model has been designed and manufactured in Tianjin University. Figure 1 shows the picture of MRF-04K damper, in which the electromagnetic coils, containing the wire of about 500m and wired in series, are wounded in two sections on the piston, which results the effective magnetic poles of 8cm. The damper has an inside diameter of 12.5cm and a stroke of ±4cm, and the completed damper is approximately 0.5m long and with a mass of 50kg and the MR fluid used in this damper is the type of MRF-04K provided by Chongqing Instrument Materials Research Institute of China. The maximum force at a full magnetic field strength is about 20kN. The responses of the MRF-04K damper due to 2.0Hz sinusoid with amplitude of 10mm are shown in Figure 2 for five constant current levels, 0A, 0.5A, 1.0A, 1.5A and 2.0A.



(a) Construction

(b) Outside view

Figure 1. MRF-04K damper

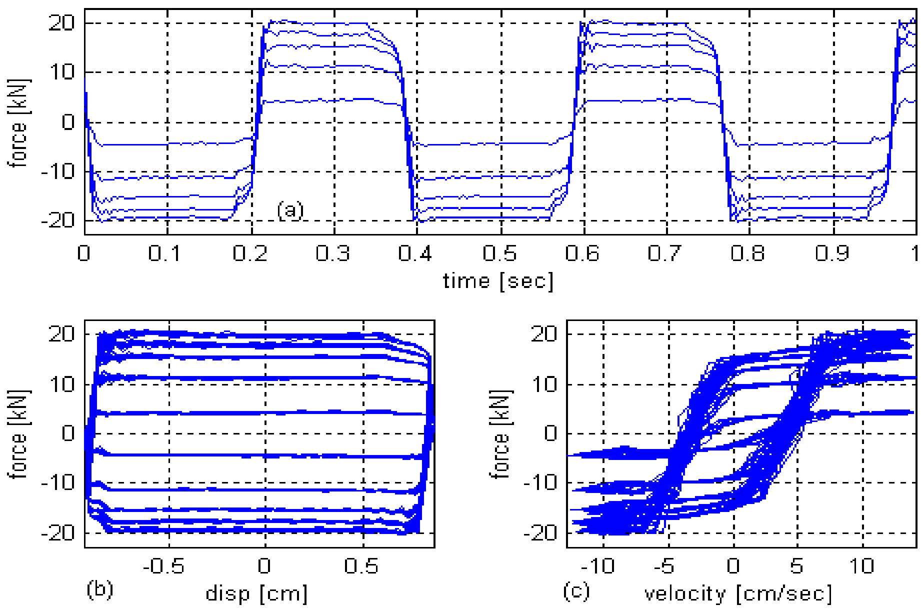


Figure 2. Measured force responses: (a) versus time, (b) versus displacement, (c) versus velocity

CONTROL SYSTEMS DESIGN

For the vibration control systems design, the key problem is how to determine the optimal control action $\mathbf{u}(t)$, while the control algorithms are the basis. Consider an *n*-degree of freedom linear building with *r* control devices subjected to seismic excitation $\ddot{u}_g(t)$, the equation of motion in state-space form can be written as,

$$\dot{\mathbf{z}}(t) = \mathbf{Az}(t) + \mathbf{Bu}(t) + \mathbf{W}\ddot{u}_g(t) \tag{1}$$

where the $2n \times 1$ state vector $\mathbf{z}(t)$, the $2n \times 2n$ system matrix $\mathbf{A}$, the $2n \times r$ input matrix $\mathbf{B}$ and the $2n \times 1$ disturbance matrix $\mathbf{W}$ are, respectively, given by,

$$\mathbf{z}(t) = \begin{Bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{Bmatrix} \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I_n} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{E} \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{I_1} \end{bmatrix} \tag{2}$$

where $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are respectively the *n*×*n* mass, stiffness and damping matrices, $\mathbf{x}(t)$ and $\dot{\mathbf{x}}(t)$ are respectively the $n \times 1$ displacement and velocity vectors, $\mathbf{u}(t)$ is the $r \times 1$ control action, $\ddot{u}_g(t)$ is the external excitation and the $n \times r$ matrix $\mathbf{E}$ is location matrix of controllers, $\mathbf{I_1}$ and $\mathbf{I_n}$ denote, respectively, the $n \times 1$ and $n \times n$ unity matrices.

Instantaneous Optimal Control (IOC)

Different control algorithm based on different control design criteria has its own feature. In IOC algorithm, the time-dependent performance index $J(t)$ is chosen as follows,

$$J(t) = \mathbf{z}^T(t)\mathbf{Qz}(t) + \mathbf{u}^T(t)\mathbf{Ru}(t) \tag{3}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are respectively the $2n \times 2n$ and $r \times r$ weighting matrices, which define the trade-off between regulation performance and control effort.

Minimizing equation (3) subject to the constraining equation (1), and the closed-loop control law is adopted, the optimal control force $\mathbf{u}(t)$ is solved as (Soong [7]),

$$\mathbf{u}(t) = -\frac{\Delta t}{2}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{Qz}(t) = -\mathbf{Gz}(t) \tag{4}$$

where Δt is the time interval, the control gain $\mathbf{G}$ is,

$$\mathbf{G} = \frac{\Delta t}{2}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{Q} \tag{5}$$

IOC control algorithm is a very simpler control design, which does not require solution of the Riccati equation.

Classical Linear Optimal Control (COC)

In COC algorithm, the quadratic performance index $J(t)$ is defined as,

$$J = \int_0^{t_f} [\mathbf{z}^T(t)\mathbf{Qz}(t) + \mathbf{u}^T(t)\mathbf{Ru}(t)]dt \tag{6}$$

where t_f represents the duration of excitation. Minimizing equation (6) subject to the constraining equation (1), and the closed-loop control law is adopted, the optimal control vector $\mathbf{u}(t)$ is gained as (Soong [7]),

$$\mathbf{u}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}(t)\mathbf{z}(t) = -\mathbf{Gz}(t) \tag{7}$$

where the control gain $\mathbf{G}$ is,

$$\mathbf{G} = \frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P}(t) \tag{8}$$

It has been proved that the Riccati matrix $\mathbf{P}(t)$ remains constant over the control interval $[0, t_f]$, which is the solution of following Riccati equation,

$$\mathbf{PA} - \frac{1}{2}\mathbf{PBR}^{-1}\mathbf{B}^{T}\mathbf{P} + \mathbf{A}^{T}\mathbf{P} + 2\mathbf{Q} = \mathbf{0} \tag{9}$$

Linear-Quadratic Gaussian (LQG) Control

From equation (4) and (7), it can be seen that the optimal control force can not be implemental without full state measurement, that is all the displacement and velocity responses of building can be measured, so the application of control systems to civil structure will be difficult. The LQG control strategy, only require the acceleration of building as the feedback information, will be presented. Figure 3 illustrates the LQG control problem, the system is assumed to be linear, time-invariant and single-input, single-output. The external disturbance w and the measurement noise $\mathbf{v}$ are assumed to be Gaussian white noise processes, and their covariance matrices $\mathbf{W_1}$ and $\mathbf{V}$ are assumed to be uncorrelated. The above assumption is just for the convenience of control law design, but the actual seismic loads are not exactly Gaussian white noise processes. The controlled plant is actuated by the external disturbance w and the control action $\mathbf{u}$ produced by the controller according to the measurement signal like the following equation,

$$\mathbf{y_v}(t) = \mathbf{y}(t) + \mathbf{v}(t) \tag{10}$$

In civil engineering applications, the acceleration measurements are easier to obtain, so here the acceleration of structure is used as the feedback, the output equation can be written as,

$$\mathbf{y}(t) = \mathbf{C_a z}(t) + \mathbf{D_a u}(t) + \mathbf{W_a}\ddot{u}_g(t) \tag{11}$$

where $\mathbf{y}(t)$ represents the output acceleration response vector, $\mathbf{v}(t)$ is the measurement noise vector, and,

$$\mathbf{C_a} = [-\mathbf{M}^{-1}\mathbf{K} \quad -\mathbf{M}^{-1}\mathbf{C}] \qquad \mathbf{D_a} = \mathbf{M}^{-1}\mathbf{E} \qquad \mathbf{W_a} = \mathbf{M}^{-1}\mathbf{F} \tag{12}$$

where $\mathbf{F}$ is the n×1 location matrix of the excitation.

LQG control method is a modern state-space technique for designing optimal dynamic regulators, which can be decomposed into two problems according to the separation theorem (Michael [8]): (i) design of a stable state-feedback controller, and (ii) design of a nonlinear observer.

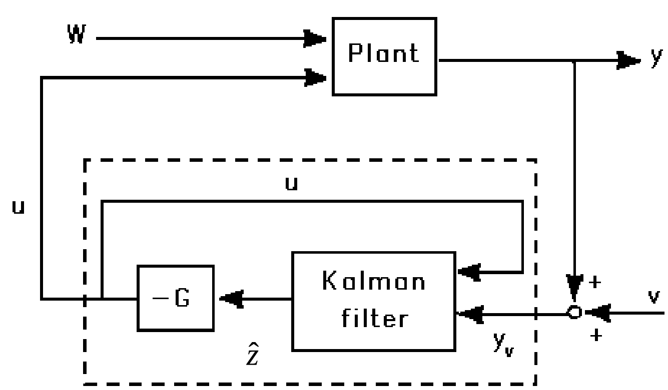


Figure 3. Illustration of LQG control problem

Design of controller

The LQG control problem is to devise a control law with constant gain

$$\mathbf{u}(t) = -\mathbf{G}\mathbf{z}(t) \tag{13}$$

to minimizes the following quadratic cost function,

$$J = \frac{1}{2}\lim_{T\to\infty} E\left\{\int_0^T (\mathbf{z}^T(t)\mathbf{Q}\mathbf{z}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t))dt\right\} \tag{14}$$

The solution of the problem is (André [9]),

$$\mathbf{G} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{15}$$

where **P** is the symmetric positive definite solution of the algebraic Riccati equation,

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} + \mathbf{Q} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} = \mathbf{0} \tag{16}$$

Design of observer

The optimal controller (13) is not implemental without the full state measurement. However, we can derive state estimate $\hat{\mathbf{z}}$ such that $\mathbf{u}(t) = -\mathbf{G}\hat{\mathbf{z}}(t)$ remains optimal based on the measurements of the accelerations. This state estimate is generated by the Kalman filter,

$$\begin{aligned}\dot{\hat{\mathbf{z}}}(t) &= \mathbf{A}\hat{\mathbf{z}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}(\mathbf{y}_\mathbf{v}(t) - \mathbf{C}_\mathbf{a}\hat{\mathbf{z}}(t) - \mathbf{D}_\mathbf{a}\mathbf{u}(t)) \\ &= (\mathbf{A} - \mathbf{L}\mathbf{C}_\mathbf{a})\hat{\mathbf{z}}(t) + (\mathbf{B} - \mathbf{L}\mathbf{D}_\mathbf{a})\mathbf{u}(t) + \mathbf{L}\mathbf{y}_\mathbf{v}(t)\end{aligned} \tag{17}$$

in which $\hat{\mathbf{z}}(t)$ is the estimate of the state $\mathbf{z}(t)$, and $\mathbf{y}(t)$ is the output vector. The filter gain **L** is,

$$\mathbf{L} = \mathbf{P}_\mathbf{1}\mathbf{C}_\mathbf{a}{}^T\mathbf{V}^{-1} \tag{18}$$

The estimation error variance $\mathbf{P}_\mathbf{1}$ is a solution of the filter Riccati algebraic equation,

$$\mathbf{AP}_\mathbf{1} + \mathbf{P}_\mathbf{1}\mathbf{A}^T + \mathbf{WW}_\mathbf{1}\mathbf{W}^T - \mathbf{P}_\mathbf{1}\mathbf{C}_\mathbf{a}{}^T\mathbf{V}^{-1}\mathbf{C}_\mathbf{a}\mathbf{P}_\mathbf{1} = \mathbf{0} \tag{19}$$

where $\mathbf{W}_\mathbf{1}$ and $\mathbf{V}$ are covariance matrices of the noise *w* and **v**, respectively. For simplicity, we restrict our attention to the case where $\mathbf{W}_\mathbf{1}$ and $\mathbf{V}$ are constant matrices.

The full state-space equation of LQG control strategy is as follows,

$$\begin{aligned}&\dot{\hat{\mathbf{z}}}(t) = (\mathbf{A} - \mathbf{L}\mathbf{C}_\mathbf{a})\hat{\mathbf{z}}(t) + (\mathbf{B} - \mathbf{L}\mathbf{D}_\mathbf{a})\mathbf{u}(t) + \mathbf{L}\mathbf{y}_\mathbf{v}(t) \\ &\mathbf{u}(t) = -\mathbf{G}\hat{\mathbf{z}}(t)\end{aligned} \tag{20}$$

In practical application, $\mathbf{y}(t)$ in equation (11) should be replaced by the real measured acceleration vector.

For IOC, COC, and LQG control algorithms, the weighting matrices are selected as follows,

$$\mathbf{Q} = \begin{bmatrix}\mathbf{K} & \mathbf{C}/2 \\ \mathbf{C}/2 & \mathbf{M}\end{bmatrix} \qquad \mathbf{R} = \alpha\mathbf{I}_2 \tag{21}$$

where α is a coefficient, and $\mathbf{I}_2$ is $r \times r$ unity matrix.

Control Law Design

In practical application, the control force $f(t)$ produced by the MRF-04K damper can be controlled by adjusting the current $i(t)$ applied to the DC power supply connected with the damper, so the command signal $i(t)$ is selected as follows,

$$i(t) = \begin{cases} i_{\max} & \text{while } |f(t)| < |u(t)| \\ 0 & \text{other}\end{cases} \tag{21}$$

where $i_{\max}$ is the maximum applied current, for MRF-04K damper $i_{\max} = 2.0$ A. If the magnitude of the force $f(t)$ produced by the MRF-04K damper is smaller than the magnitude of the desired force $u(t)$ calculated by control algorithms at time t, the current applied to the damper is increased to the maximum level, otherwise, the command current is set to zero.

SHAKINGTEST TABLE TEST ON SEISMIC RESPONSE CONTROL

To demonstrate the effectiveness of seismic response control for buildings using MRF-04K damper, a series of shaking-table tests on a 3-story, single bay, steel frame model under different seismic input were carried out. Figure 4 shows the model structure test control system, MRF-04K damper is placed between the ground and first floor. Accelerometers located on the shaking table and each floor of model structure provide measurements of the absolute accelerations, and a LC0502 force transducer is placed in series with MRF-04K damper to measure the control force being applied to the structure.

During the experiment, the measured signal was sent to the Charge Amplifier, after shaping and filtering the displacement and velocity responses were calculated by integral circuit. They were sent to the control computer through the A/D conversion board of PCI-1710 multifunction data acquisition (DAS) card, according to the control algorithms the control command was calculated and sent to the PCLD-786 relay driver board to drive the switch of relay, which was used to control the on-off of applied current circuit made up of a DC power supply and MRF-04K damper, through the digital output (DO) channel of PCI-1710 DAS card. The control software was written in Visual c^{++}6.0, and the test was conducted on a 3m×3m horizontal and unilateral shaking table.

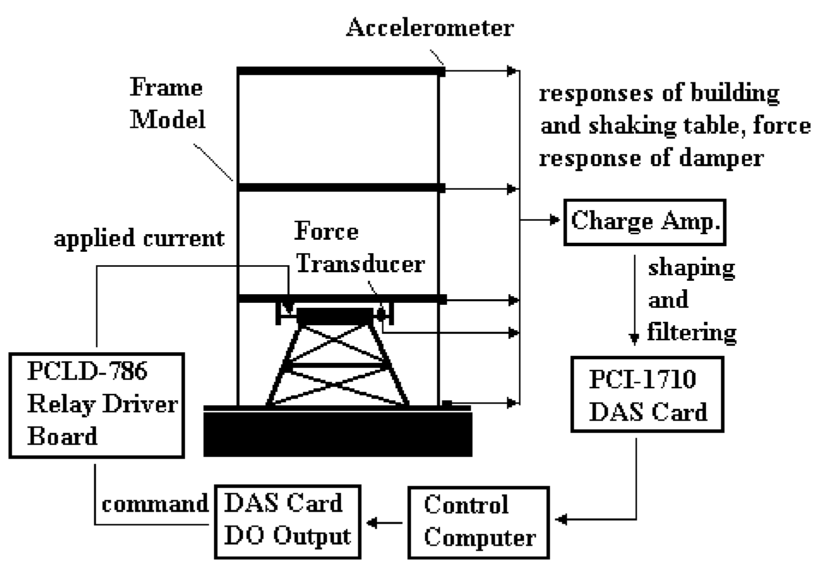


Figure 4. Test control system

Model Structure

The test structure used in this experiment is a 3-story, single bay, steel frame model with the plane size of 1.196m×1.096m and the story height of 1.1m, shown in Figure 5. In order to estimate the system characteristics, the bare frame of the experimental building was first tested on the shaking table under random excitation with the range of frequency from 1Hz to 50Hz and the peak acceleration of 0.15g. The amplitude frequency characteristics curve of the roof displacement response was used to identify both the stiffness matrix and the damping matrix of the system. Adding 320 kg mass on each floor, and the identified structural parameters are shown in Table 1.

Figure 5. Model structure on the Shaking-Table

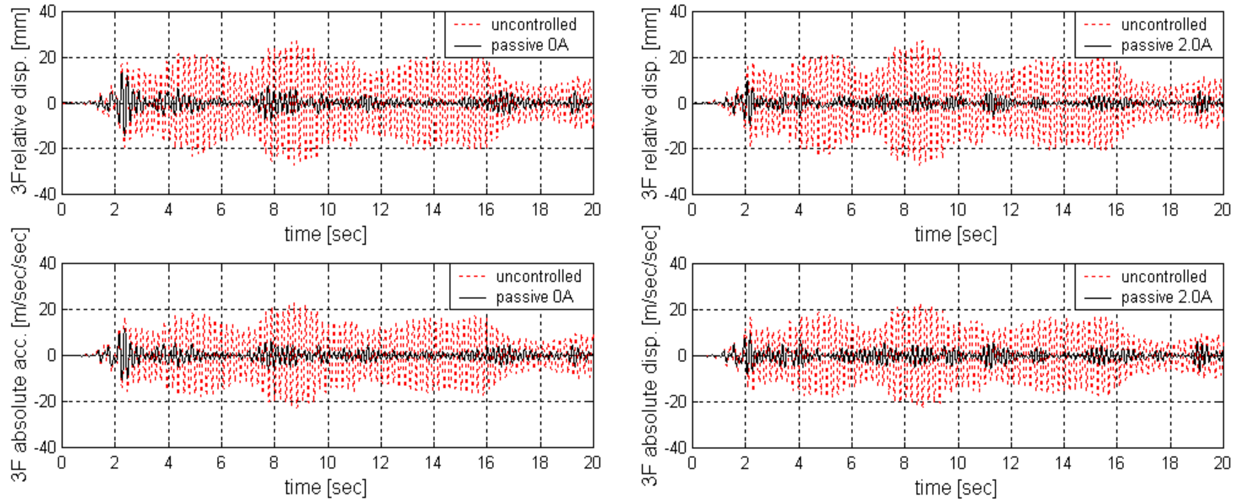
Table 1. Parameters of the 3-story model frame structure

Mass matrix (kg)	Stiffness matrix (10^6N/m)	Damping matrix (N.s/m)
$\begin{bmatrix} 443 & 0 & 0 \\ 0 & 443 & 0 \\ 0 & 0 & 428 \end{bmatrix}$	$\begin{bmatrix} 3.969 & -1.984 & 0 \\ -1.984 & 3.969 & -1.984 \\ 0 & -1.984 & 1.984 \end{bmatrix}$	$\begin{bmatrix} 507.99 & -176.94 & 0 \\ -176.94 & 507.99 & -176.94 \\ 0 & -176.94 & 325.83 \end{bmatrix}$
Natural frequency (Hz)	4.79, 13.67, 18.95	

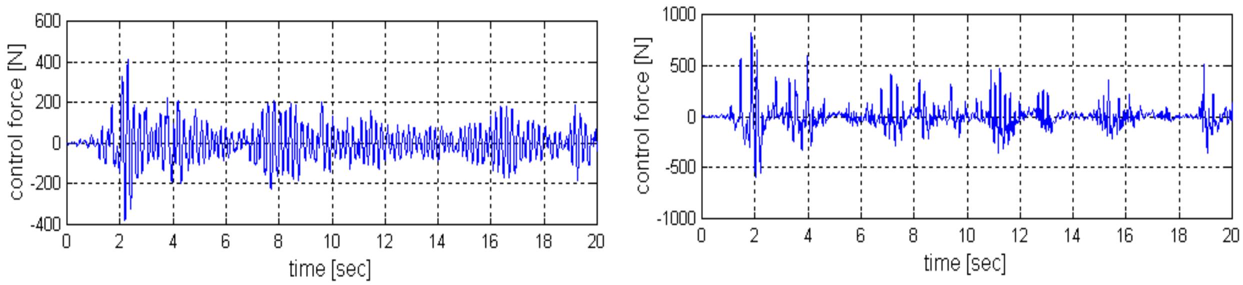
Test Results

During the shaking table tests, three typical earthquake records, El-Centro wave (1940, NS), Taft wave (1952, SE), and Tianjin wave (1976, EW), all scaled to the peak acceleration of 0.15g, 0.30g and 0.40g, were used as input ground excitations. And the validity of two passive control situations, in which the current applied to the MRF-04K damper was fixed at the minimum value of 0A and the maximum value of 2.0A respectively, and semi-active control systems based on IOC algorithm, COC algorithm, and LQG control algorithm only using the accelerations of structure as the feedback information, was examined. Limited by the paper, here only give parts of test results.

Figure 6 to 9 shows the time histories of the roof absolute acceleration and relative displacement of test structure subjected to El-Centro earthquake excitation with the acceleration normalized to 0.4g, and the time history of control force produced by MRF-04K damper and applied current for different control situations. And the response values and control effectiveness of the roof floor subjected to 0.4g different seismic excitations are shown in Table 2.

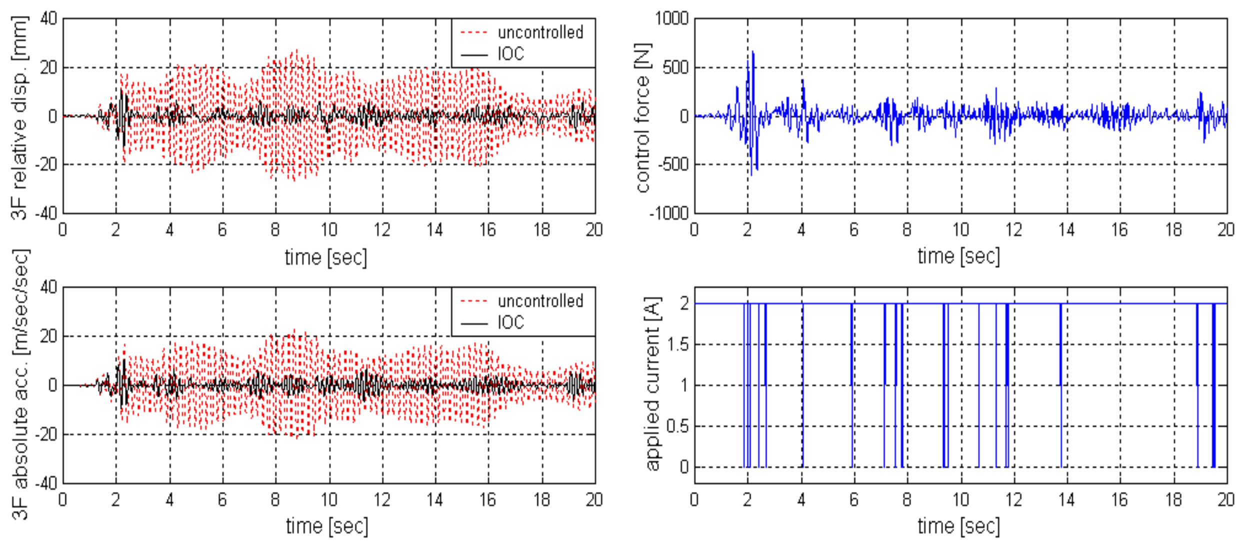


(a) Roof responses for 0A **(b)** Roof responses for 2.0A



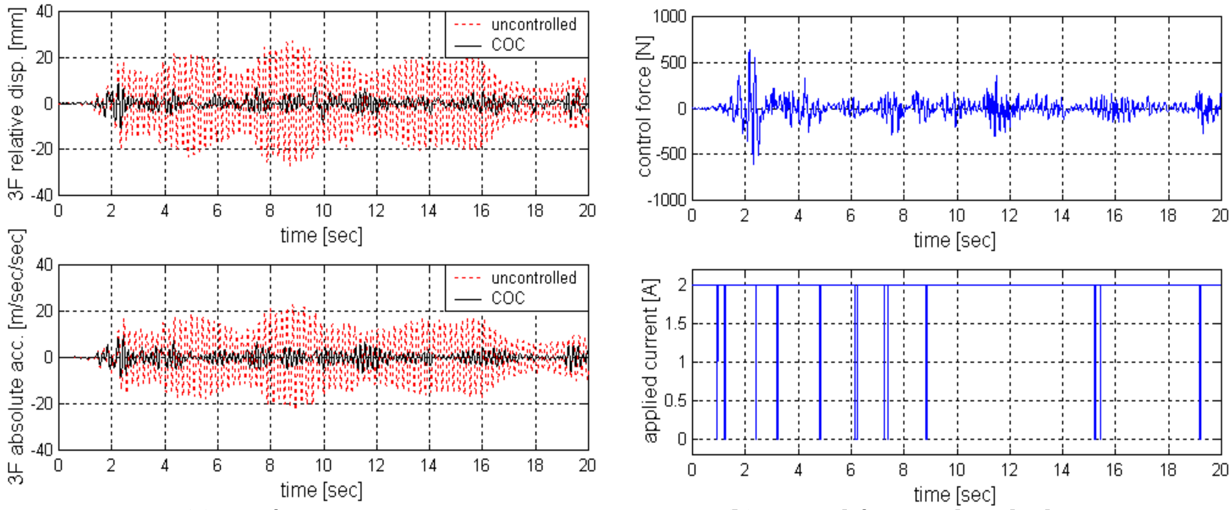
(c) Control force for 0A **(d)** Control force for 2.0A

Figure 6. Roof responses and control force for passive control with current 0A or 2.0A



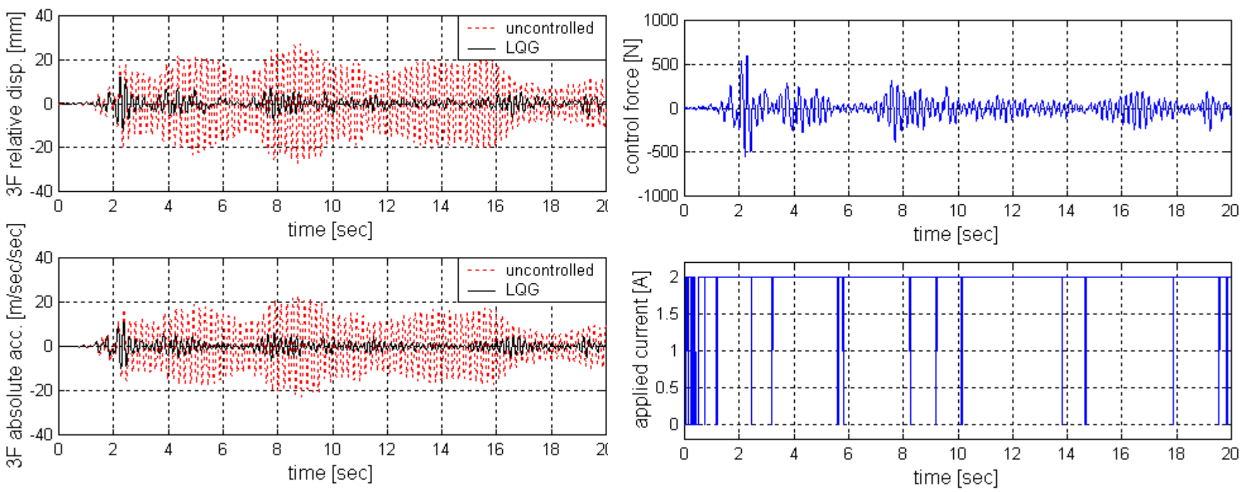
(a) Roof responses **(b)** Control force and applied current

Figure 7. Roof responses, control force and applied current for IOC semi-active control



(a) Roof responses **(b)** Control force and applied current

Figure 8. Roof responses, control force and applied current for COC semi-active control



(a) Roof responses **(b)** Control force and applied current

Figure 9. Roof responses, control force and applied current for LQG semi-active control

From Figure 6-9 and Table 2, it can be seen that the MRF-04K damper is very effective, either two passive control situations or the semi-active control systems based on different control algorithms are all able to significantly reduce the seismic responses of the model structure.

In most cases of these tests the controlled system achieved more than 50% reduction in the peak responses, and more than 70% reduction in the RMS (root of mean square) responses over the uncontrolled system (without MRF-04K damper), while the semi-active control systems can effectively utilize the behavior of MRF-04K damper and achieve a better control effectiveness, and less control forces are needed compared with the case of passive 2.0A. Especially, the LQG controller, in which only the acceleration responses of test structure were used as the feedback, can reach almost the same control effectiveness as using IOC and COC control algorithms with the full state feedback, which make the realization of semi-active control system more simple and enhance the feasibility of applying the seismic responses control systems based on MRF-04K damper to civil structures.

Table 2. Roof responses and its reduction of the model structure and the maximum control force for different control cases due to 3 earthquakes with accelerations normalized to 0.4g

Cases	Excitation	Roof relative displacement [mm]		Roof absolute acceleration [m/s^2]		Reduction of relative disp. [%]		Reduction of absolute acce. [%]		Maximum control force [N]
		Peak	RMS	Peak	RMS	Peak	RMS	Peak	RMS	
Without control	El Centro	27.20	11.41	22.75	9.34	-	-	-	-	-
	Taft	28.52	10.01	23.45	7.86	-	-	-	-	-
	Tianjin	35.44	7.58	21.07	5.32	-	-	-	-	-
Passive 0A	El Centro	14.00	2.45	11.93	2.11	48.51	78.56	47.56	77.45	413.9
	Taft	13.49	3.09	11.31	2.57	52.70	69.11	51.76	67.33	352.9
	Tianjin	15.70	2.73	10.25	2.06	55.70	64.05	51.32	61.22	557.4
Passive 2.0A	El Centro	9.65	1.89	9.18	2.13	64.52	83.41	59.64	77.17	821.2
	Taft	9.79	2.72	10.01	3.00	65.67	72.86	57.31	61.83	753.2
	Tianjin	10.76	2.08	8.87	1.69	69.64	72.59	57.88	68.25	518.5
IOC Semi-active control	El Centro	12.47	2.34	10.69	2.21	54.15	79.47	53.01	76.36	667.3
	Taft	10.40	3.00	10.0	2.78	63.55	70.02	57.35	64.67	638.7
	Tianjin	11.41	2.02	8.32	1.56	67.81	73.30	60.53	70.58	505.3
COC semi-active control	El Centro	10.98	2.39	9.46	2.24	59.63	79.15	58.43	76.02	638.2
	Taft	10.0	2.81	10.21	2.89	64.93	71.87	56.47	63.31	613.8
	Tianjin	11.09	2.02	8.48	1.58	68.71	73.36	59.75	70.19	508.5
LQG semi-active control	El Centro	12.15	2.43	11.14	2.04	55.34	78.68	51.05	78.14	600.3
	Taft	11.31	2.89	9.53	2.39	60.35	71.09	59.34	69.66	502.4
	Tianjin	14.55	2.52	9.64	1.89	58.95	66.71	54.24	64.40	480.6

CONCLUSIONS

MR fluid has the distinguished ability to change its rheological properties in a rapid and reversible manner on application of an external magnetic field. To take full use of the advantages of MR fluid to devices, a double-ended, shear mode combined with valve mode MRF-04K damper model had been designed and manufactured, the maximum force at a full magnetic field strength is about 20kN while the maximum power required is less than 50w.

To demonstrate the effectiveness of seismic response control for buildings using MRF-04K damper, a series of shaking-table tests on a 3-story frame model under different seismic input were conducted on a 3m×3m horizontal and unilateral shaking table. In these tests, the validity of five control cases, passive 0A, passive 2.0A, semi-active control base on IOC algorithm, COC algorithm, and LQG algorithm using the accelerations of test structure as the feedback, were verified.

In most cases of these tests the controlled system achieved more than 50% reduction in the peak responses, and more than 70% reduction in the RMS responses over the uncontrolled system, while the semi-active control systems can effectively utilize the behavior of MRF-04K damper and achieve a better control effectiveness, and less control forces are needed compared with the case of passive 2.0A.

Especially, the LQG controller, in which only the acceleration responses of test structure were used as the feedback, can reach almost the same control effectiveness as using IOC and COC control algorithms with the full state feedback, which make the realization of semi-active control system more simple and enhance the feasibility of applying the seismic responses control systems based on MRF-04K damper to civil structures.

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REFERENCES

1. Spencer Jr BF, Carlson JD, Sain MK, Yang, G. "On the Current Status of Magnetorheological Dampers: Seismic Protection of Full-Scale Structures". Proc. American Control Conf., Albuquerque, New Mexico, USA, 1997:458-462.
2. Dyke SJ, Spencer Jr BF, et al. "Seismic Response Reduction using Magnetorheological Dampers". Proc. IFAC World Congress, San Francisco, CA, USA,1996:331-336
3. Carlson JD, Spencer Jr BF. "Magneto-Rheological Fluid Dampers for Semi-Active Seismic Control". Proc. 3rd Int. Conf. on Motion and Vib. Control, China, Japan, 1996:III, 35-40.
4. Hiemenz GJ, Choi YT, Wereley NM. "Seismic Control of Civil Structures Utilizing Semi-active MR Braces". Comput-aided CIV INF, 2003:18(1), 31-44.
5. Gordaninejad F, Saiidi M, et al. "Magneto-rheological Fluid Dampers for Control of Bridges". J Intel. Mat. Syst. Str., 2002:13(2-3), 167-180.
6. Xu LH, Li ZX, Zhou Y. "Optimum And Simulation of Semi-active Control System Based on MRFD." Spencer Jr BF, Hu YX editors, Earthquake Engineering Frontiers in the New Millennium, Lisse: A.A.Balkema Publishers, 2001:317-320.
7. Soong TT. Active Structural Control: Theory And Practice. Essex: Longman Scientific and Technical, 1990.
8. Michael GS. Stability And Robustness of Multivariable Feedback Systems. Cambridge: The MIT Press, 1980
9. André P. Vibration Control of Active Structures. Boston: Kluwer Academic Publishers, 1997