NON-LINEAR ANALYSIS OF SOIL-PIPELINE INTERACTION IN UNSTABLE SLOPES

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SUMMARY

The interaction occurring between buried pipes and surrounding soil during slope movements is investigated. This allows to assess the pipe failure risk, especially during a seismic event when many unstable slopes exhibit large movements. Generally, the analysis of laterally loaded pipes is carried out using an elastic theory or a rigid-plastic behaviour. However, it is known that the soil shows a non-linear behaviour even at small strain so an elasto-plastic model should be assumed for a better evaluation of soil-pipe interaction and for design. A non-linear model to analyze the soil-pipe interaction in a sliding slope introducing non-linear load transfer $p-y$ curves will be presented. The results of the analytical solution proposed, are exposed in a dimensionless form for different soil parameters.

INTRODUCTION

Pipelines for the transportation of liquid fuel, gas, oil, etc. sometimes cross unstable slopes, where earthquake-induced slope movements can easily take place. These movements can induce strains and stress in the pipe which can compromise the integrity and the safety of the structure. The aim of the pipe stress prediction is to assess the failure risk and eventually to plan monitoring. The evaluation of seismic response of buried pipelines will depend on various factors, such as the direction of the ground movement, the entity of the earthquake-induced slope motion, the dimension and the stiffness of the pipe. The study of the soil-pipe interaction presents a considerable complexity for the number of parameters to be introduced in the theoretical analysis and for the great difference in stiffness and strength characteristics of the soil-pipe system. During the soil movement of the slope, the relative displacement between soil and pipe increases and a load is transferred to the pipe (figure 1). However, because of the elasto-plastic behaviour of the soil-pipe system, this load cannot increase indefinitely with the increasing of the slope movement thus an ultimate value for the load will be achieved; this occurs when a limit value for the relative displacement between soil and pipe is attained.

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Many researches have been carried on to evaluate the soil-pipe response due to slope motion. An extensive research program was developed by Bruschi et al. [1] and [2] with the aim to study interaction phenomena occurring between pipelines and the surrounding soil in slowly deforming slopes. The analysis of a real case of soil-pipe interaction was performed using a numerical model, after a field and laboratory testing programme. The soil-pipe interaction analysis is often carried on with a finite element discretization of the pipe and a non-linear spring-type model to represent the soil response.

Lateral force-displacement response of buried pipes can be modeled with the method of the load transfer curves that relates the relative displacement between soil and pipe ($y_s-y$) to the applied load. However some uncertainties arise in the evaluation of such load transfer curves which, in an elastic–perfectly plastic formulation, is characterized by only two parameters i.e. by the modulus of sub-grade reaction and by the ultimate load.
Audibert and Nyman [3] gave indication for the evaluation of the modulus of sub-grade reaction are for the design of buried conducts. Various models for evaluating the maximum lateral resistance of buried pipes have been proposed. Trautmann and O’Rourke [4] and [5] presented the results of an experimental program to assess the response of buried pipes to lateral movements. In the technical literature analytical solutions for beams on elasto-plastic foundation were developed and characteristic dimensionless load-displacement and stress-displacement relationships were given. The results were presented in a form of dimensionless charts for a fast design of pipelines or piles. Rajani and Mongenstern [6] remarked that similar interactions are present in a variety of situations like a laterally loaded pile embedded in a stiff soil, a pipeline subjected to fault movement, a pipeline subjected to landslide movement and so on. According to the load transfer functions, the ground around the pipeline can be modeled, in the most general case, in three space directions with non-linear springs for each direction. Two springs are normal to the pipe axis, while the spring parallel to the pipe axis supplies the force in the longitudinal force (figure 2). In general, the problem should be considered in all the space directions, but in most practical applications only a direction normal to the pipe axis can be considered. Thus in this paper only the case where the ground motion is parallel to the slope will be analyzed. No movements will be considered in the direction of the pipeline axis.

The evaluation of seismic response of pipelines can be divided into two different steps:

i) Prediction of the slope permanent displacements caused by the seismic loading;

ii) Evaluation of displacement, rotation and bending moment distribution along the pipe subjected to the normal load induced by the slope movement.

The determination of the permanent displacements in the slope caused by seismic action, can be carried out using the Newmark [7] method. To evaluate the pipe response to the ground motion, the load transfer method is adopted. This approach is similar to those of a lot of other soil-structure interaction problems, for example, laterally loaded pile in elastic medium (Matlock and Reese [8] and Matlock [9]), or in an elastic-perfectly plastic soil (Motta [10] and Motta [11]). The ground around the pipeline is then modeled by a series of springs in the direction of the slope movement, with non-linear behaviour. The method gives analytical solutions in a closed form, by solving a fourth order differential equation.

SEISMIC INDUCED DISPLACEMENTS IN SLOPES

The soil-pipe interaction considered in this paper is limited to a ground motion perpendicular to the direction of the pipeline axis. The forces transmitted by the surrounding soil to the pipe depend on the amount of the ground movement. If the inertial effects prevail on the soil shear strength, an instability occurs, characterized by an accumulation, during the seismic motion, of permanent deformations. In this case the sliding block model introduced by Newmark, can be suitably used for the prediction of permanent displacements in the slope.

The earthquake-induced instability which can take place in slopes during a seismic event may have a limited cross-sectional extension so that the boundary effects can be significant and the slope stability as well as the slope movements, should be evaluated with a three dimensional analysis.

In the Newmark method, the relative displacement between unstable and stable mass occurs when the value of the seismic acceleration exceeds the critical value. The critical acceleration represents the value of the acceleration that brings the slope to a limit condition along the potential failure surface. This method can be applied using accelerograms recorded during severe earthquakes occurred in the reference region. Baziar [12] confirmed the general validity of such approach showing that the scheme of the slope constituted by a single rigid block supplies values more conservative for the critical acceleration. For a given seismic acceleration, the evaluation of permanent displacements needs the solution of a differential equation of motion or, alternatively, a finite difference approach.

The value of the critical acceleration is related to the shear strength of the soil along the potential sliding surface, but also to the failure mechanism assumed in the analysis. Although the Newmark approach was
thought for a plane sliding, the generality of the method is obvious for a three dimensional analysis of slope stability.

Figure 2 – Three dimensional analysis of an infinite slope: a) slope profile; b) actual cross section; c) regularized cross section.

According to figure 3a a simple expression of the factor of safety for a three dimensional (3D) analysis of an infinite slope can be deduced if the actual cross section indicated in figure 3b is regularized as in figure 3c. It is possible to show that the 3D factor of safety for an infinite slope with a width $D$ is:

$$
F_{S_{3D}} = \frac{c}{\gamma H \cos i \left[ (1-k_y) \sin i + k_x \cos i \right]} + \frac{\left[ (1-k_y) \sin i + k_x \cos i \right] \tan \phi}{\left[ (1-k_y) \sin i + k_x \cos i \right]} + \frac{2c}{\gamma d \left[ (1-k_y) \sin i + k_x \cos i \right]} + \frac{K_0H \cos i \tan \phi}{d \left[ (1-k_y) \sin i + k_x \cos i \right]}
$$

(1)

Where:

$c$ = cohesion

$\phi$ = angle of friction

$i$ = angle of the sliding surface

$k_x$ = seismic coefficient in the horizontal direction

$k_y$ = seismic coefficient in the vertical direction

$K_0$ = lateral coefficient of earth pressure
\( \gamma \) = unit weight of the soil  \\
\( H \) = depth of the sliding surface  \\
\( d \) = width of the sliding mass

d = width of the sliding mass

from eq. (1), assuming a ground acceleration parallel to the slope and a cohesionless soil, one obtains for the critical acceleration:

\[
\mathbf{a_g} = g \cdot \left[ 1 + \frac{K_0 H}{d} \right] \cos \phi \tan i - \sin i
\] (2)

where \( g \) is the gravity acceleration.

Once the value of \( a_c \) is determined, the displacement analysis for a 3D slope can be carried out in the time domain and a prevision of the amount of the slope movement \( y_s \) can be made.

**ANALYTICAL APPROACH FOR SOIL-PIPELINE INTERACTION**

As discussed above, the stress on the pipe generated by the ground motion is a function of the soil characteristics, of the amount of displacement, of the direction of motion, of the pipeline features. The action due to the ground motion is transferred to the pipeline applying the values of the displacements deduced by the Newmark method as discussed before. With the increasing of the relative displacements between soil and pipe, the load transferred to the conduct will increase consequently and can be treated by means of the elasto-plastic load transfer functions until the achievement of the limit value of the soil resistance. The ultimate load depends on the nature of the soil: for a cohesive soil the limit load can be related to the undrained shear strength \( C_u \), while for a cohesionless soil it will depend on the angle of shearing resistance. If the pipe is assumed as elastic then the following differential equation can be applied:

\[
EI y'' = P
\] (3)

Where \( P \) is function of the relative displacement \( (y_s - y) \) between the soil \( y_s \) and pipe \( y \).

According to the method of the load-transfer function, if one assumes an elastic-perfectly plastic law, the load on the pipe can be expressed as:

\[
P = E_s (y_s - y) \quad \text{for} \quad y_s - y \leq P_{lim} / E_s
\] (4)

\[
P = P_{lim} \quad \text{for} \quad y_s - y > P_{lim} / E_s
\] (5)

Then in an elasto-plastic analysis the soil-pipe parameters to be determined are:

- the limit resistance \( P_{lim} [F/L] \) of the soil at large relative displacement \( (y_s - y) \)
- the stiffness \( E_s [F/L^2] \) that expresses the slope of the elastic behaviour.

A simple schematization of the problem for initial (a) and deformed (b) configuration is shown in the figure 4.
The origin of the reference system is located at the boundary between stable and unstable zones. We can distinguish three zones depending on the amount of the relative displacement between pipe and soil.

In the zone 1 an elastic behaviour for the soil-pipe interaction is assumed, because, due to the large deformation $y$ of the pipe in this zone, the relative soil-pipe displacement $(y_s - y)$ is less than the critical value $(y_s - y)_c = P_{LIM} / E_{s1}$ for which the condition of yielding for the soil is reached. Thus in zone 1 the load applied on the pipe is $P_{LIM} = E_{s1} \cdot (y_s - y)$.

In the zone 2 it is assumed a ultimate load on the pipe because the relative soil-pipe displacement $(y_s - y)$ is greater than the critical value $(y_s - y)_c = P_{LIM} / E_{s1}$.

In the zone 3 the pipe is in the stable slope: here it is hypothesized an elastic behaviour either for the soil, because of its higher shear strength parameters, or for the pipe.

The separation between plastic and elastic zones is indicated as $x_p$.

For the zone 1 ($x \leq x_p$) we have:

$$P = E_{s1} \cdot \frac{P_{LIM}}{E_{s1}} \cdot (y_s - y)$$
\[ EI y_1^{IV} + E_s y_1 \left( y_1 - y_s \right) = 0 \quad (6) \]

being \( E_s \) the modulus of sub-grade reaction for the soil in the unstable slope and \( y_1 \) the pipe displacement in the zone 1. Introducing \( \lambda_1 = \sqrt{\frac{4EI}{E_s}} \) the solution can be written as follows:

\[
y_1 = y_s + e^{\lambda_1 x} \left( C_1 \cos \frac{x}{\lambda_1} + C_2 \sin \frac{x}{\lambda_1} \right) + e^{-\lambda_1 x} \left( C_3 \cos \frac{x}{\lambda_1} + C_4 \sin \frac{x}{\lambda_1} \right) \quad (7)
\]

however if the width \( d \) is large enough a good approximation of equation (7) is:

\[
y_1 = y_s + e^{\lambda_1 x} \left( A \cos \frac{x}{\lambda_1} + B \sin \frac{x}{\lambda_1} \right) \quad (8)
\]

Posing \( X_1 = \frac{x}{\lambda_1} \), the equation (8) can be written:

\[
y_1 = y_s + e^{X_1} \left( A \cos X_1 + B \sin X_1 \right) \quad (9)
\]

rotation, bending moment and shear force are obtained as follows:

\[
y_1' (X_1) = \frac{\partial y_1}{\partial x} = \frac{1}{\lambda_1} \left[ e^{X_1} \left( A \cos X_1 + B \sin X_1 \right) + e^{-X_1} \left( -A \sin X_1 + B \cos X_1 \right) \right] \quad (10)
\]

\[
M_1 = EI y_1'' (X_1) = EI \frac{\partial^2 y_1}{\partial x^2} = EI \frac{2}{\lambda_1^2} \left[ e^{X_1} \left( -A \sin X_1 + B \cos X_1 \right) \right] \quad (11)
\]

\[
T_1 = -EI y_1''' (X_1) = -EI \frac{\partial^3 y_1}{\partial x^3} = -\frac{2}{\lambda_1^3} \left[ e^{X_1} \left( -A \sin X_1 + B \cos X_1 \right) - e^{-X_1} \left( B \sin X_1 + A \cos X_1 \right) \right] \quad (12)
\]

In the range \( x_p \leq x \leq 0 \) a plastic behaviour is assumed for the unstable zone 2 and the differential equation which describe mechanical response can be written in the form:

\[
y_2^{IV} = \frac{P_{lim}}{EI} \quad (13)
\]

being \( P_{lim} [F/L] \) the ultimate soil resistance attributed to unstable zone and \( y_2 \) the pipe displacement in the zone 2. Introducing the non-dimensional parameter:

\[
p = \frac{4P_{lim}}{D \cdot E_s} \quad (14)
\]
where \(D\) is the pipe diameter. The equation (13) is written in the following form:

\[
y^{iv}_{12} = \frac{p \cdot D}{\lambda_i^4} \tag{15}\]

The solution is:

\[
y_2 = \frac{p \cdot D}{24 \lambda_i} x^4 + \frac{C_1}{\lambda_i^3} x^3 + \frac{C_2}{\lambda_i^2} x^2 + \frac{C_3}{\lambda_i} x + C_4 \tag{16}\]

Posing then \(X_1 = \frac{x}{\lambda_i}\), equation (16) becomes:

\[
y_2 = \frac{p \cdot D}{24} X_1^4 + C_1 X_1^3 + C_2 X_1^2 + C_3 X_1 + C_4 \tag{17}\]

rotation, bending moment and shear force are then obtained:

\[
\varphi_2 = y_2''(X_1) = \frac{\partial y_2}{\partial x} = \frac{1}{\lambda_i^2} \left[ \frac{p \cdot D}{6} X_1^3 + 3 C_1 X_1^2 + 2 C_2 X_1 + C_3 \right] \tag{18}\]

\[
M_2 = EI y'''(X_1) = EI \frac{\partial^2 y_2}{\partial x^2} = \frac{EI}{\lambda_i^2} \left[ \frac{p \cdot D}{2} X_1^2 + 6 C_1 X_1 + 2 C_2 \right] \tag{19}\]

\[
T_2 = -EI y'''''(X_1) = -EI \frac{\partial^3 y_2}{\partial x^3} = -\frac{EI}{\lambda_i^3} \left[ p \cdot DX_1 + 6 C_1 \right] \tag{20}\]

At the same way, for \(x \geq 0\), in zone 3, where \(y_s = 0\), the differential equation can be expressed in the form:

\[
EI y^{iv}_3 + E_{s3} y_3 = 0 \tag{21}\]

being \(E_{s3}\) the modulus of soil reaction for the stable zone of the slope.

The solution of the differential equation is

\[
y_3 = e^{\frac{x}{\lambda_3}} \left( C_5 \cos \frac{x}{\lambda_3} + C_6 \sin \frac{x}{\lambda_3} \right) + e^{-\frac{x}{\lambda_3}} \left( C_7 \cos \frac{x}{\lambda_3} + C_8 \sin \frac{x}{\lambda_3} \right) \tag{22}\]

where \(\lambda_3 = \sqrt[4]{\frac{4EI}{E_{s3}}}\).

Since the pipe can be considered infinitely long in the zone 3, equation (22) simply reduces to:
\[ y_3 = e^{-\frac{x}{\lambda_3}} \left( F \cos \frac{x}{\lambda_3} + G \sin \frac{x}{\lambda_3} \right) \]  \hspace{1cm} (23)

Posing then \( \beta X_1 = \frac{x}{\lambda_3} \) we obtain:

\[ y_3 = e^{-\beta X_1} \left( F \cos \beta X_1 + G \sin \beta X_1 \right) \]  \hspace{1cm} (24)

The parameter \( \beta = \frac{\lambda_1}{\lambda_3} \) expresses a stiffness ratio between stable and unstable zone.

Rotation, bending moment and shear force are then obtained:

\[ \phi_3 = y_3' (\beta X_1) = \frac{\partial y_3}{\partial x} = -\frac{1}{\lambda_3} \left[ \frac{e^{-\beta X_1} (F \cos \beta X_1 + G \sin \beta X_1)}{\lambda_3} + \frac{e^{-\beta X_1} (F \sin \beta X_1 - G \cos \beta X_1)}{\lambda_3} \right] \]  \hspace{1cm} (25)

\[ M_3 = EIy''_3 (\beta X_1) = EI \frac{\partial^2 y_3}{\partial x^2} = -EI \frac{2}{\lambda_3^2} \left[ e^{-\beta X_1} (-F \sin \beta X_1 + G \cos \beta X_1) \right] \]  \hspace{1cm} (26)

\[ T_3 = -EIy'''_3 (\beta X_1) = -EI \frac{\partial^3 y_3}{\partial x^3} = -EI \frac{2}{\lambda_3^3} \left[ e^{-\beta X_1} (G \cos \beta X_1 - F \sin \beta X_1) \right] \]  \hspace{1cm} (27)

In order to evaluate the 8 unknown integration constants the following boundary conditions are imposed:

\[ y_1 (x = x_p) = y_2 (x = x_p) \]  \hspace{1cm} (28)

\[ \phi_1 (x = x_p) = \phi_2 (x = x_p) \]  \hspace{1cm} (29)

\[ M_1 (x = x_p) = M_2 (x = x_p) \]  \hspace{1cm} (30)

\[ T_1 (x = x_p) = T_2 (x = x_p) \]  \hspace{1cm} (31)

\[ y_2 (x = 0) = y_3 (x = 0) \]  \hspace{1cm} (32)

\[ \phi_2 (x = 0) = \phi_3 (x = 0) \]  \hspace{1cm} (33)

\[ M_2 (x = 0) = M_3 (x = 0) \]  \hspace{1cm} (34)

\[ T_2 (x = 0) = T_3 (x = 0) \]  \hspace{1cm} (35)

Moreover a further unknown is the dimensionless extension \( X_{1p} = \frac{x_p}{\lambda_1} \) of the plastic zone.

To solve this further indeterminateness it is possible to reference to the elasto-plastic law for the unstable zone. Particularly, at the distant \( X_{1p} = \frac{x_p}{\lambda_1} \), one has \( y_s - y_i = \frac{P_{LM}}{E_s} \) is valid. By utilizing this further condition and substituting the equation (8) for \( y_i \), the dimensionless extension \( X_{1p} \) can be found as a function of the stiffness ratio \( \beta \), as shown in figure 5 where \( X_{1p} \) is plotted versus the slope movement \( y_s \).
**Analytical solutions**

Once the value $X_{lp}$ is known, the integration constants for the evaluation of rotation, bending moment and shear force for the pipe, can be derived. The non-linearity of the load transfer curves produces results that depend on $y_s$ and $p$.

**Figure 5 - Yielding distance varying $y_s$ for $p=0.04$ and $D=1$ m**

**Figure 6 – Elasto-plastic analysis of buried pipe in moving slope: pipe displacement for $p=0.04$ and $y_s=0.05$ m.**
Figure 7—Elasto-plastic analysis of buried pipe in moving slope: pipe rotation for \( p=0.04 \) and \( y_s=0.05 \) m.

Figure 8—Elasto-plastic analysis of buried pipe in moving slope: bending moment for \( p=0.04 \) and \( y_s=0.05 \) m.
Figure 9 – Elasto-plastic analysis of buried pipe in moving slope: shear force for $p=0.04$ and $y_s=0.05$ m.

Figure 10 – Elasto-plastic analysis of buried pipe in moving slope: pipe displacement for $p=0.08$ and $y_s=0.20$ m.
Figure 11 – Elasto-plastic analysis of buried pipe in moving slope: pipe rotation for $p=0.08$ and $y_s=0.20\ m$.

Figure 12 – Elasto-plastic analysis of buried pipe in moving slope: bending moment for $p=0.08$ and $y_s=0.20\ m$. 
The non-dimensional plots presented in figures 6-13 show that the stress on the pipe is influenced by the stiffness ratio $\beta$, particularly at the boundary between the stable and the unstable zone, where bending moment and shear force assume high value if the characteristics of deformability of the stable and unstable zone are very different. The stress characteristics however strongly reduce with the increasing of the distance from the boundary.

CONCLUSIONS

An analytical approach for the assessment of stress and deformation of a buried pipe crossing an unstable slope has been presented. The evaluation of the seismic response of a buried pipe can be carried out into two different steps: i) the prediction of permanent displacements caused in the slope by the seismic loading, according to the Newmark method and ii) the stress and strain evaluation with the method of the load transfer curves. Since the earthquake-induced slope movements are depending on the critical acceleration of the slope, an analytical solution for the critical acceleration in a 3D sliding block has been given, assuming a simple scheme of infinite slope. Usually the critical acceleration evaluated in a 3D analysis is greater than that obtained by a 2D conventional stability analysis, thus a 2D Newmark displacement analysis could result uneconomical because of the greater displacements deduced.

The soil-pipe interaction has been investigated with the load transfer function technique, assuming an elastic perfectly-plastic behaviour for the load-displacement curves in the unstable slope. Results obtained from the analytical approach are presented in non dimensional plots, for different stiffness and soil strength. The analysis shows that the bending moment and the shear force along the pipe are larger at the boundary between the stable and unstable zone and they are greatly influenced by the stiffness ratio $\beta$. However bending moment and shear force rapidly decrease with the increasing distance from the boundary between stable and unstable zone.
REFERENCES