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RESPONSE OF SLANT LEGGED SKEW BRIDGE UNDER DYNAMIC LOADING

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SUMMARY

All real structures, when subjected to loads or displacements, behave dynamically. One of the most important problems accounted in structural engineering is the dynamic behaviour of bridges subjected to moving loads. Historically, the analysis of moving loads on bridges goes back to 19th century, when railroad construction was first initiated. Research on this subject is still in progress, especially due to development of numerical techniques. Slant legged skew bridge is one of the most widely used highway bridges, which eliminates the need of concrete piers. In this study, the bridge was idealized space bar system. Important parameters considered are (a) Speed of the moving vehicles (b) span of the bridge (c) Skew of the bridge. Here, moving vehicle is treated as triangular time dependant pulse. Programmes were developed in MatLab 6.5 with the help of Simulink and EWM (Exponential Window Method) is adopted to find natural frequencies, mode shapes and response of the bridge in the form of dynamic load factor.

Main conclusions in the above study are:

(1) Largest dynamic load factor occur at center of mid span.
(2) Dynamic load factor increases with increase in vehicle speed.
(3) Dynamic load factor increases with Span of the bridge.
(4) Different cross sections like Tee , unsymmetrical I section and Box sections are considered in evaluating dynamic load factor. Irrespective of cross section used in the bridge system, there is no appreciable change in the dynamic load factor.
(5) Skew has no effect on dynamic load factor.

INTRODUCTION

Slant legged bridge is one of the most widely used highway bridges. This type eliminates the need for concrete piers and position the supports away from lower roadway. The response of a bridge under a

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moving (force) vehicle is a complex phenomenon because of the interaction between bridge and the vehicle. Bridge structures that have long service years or long spans are frequently subjected to heavier loadings than their design loads are greatly effected by heavy traffic – induced vibrations. Many bridge design engineers treat such vibration problems by considering only IMPACT FACTORS specified in their current design codes, even though the vibrations may depend on Vehicle dynamic characteristics and Bridge dynamic characteristics.

The study of Dynamic response of Highway bridges due to moving force/loads as well as during an Earthquake is an essential pre-requisite for the design of any bridge. Critical review of the available literature reveals that, Dynamic response study due to vertical force with constant velocity dates back to Krylov [1]

The problem of moving load was first tackled for the case in which the beam mass was considered small as against the mass of the single constant load. The approximate mathematical solution was given by Willis [2] and Stokes [3]. Some of the research workers considered that the mass of moving load as small when compared with the beam mass. This was examined by Krylov [1], Timoshenko [4], Lowan [5] & Bonder [6].

The above authors solved the problem by applying Eigen Functions, Green functions and integral equations,where the moving force was treated as a constant force. The problem involving both load mass and beam mass is complicated and was not solved earlier to 1929. Jeffcott [7], Schallenkamp [8] and Steuding [9] attempted this problem of single concentrated load of constant magnitude through iterative methods and integral equations. Inglis [10] solved some problems connected to railway bridges by introducing Harmonic Analysis to solve dynamic calculation of Railway bridges traversed by steam locomotives.The solution of the motion of the beam is due to Hillerberg [11], who adopted Fourier method and Finite differences. Further, the problem was solved by Biggs et al [12], using Inglis [10] method and Tung et al [13], using Hillerberg's [11], method. In all the above methods, solutions are classical and have treated the bridge as one-dimensional case keeping in view the support conditions as simply supported and mass distribution as continuous and having elastic properties. Looney C.T.G [14] and Biggs J.M et al [12], represented the bridge as a simple beam of which only first or fundamental mode of vibration is considered and deflected shape of the bridge represented by a half sine wave. But their approach cannot be applied to find dynamic response of continuous highway bridges as deflected shape of the bridge is approximated by a half sine wave.

Fleming and Romualdi [15] treated the bridge as simply supported beam by lumping masses at convenient points, subjected to a moving force with constant velocity. Gillespie and Liaw [16], formulated dynamic equations for frequency analysis by matrix formulation using the above approach. Fernando Venancio Filho [17], developed a general method for calculating response of beams and frames to a constant force moving with a constant velocity using discrete model for mass distribution of structure and moving force, mass distribution constituted by lumped masses with transnational degrees of freedom. Moving force is idealized as triangular pulse, which excite the lumped masses. He adopted Laplace transforms to get closed form solution for dynamic influence lines and applied for simply supported, continuous, cantilever and orthogonal rigid frames. Shah [18] treated the whole bridge as rigid structure and adopted lumped mass approach. Wen,R.K., & Toridis [19], studied dynamic behaviour and free vibration of three span symmetrical Cantilever bridge using lumped mass approach. Inglis C.E [10] obtained solutions for the case of single moving load consisting of a rolling mass and a pulsating force. The procedure was based on the assumption, that the elastic deflection and load deflection curve can be approximated by Fourier series representation. A.S.Veletsos and T.Huang [20], described the most general method of analysis developed at university of Illinois. Within the limitations of the representation of the bridge as a single beam, it was analyzed exactly as a multi degree of freedom system and linearly elastic system having distributed flexibility and concentrated point masses. The vehicle was represented

realistically as a three-axle spring load unit. They also considered inter leaf friction in its suspension system. The most important feature of this method is that the transverse flexibility is neglected which is not correct, more so when there is considerable relationship existing between span and width of the bridge. The assumptions have been fully verified and the solutions are based on Continuum approach and Discrete approach. Suitable mathematical idealisation have been adopted. This has been studied under highway and railway bridges.

REVIEW OF LITERATURE - SLANT AND SKEW LEGGED BRIDGES

The research work so far carried out on slant legged bridges are limited in nature. Also, parameters considered in the evaluation of the dynamic response of slant legged bridges are also limited. From the review it can be observed that, not much work has been done on dynamic analysis of skew bridges and slant legged bridges, which are of current interest.

Ghobarah and Tos [21] analyzed the foothill Boulevard under crossing southeast bridge. The bridge has a skew of 60° ,under vertical component of earthquake motion. They modeled the bridge deck as a beam fixed to the abutment walls at its ends and concluded that the torsional and flexural motions of the bridge deck dominated the failure. Ronald et al [22] studied the dynamic behaviour of skewed reinforced concrete box girder bridge under the effect of earthquake excitations. They considered linear as well as non-linear models. Skew angles were varied approximately from 55° – 65°. The built-up model of the bridge had 6000 degrees of freedom. Eigen value analysis was carried out for getting natural frequencies of first four modes of vibrations. The first three modes are rigid body motions with little bending in the deck. By comparison of results of the free vibration analysis of the two models, it was observed by the two models differ by less than 30% of each of four modes. Tong Lo Wang and Dongzhou Huang [23] investigated the dynamic response of a slant legged rigid frame bridge to one or two tracks (side by side) passing over the rough bridge deck. The bridge was modeled as a space bar system. In the free vibration study, each of the longitudinal girders were divided into fifty two elements and each of the leg into five elements. Since, the Mechanical behaviour of slant legged rigid frame bridge is same as arch bridge, the first mode shape is anti symmetrical mode. The second and fourth modes are vertical bending, lateral bending and torsional vibration modes. The third and fifth modes are symmetric vertical bending modes. Dongzhou Huang et al [24] in their study on dynamic response of a rigid frame bridge to two trucks (side by side) passing over the rough bridge deck modeled the bridge as a space bar system. The dynamic response of the bridge is analysed by Finite Element method. The analytic truck is modeled as a non-linear system with one degree of freedom according to the HS20-44 truck design loading specified in the AASHTO specifications.

METHOD OF ANALYSIS

Fernando Venancio Filho [17], developed a general method for calculating response of beams and frames to a constant force moving with a constant velocity using discrete model for mass distribution of structure and moving force, mass distribution constituted by lumped masses with transnational degrees of freedom. Moving force is idealized as triangular pulse, which excite the lumped masses. He adopted Laplace transforms to get closed form solution for dynamic influence lines and applied for simply supported, continuous, cantilever and orthogonal rigid frames.

The bridge considered (Fig. 1) is traversed by a force that has triangular pulse time variation as shown in Fig. 2 for a generic mass point m_i , in which x_i is the distance of the point to the origin of the beam. The mass points m_i (i=1,2,3,…..n) are excited by this force.

Methodology developed by Filho (Ref. 1) has been made use of and is programmed in MatLab.

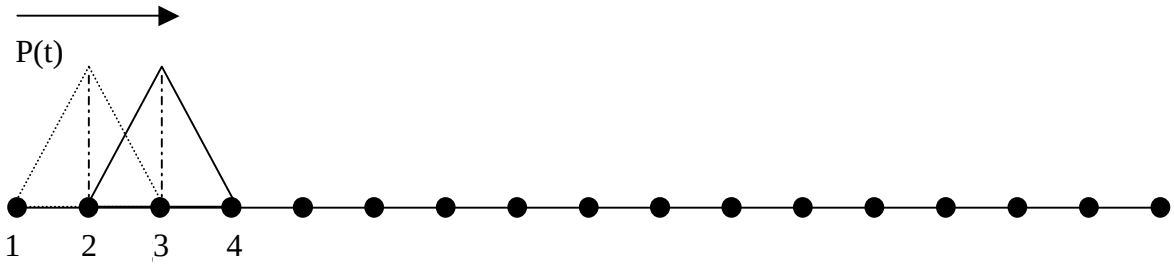


Figure 1. Moving triangular pulse on the bridge

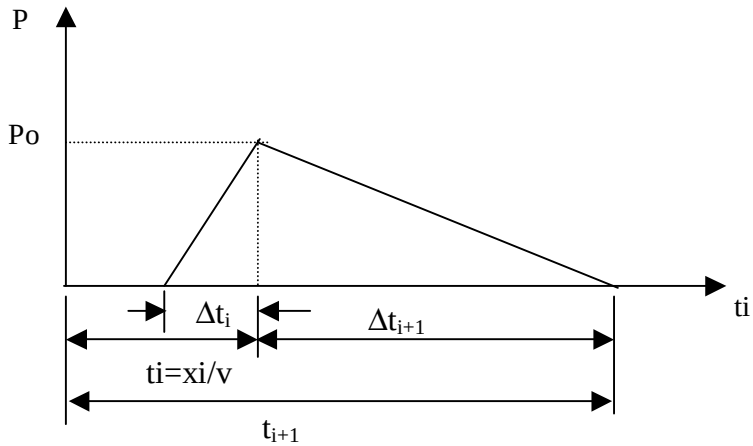


Figure 2. Time variation of force acting on mass point

The basic dynamic equilibrium equation derived by using Newton's Second Law of Motion can be shown to be equal to

$$[M]\,[\ddot{y}] + [C]\,[\acute{y}] + [K]\,[y] = [F(t)] \qquad \text{---------- (1)}$$

Where,

[M] = Diagonal mass matrix of size n x n where n is the degrees of freedom of the structure.

[K] = Total structural stiffness matrix of size n x n given in equation-where n is degrees of freedom.

[C] = Damping coefficient

[F (t)] = Column matrix of the applied time dependent forces.

Deflections associated with degrees of freedom of the system at any time `t' is defined by column matrix;

$$\{ d(t) \} = [W]\,\{\xi\,(t)\} \qquad \text{-------- (2)}$$

Where [W] = (n x n) matrix of amplitudes of normal modes.

$\{\xi\,(t)\}$ = Column matrix of dynamic factors related to each normal mode.

In equation 2. value of ξ (t) is obtained by matrix differential equation.

$$[\Lambda]\,\{\xi\,(t)\} + [\omega^2]\,[\Lambda]\{\xi\,(t)\} = \{E(t)\} \qquad \text{------ (3)}$$

Where,

$[\Lambda]$ - (n x n) Diagonal matrix of generalized elastic mass of mode k
$\{\xi(t)\}$ - (n x 1) Column matrix
$[\omega^2]$ - (n x n) Square matrix of the squares of natural frequency of mode k
$\{E(t)\}$ - (n x 1) Column matrix of generalized elastic forces

But,

$$[\Lambda] = [W] * [M] [W] \qquad \text{------} \quad (4)$$

Hence,

$$\{E(t)\} = [W] * \{P(t)\} \qquad \text{-------} \quad (5)$$

From equations 4 and 5, differential equations for dynamic factor relative to k^{th} Normal mode $\xi(t)$ is given by;

$$[\Lambda_k]\{\xi_k(t)\} + +\omega_k^2 \ \Lambda_k]\{\xi_k(t)\} = W_{ik} \ P(t) \qquad \text{-------} \quad (6)$$

Solution of equation 5 is obtained by using Laplace Transform, which gives time deflection relation for d_i (t) as

$$\{\xi_k(t)\} = (P_o / [\Lambda_k] * \omega_k^2) \text{ Sigma}\{ W_{ik} \ (U(t-t_{i-1})/\Delta t_i \left[(t-t_{i-1}) - \sin \omega_k (t-t_{i-1}) \right] - \left(1/\Delta t_i + 1/\Delta t_{i+1}\right)$$

$$U(t-t_i) \left(\frac{(t-t_i) \ \sin \omega_k (t-t_{i-1})}{\omega_k} \right) + \ldots\ldots \qquad \text{-------} \quad (7)$$

Where

$[\Lambda]$ - (n x n) Diagonal matrix of generalized elastic mass of mode k
$\{\xi(t)\}$ - (n x 1) Column matrix
$[\omega^2]$ - (n x n) Square matrix of the squares of natural frequency of mode k
P_o - Magnitude of moving force
W_{ik} - Amplitude associated to ith degree of freedom in normal mode k.
t - Time
t_i - Time in which moving force passes over mass point m_i
$U(t-t_{i-1})$- Unit Step function staring at $t = t_i$

In equation 7, terms such as (t-ti) represents forced part of response, terms like 1/ωk sin wk (t-ti) corresponds to SINUSUIDAL vibration representing free vibration part of the response.

The dynamic effect of moving triangular force [as shown Fig. 2] and static influence line of deflections are obtained for the triangular force using Modal superposition method with the help of the relation

$$[D] = [w] [\Lambda]^{-1} [\omega^2]^{-1} [w]^* \qquad \text{-------} \quad (8)$$

The equation 8 has been programmed in MatLab 6.5 and $d_i(t)$ is calculated.

NUMERICAL ANALYSIS

Mathematical model of the bridge (modeled using SAP 2000) is as shown in Fig 3. Different spans considered for analysis are 16-32-16 (64) M , 20-32-20 (72) M, 20-40-20 (80) M respectively. Center to center distance between the nodes is 4 M. Cross sections adopted for the study are Tee section , Unsymmetrical I section and Box section as shown Fig 4 (a,b,c).

After developing the programme in MatLab 6.5 with the help of Simulink & Exponential window method it was run for all the different cross sectional properties, different spans under the influence of a moving triangular force.

Speed considered is in the range of 30 kmph, 45 kmph, 60 kmph, 90 kmph to 120 kmph, for which frequencies were evaluated at different node positions of the triangular pulse and DLF is determined for central position of the load by using equation (7).

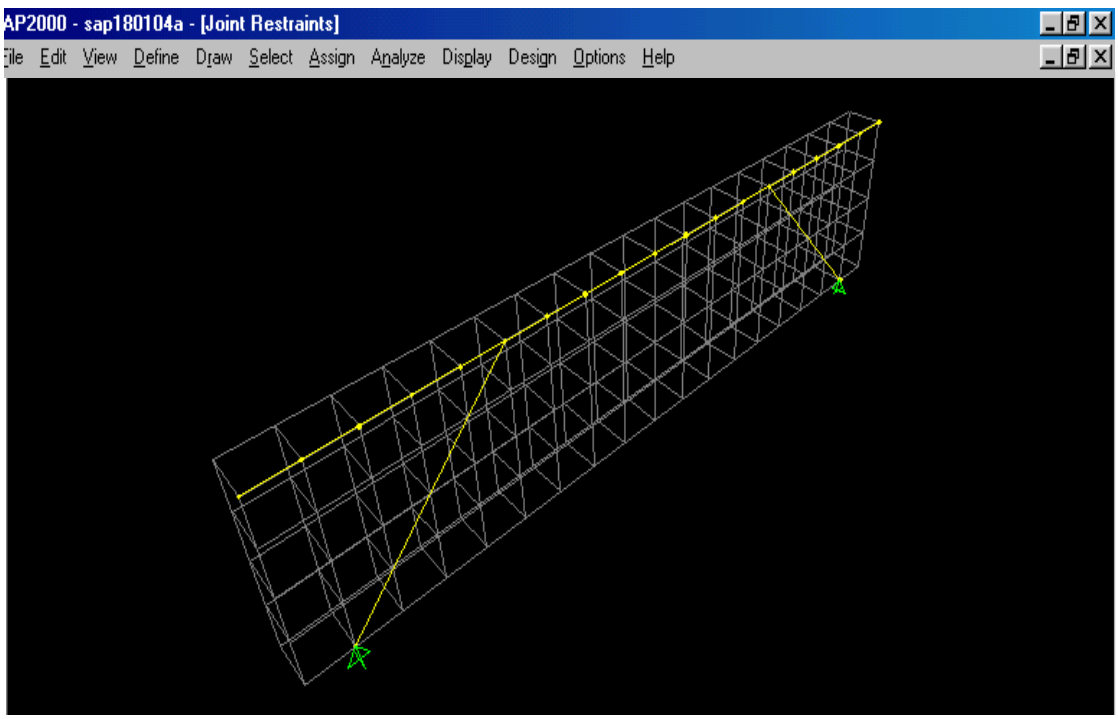
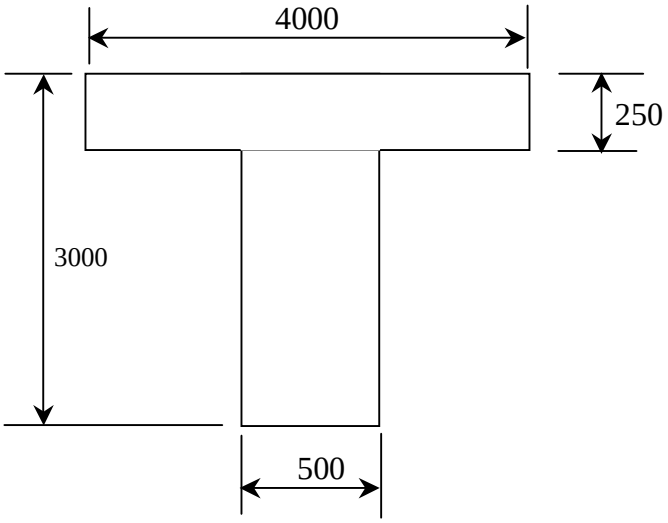


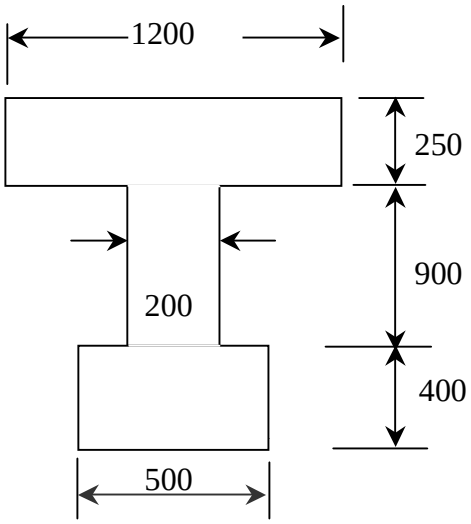
Figure 3. Mathematical model of the bridge

Area : 2.375 E 06 mm^2
Ix : 2.125 E 12 mm^4



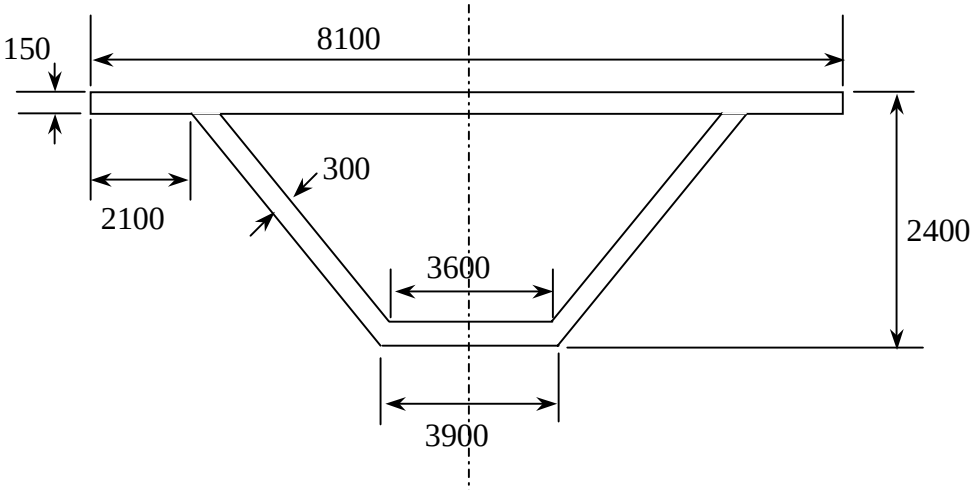
(a) Tee Section

Area : 0.68 E06 mm2
Ix : 0.2 E 12 mm^4



(b) I - Section (Unsymmetrical)

Area : 4.895E06 mm^2
Ix :2.37 E 12 mm^4



(c) Box Section

Figure 4 (a),(b),(c) Cross Sections Adopted

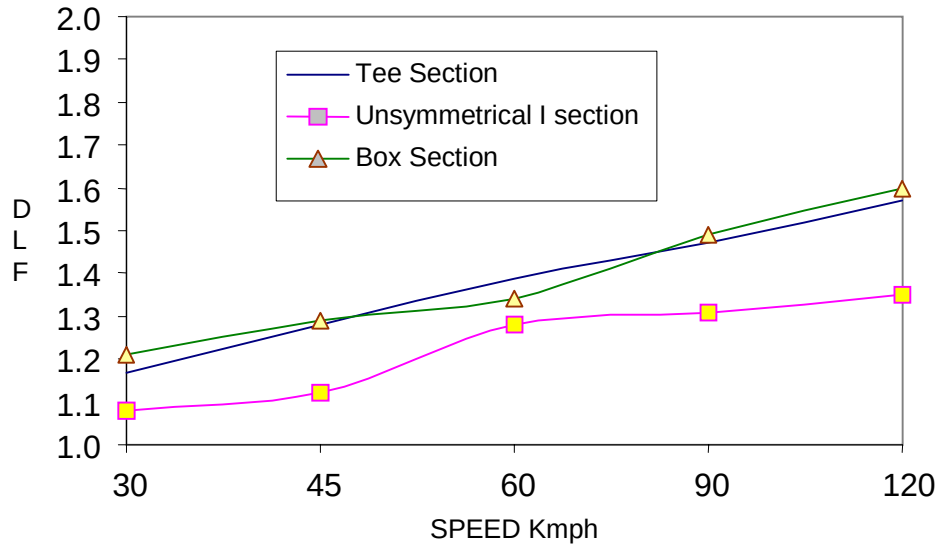


Figure 5. Variation of DLF v/s Speed, Span 16-32-16 (64M) - (Without Skew)

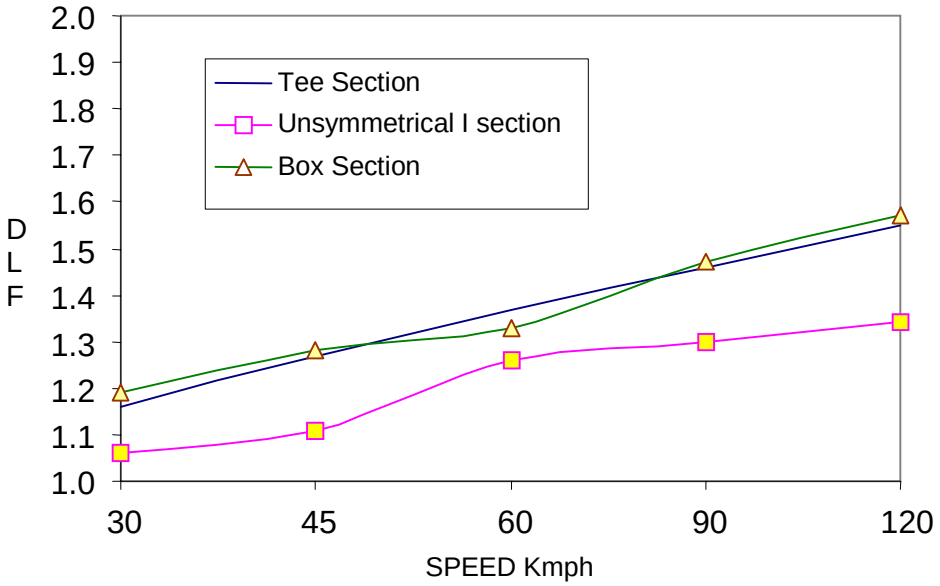


Figure 6. Variation of DLF v/s Speed, Span 16-32-16 (64M) (With Skew of 30°)

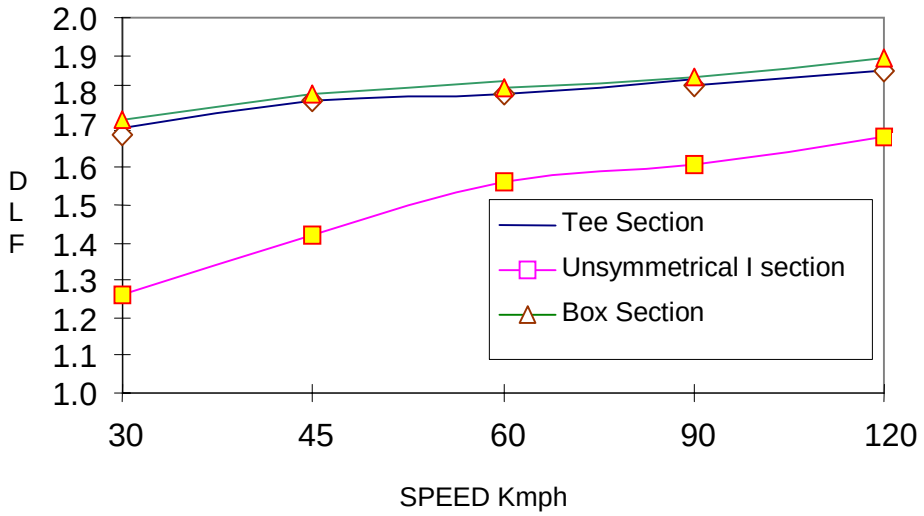


Figure 7. Variation of DLF Vs Speed, Span 20-32-20 (72M) (Without Skew)

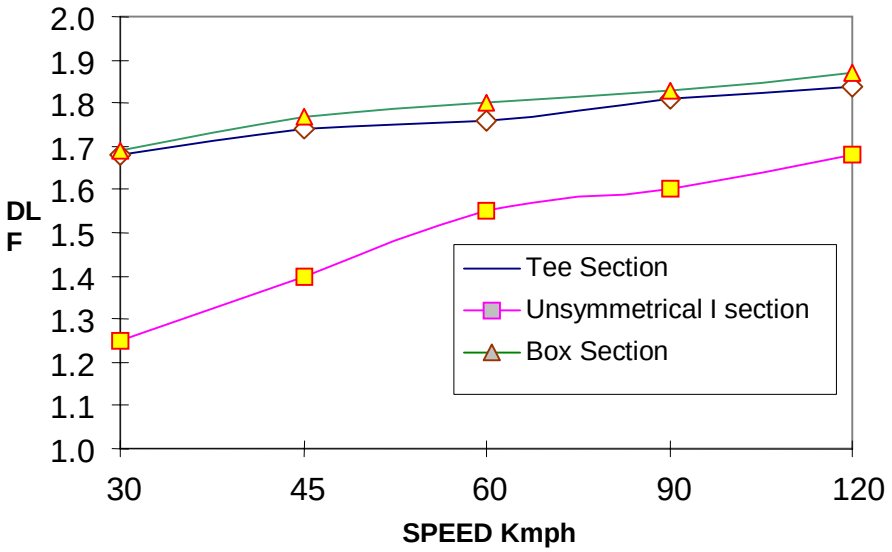


Figure 8. Variation Of DLF v/s Speed , Span 20-40-20 (80M) (With Skew of 30°)

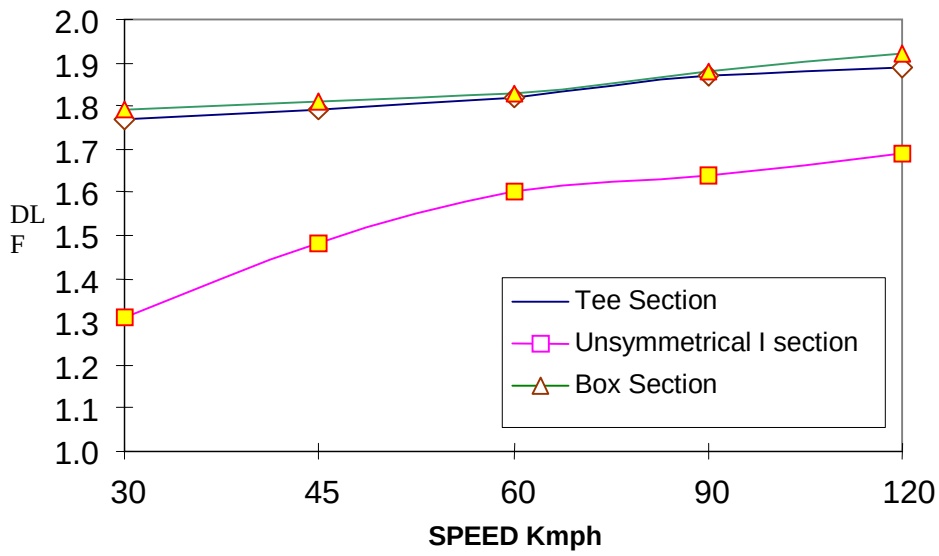


Figure 9. Variation Of DLF v/s Speed, Span 20-40-20 (80M) (Without Skew)

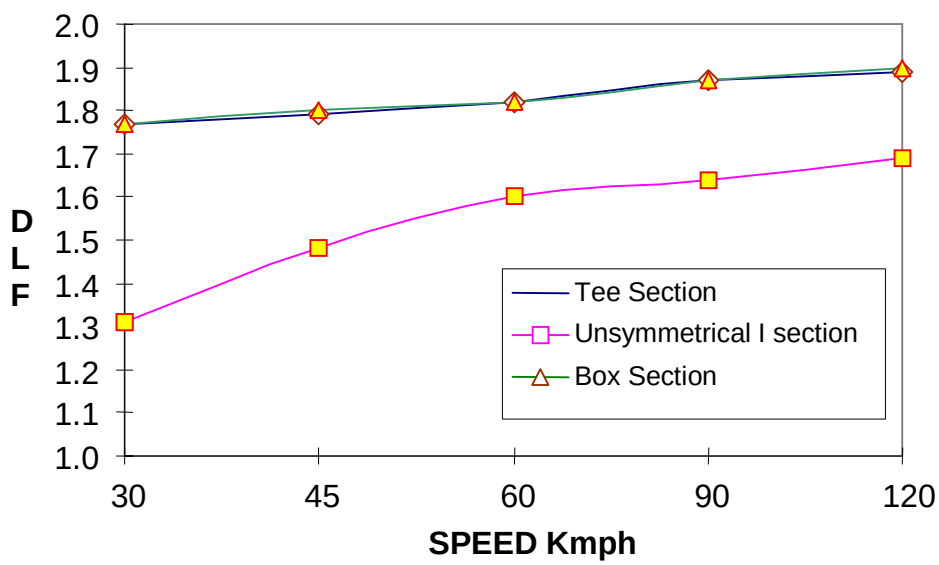


Figure 10. Variation of DLF v/s Speed, Span 20-40-20 (80M) (With Skew of 30°)

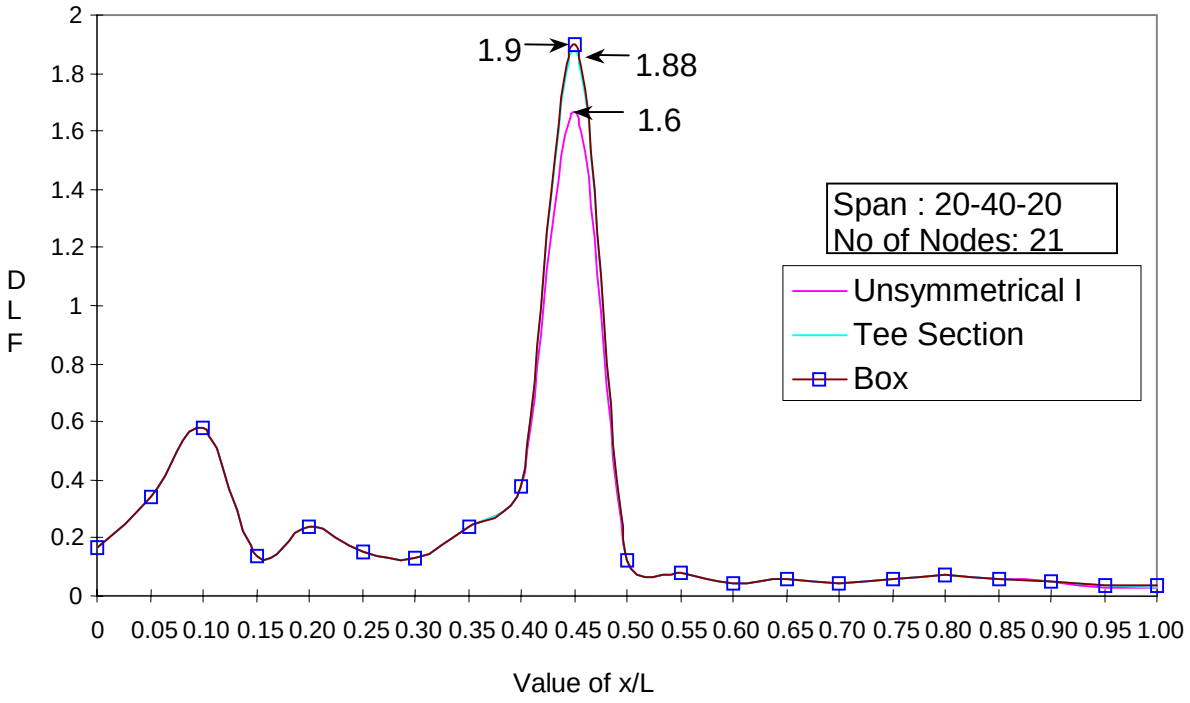


Figure 11. Variation of DLF

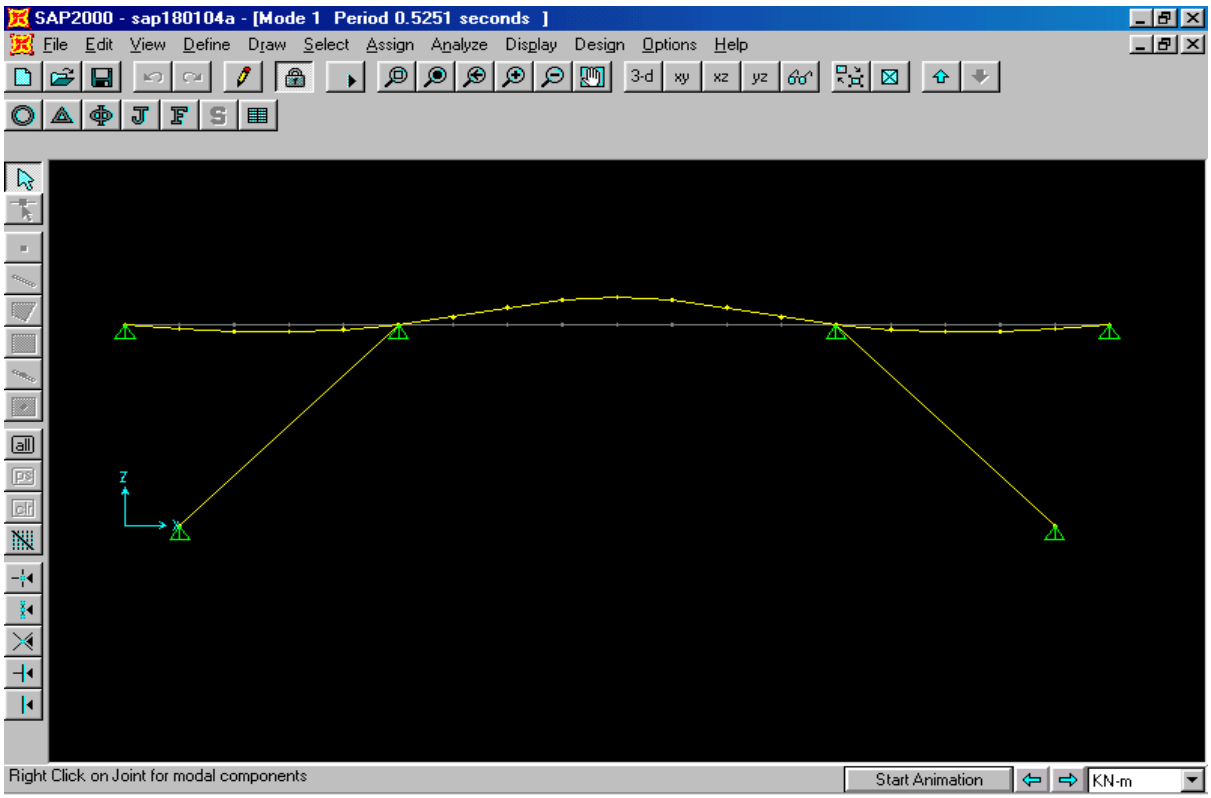


Figure 12. Free vibration of the bridge – Fundamental Mode

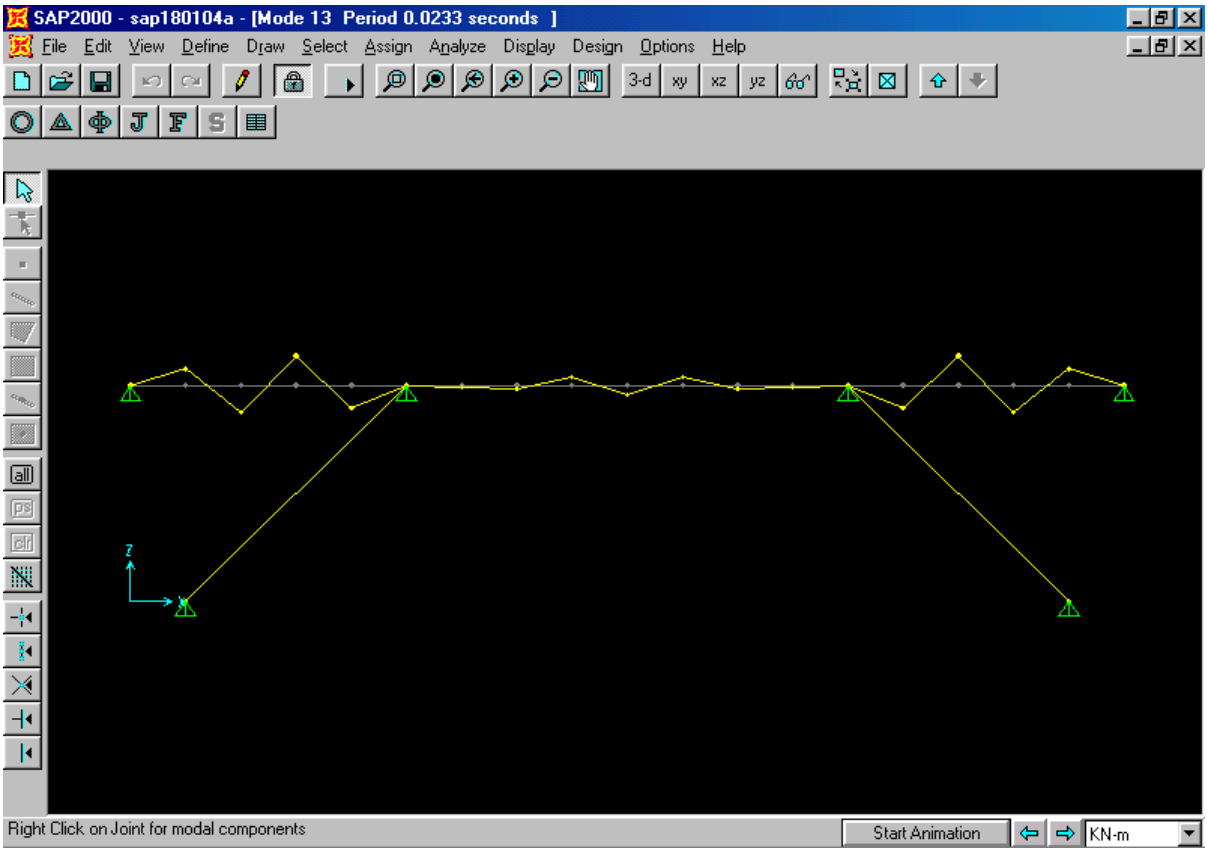


Figure 13. Free vibration of the bridge – Mode 13

RESULTS

Table No 1. Values of DLF – Without Skew

SPAN	SPEED	TEE SECTION	I SECTION	BOX SECTION
		A =2.375 E 06 mm^2	A = 0.68 E 06 mm^2	A = 4.895 E 06 mm^2
		M.I = 2.125 E 12 mm^4	M.I = 0.2 E 12 mm^4	M.I = 2.37 E 12 mm^4
		T	I	BOX
16-32-16 (64 M)	30	1.17	1.08	1.21
	45	1.28	1.12	1.29
	60	1.39	1.28	1.34
No. of Nodes = 17	90	1.47	1.31	1.49
	120	1.57	1.35	1.60
20-32-20 (72 M)	30	1.69	1.26	1.71
	45	1.76	1.42	1.78
	60	1.78	1.56	1.81
No. of Nodes = 19	90	1.82	1.61	1.84
	120	1.86	1.68	1.89
20-40-20 (80 M)	30	1.77	1.31	1.79
	45	1.79	1.48	1.81
	60	1.82	1.60	1.83
No. of Nodes = 21	90	1.87	1.64	1.88
	120	1.89	1.69	1.92

Table No 2. Values of DLF – With Skew of 30°

SPAN	SPEED	TEE SECTION	I SECTION	BOX SECTION
		A =2.375 E 06 mm^2	A = 0.68 E 06 mm^2	A = 4.895 E 06 mm^2
		M.I = 2.125 E 12 mm^4	M.I = 0.2 E 12 mm^4	M.I = 2.37 E 12 mm^4
		T	I	BOX
16-32-16 (64 M)	30	1.16	1.06	1.19
	45	1.27	1.11	1.28
	60	1.37	1.26	1.33
No. of Nodes = 17	90	1.46	1.30	1.47
	120	1.55	1.34	1.57
20-32-20 (72 M)	30	1.68	1.25	1.69
	45	1.74	1.40	1.77
	60	1.76	1.55	1.80
No. of Nodes = 19	90	1.81	1.60	1.83
	120	1.84	1.68	1.87
20-40-20 (80 M)	30	1.76	1.30	1.77
	45	1.78	1.46	1.80
	60	1.81	1.59	1.82
No. of Nodes = 21	90	1.86	1.63	1.87
	120	1.88	1.67	1.90

DISCUSSIONS

Table no 1 shows the variation of DLF with out skew for spans 64M, 72M & 80M - at different vehicle speeds of 30,45,60,90 & 120 Kmph along with various types of cross sections ie, Tee section, Unsymmetrical I section and Box section.

Table no 2 shows the variation of DLF with skew (10°,15°,20°,25° & 30°) for spans 64M, 72M & 80M – at different vehicle speeds of 30,45,60,90 & 120 Kmph along with various types of cross sections ie, Tee section, Unsymmetrical I section and Box section.

Fig. 12 & 13 shows free vibration of the bridge . Fundamental mode can be observed in Fig 12 and higher mode (Mode no 13) is shown in Fig 13.

1. As seen in table no 1 & 2, the span is varied as 16-32-16 (64 M), 20-32-20 (72 M), 20-40-20 (80 M). It is observed that, as span of the bridge increases, DLF increases.
2. The different cross sections considered as shown in table no 1 & 2 are Tee Section, Unsymmetrical I section and Box section. It is observed that, irrespective of the cross sections used in the bridge system, there is no appreciable change in DLF.
3. The different vehicle speeds considered for the analysis are 30, 45, 60, 90 & 120 Kmph (as shown in table no 1). It is observed that DLF increases with increase in vehicle speed.
4. From Fig 11, it can be observed that maximum DLF occurs at mid span.
5. Skew is varied as 10°,15°,20°,25° & 30° . From figures 6, 8 & 10, it can be observed that, skew has no effect on DLF with respect to span of the bridge. Figures 5,7 & 9 shows the variation of DLF with respect span of the bridge without skew.

CONCLUSIONS

1. Largest dynamic load factor occur at center of mid span.
2. Dynamic load factor increases with increase in vehicle speed.
3. Dynamic load factor increases with Span of the bridge.
4. Different cross sections like Tee , unsymmetrical I section and Box sections are considered in evaluating dynamic load factor. Irrespective of cross section used in the bridge system, there is no appreciable change in the dynamic load factor.
5. Skew has no effect on dynamic load factor.

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