13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 3129

SEISMIC RESPONSE OF INDUSTRIAL STRUCTURES CONSIDERING SOIL-PILE-STRUCTURE INTERACTION

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SUMMARY

The seismic behavior of industrial structures can be greatly affected by non-linear soil-pile interaction during strong earthquakes. The following reactor structure with height of 235 feet is examined as a typical industrial structure supported on a pile foundation with different base conditions. Dynamic analysis of the structure is carried out in two ways, time history analysis and response spectrum analysis.

The soil-pile-structure interaction is accounted for by means of the substructure method. The dynamic response of the reactor structure is calculated using FEM program. The stiffness and damping of pile foundation are generated from the program DYNAN and then input into the finite element model. Novak's method of soil-pile interaction is improved by modeling a non-reflective boundary between the near field and far field.

INTRODUCTION

Many industrial structures supported on pile foundations are constructed in soft soil in seismically active areas. The behavior of such structures can be greatly affected by non-linear soil-pile interaction during strong earthquakes. An evaluation of soil-pile-structure interaction is needed in order to establish the forces expected to act on the structure and the piles in a seismic event. The dynamic behavior of piles, of course, is very complex and this might have lead Terzaghi and Peck [19] to state that "… theoretical refinements in dealing with pile problems … are completely out of place and can be safely ignored".

A theoretical analysis of soil-structure interaction is often used during design. A simple procedure based upon substructuring is adequate for routine design. The following assumptions are adopted. The input ground motion is given for the level of pile heads and is not affected by the presence of the piles and their caps. Soil-pile interaction analysis is conducted separately to yield the impedance of pile foundation. The seismic response is obtained in time domain using input of earthquake records or in frequency domain with input of response spectra. This type of analysis is know as inertial interaction (see Novak, [15]).

A number of approaches are available to account for dynamic soil-pile interaction but they are usually based on the assumption that the soil behavior is governed by the law of linear elasticity or visco-elasticity

and the soil is perfectly bonded to a pile. In practice, however, the bonding between the soil and the pile is rarely perfect and slippage or even separation often occurs in the contact area. Furthermore, the soil region immediately adjacent to the pile can undergo a large degree of straining, which would cause the soil-pile system to behave in a nonlinear manner.

In this study, the soil-pile system is simulated by a boundary zone model with non-reflective interface. The model is an approximate but simple and realistic method that accounts for the nonlinearity of a soil-pile system. DYNAN [2] computer program developed by the author is used to generate the stiffness and damping of piles in the linear and nonlinear situations. The validity of the computation method vas verified by dynamic experiments on full-scale pile foundations. The nonlinear features of the pile foundation and the group effects are examined.

The seismic response of a reactor structure with height of 235 feet supported on a piled foundation is calculated for different base conditions: (1) with rigid base, i.e. no deformation in the foundation: (2) with a linear soil-pile system; and (3) with a nonlinear soil-pile system. Dynamic analysis of the structure is carried out in two ways, time history analysis and response spectrum analysis.

SOIL-PILE-STRUCTURE INTERACTION

The governing equation of a structure under seismic loads can be written as

$$\begin{bmatrix} m & m_i & m_i h_i \\ m_i^{\mathrm{T}} & m_b + \sum_1^n m_i & \sum_1^n m_i h_i \\ (m_i h_i)^{\mathrm{T}} & \sum_1^n m_i h_i & I + \sum_1^n m_i h_i^2 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{u}_b \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0^{\mathrm{T}} & c_{xx} & c_{x\varphi} \\ 0^{\mathrm{T}} & c_{\varphi x} & c_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{u}_b \\ \dot{\varphi} \end{Bmatrix} + \begin{bmatrix} k & 0 & 0 \\ 0^{\mathrm{T}} & k_{xx} & k_{x\varphi} \\ 0^{\mathrm{T}} & k_{\varphi x} & k_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} u \\ u_b \\ \varphi \end{Bmatrix}$$

$$= -[m_i \;\vdots\; m_b + \sum_1^n m_i \quad \sum_1^n m_i h_i]^{\mathrm{T}} \ddot{u}_g(t) \qquad (1)$$

in which *{u}* is the displacement vector, $\ddot{u}_g(t)$ is horizontal ground acceleration, h_i is the height of ith floor. The matrices *[m], [k]* and *[c]* list all the mass, stiffness and damping constants of the structure, *I* is the total mass moment of inertia, and *{0}* is the null vector.

The foundation properties are incorporated as shown in equation (1). m_b is mass of foundation; u_b and φ are horizontal translation and rotation; k_{xx}, $k_{x\varphi}$, $k_{\varphi x}$ and $k_{\varphi\varphi}$ are stiffness coefficients of the foundation and c_{xx}, $c_{x\varphi}$, $c_{\varphi x}$ and $c_{\varphi\varphi}$ are damping coefficients of the foundation.

Stiffness and damping of pile foundation

A rigorous approach to the nonlinearity of a soil-pile system is extremely difficult and time consuming. Approximate theories, therefore, have to be used for practice. As an approximate analysis, the Baranov-Novak method is considered as an efficient technique for solving the problem of coupled vibration (see

Luco [14]). The relationship between the foundation vibration and the resistance of the side soil layers was derived using elastic theory by Baranov [1].

Both theoretical and experimental studies have shown that the dynamic response of piles is very sensitive to the properties of the soil in the vicinity of the pile shaft. Novak and Sheta [17] proposed including a cylindrical annulus of softer soil (an inner weakened zone or so called boundary zone) around the pile in a plane strain analysis. One of the simplifications involved in the original boundary zone concept was that the mass of the inner zone was neglected to avoid the wave reflections from the interface between the inner boundary zone and the outer zone. To overcome this problem, Velestsos and Dotson [20] proposed a scheme that can account for the mass of the boundary zone. Some of the effects of the boundary zone mass were investigated by Novak and Han [16] who found that a homogeneous boundary zone with a non-zero mass yields undulation impedance due to wave reflections from the fictitious interface between the two media.

The ideal model for the boundary zone should have properties smoothly approaching those of the outer zone to alleviate wave reflections from the interface. Consequently, Han and Sabin [8] proposed such a model for the boundary zone with a non-reflective interface.

The impedances of the composite layer are derived from the plane-strain assumption. The outer zone medium is homogeneous, isotropic, and viscoelastic, with frequency independent material damping; within the boundary zone, the complex shear modulus, $G^*(r)$, varies parabolically, as expressed by the function $f(r)$. The variation of $G^*(r)$ is continuous at the boundary, both the function itself and its derivatives, so that no reflective wave can be produced at the interface. The interface is referred to as the "non-reflection boundary".

The properties of the soil medium for each region are defined by the complex-valued modulus

$$G^*(r) = \begin{cases} G_i^* ; & r = r_o \\ G_o^* f(r) ; & r_o < r < R \\ G_o^* ; & r \geqslant R \end{cases} \tag{2}$$

and

$$G_i^* = G_i (1 + i2\beta_i) \tag{3}$$

$$G_o^* = G_o (1 + i2\beta_o)$$

in which G_i and G_o = shear modulus of the inner and outer zones; r_o = radius of pile; R = radius of boundary zone; r = radial distance to an arbitrary point; β_i and β_o = damping ratio for the two zones; and i = root (–1). The parabolic function, $f(r)$, can be expressed as

$$f(r) = 1 - m^2 ((r-R)/r_o)^2 \tag{4}$$

and

$$m^2 = (1 - G_i^* / G_o^*)/ (t_m / r_o)^2 \tag{5}$$

where t_m = thickness of boundary zone; m = a constant whose value depends on G_i^* / G_o^* and t_m / r_o as shown above.

With the impedance of the soil layer, the element stiffness matrix of the soil-pile system can be formed in the same way as the general finite element method. Then the overall stiffness matrix of a single pile can be assembled for different modes of vibration, included three translations and three rotations. The dynamic stiffness and damping of a single pile can be expressed in terms of complex stiffness, such as for vertical vibration

$$K_v = (E_p A/r_o) f_{v1} + i\omega (E_p A/V_s) f_{v2} \quad (6)$$

in which E_p is the Young's modulus of the pile; A is the cross-section area of the pile element; ω is the circular frequency; V_s is the shear wave velocity of the soil. f_{v1} and f_{v2} are the dimensionless stiffness and damping parameters.

The group effect of piles is accounted for using the method of interaction factors. The vertical static interaction factors are based on Poulos and Davies [18]. The horizontal static interaction factors are due to El-Sharnouby and Novak [4]. The dynamic interaction factors are derived from the static interaction factors multiplied by a frequency variation, and the frequency variation of interaction factors is based on the charts of Kaynia and Kausel [13].

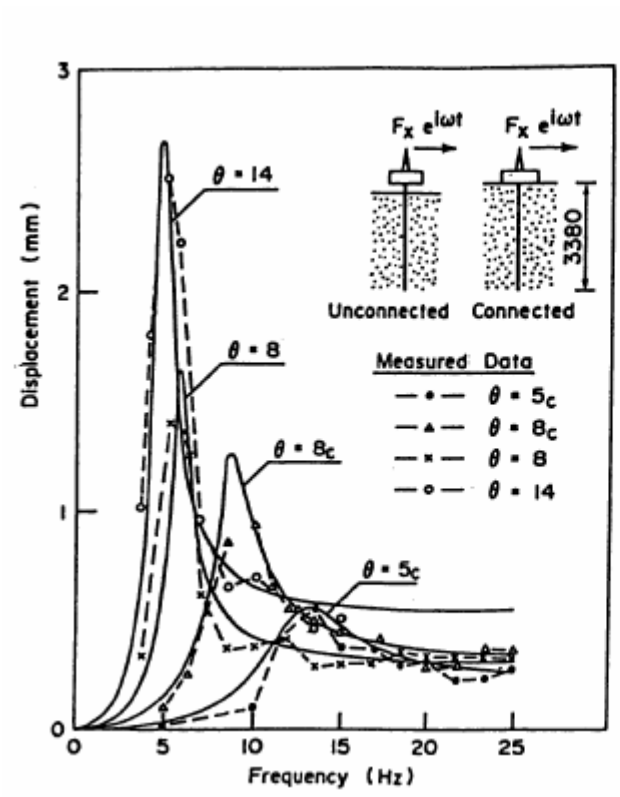
It should be explained that the foundations (or caps on piles) are assumed to be rigid. In most cases, the superstructures are flexible rather than rigid. By means of a substructure method, the dynamic response of the superstructure is calculated using a finite element program, such as SAP2000, and the stiffness and damping of the foundation can be generated from the DYNAN program. With this procedure, the seismic or dynamic response of the structure can be calculated in time domain or frequency domain (from Han [5]). This method has already been applied in practice. It was used for design of the largest table top structure supported compressor in North America (from Han [6]).

Experimental verification for pile foundation

To investigate the soil-structure interaction, a series of dynamic experiments have been done on full-scale piles. Field tests of a single pile subjected to strong harmonic excitation were conducted. The pile was a steel pipe with a diameter of 0.133 m and a length of 3.38 m. The measured dynamic response of the single pile was compared with the computed curves, and is shown in Figure 1. In the figure, the points show the measured data and the curves show the theoretical results under different harmonic loads. The horizontal exciting forces were expressed with θ, and θ = 5, 8, 14 and 28 correspond to $2.39\ f^2$, $3.79\ f^2$, $6.75\ f^2$ and $10.23\ f^2$. The unit of force is N, and f is the frequency (in Hz). The details of tests were described by Han and Novak [11], [12]. As for $\theta = 8$ and $\theta = 8_c$, the force is the same in the two cases. $\theta = 8$ means the bottom of the pile cap is separated from the soil, and $\theta = 8_c$ means the pile cap is connected with the soil.

Other dynamic investigations of piles have indicated that the boundary zone model is applicable to both granular and cohesive soils (see El-Marsafawi [3] and Han [7]). Dynamic experiments on a full-scale pile group were carried out in the field. The pile group was comprised of six cast-in-place reinforced concrete piles; pile diameter is 0.32 m and length 7.5 m. The pile slenderness ratio is l/d = 23.4 and spacing ratio is s/d = 2.81. Where l is the pile length, s is the pile spacing and d is the pile diameter. The shear wave velocity and mass density of the soil were measured for different depths, such as V_s = 130 m/s for the top layer soil and V_s = 280 m/s at depth of 8 m. Details of the tests were described by Han and Novak [9]. The results show that the resonant frequency of the pile group reduced and the resonant amplitude increased as the excitation intensity increased. The group effect is considered by means of the dynamic interaction approach. The experimental results indicate that the stiffness of the group was reduced and damping increased due to the group effect.

Under horizontal excitation, coupled horizontal and rocking vibrations are produced at the cap. The effect of embedment for the coupled vibration can be accounted for (see Han [10]). For linear vibration, the amplitude response of the group can be normalized by the excitation intensity to give dimensionless amplitudes.



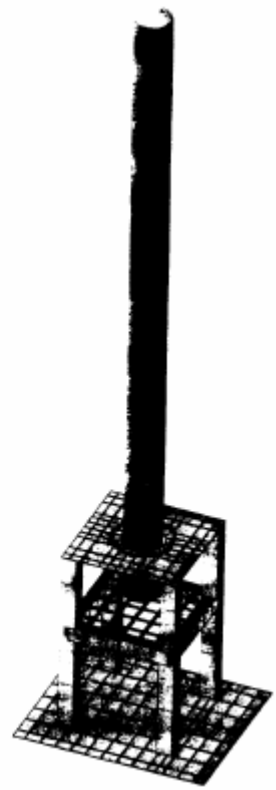


Fig.1 Horizontal response curves predicted analytically and measured in the field for a single pile.

(Han & Novak, Canada Geotechnical J., Vol.25, 1988)

Fig. 2 Reactor structure.

SEISMIC RESPONSE OF REACTOR STRUCTURE

The reactor structure shown in Fig. 2 was built for a petrochemical plant in a seismically active area of canada. The bottom portion is a concrete table top. The dimensions of structure are as follows. Four columns with cross-section of 4 x 4 (feet) and height of 47 feet are arranged rectangularly with a column center to center spacing of 26 feet. Top concrete slab is 30 x 30 (feet) with thickness of 4 feet. Four bracing beams with cross-section 4 x 3 (feet) are placed at height of 27 feet. The mat foundation is 42 x 50 (feet) and thickness is 5 feet.

A reactor vessel is installed on the concrete table top. The vessel diameter is 9 feet and the height to top is 235 feet. The thickness of vessel wall is one and half inch. The empty weight of reactor equipment is 595 kips and the operating weight is 700 kips.

The reactor vessel is modeled as an elastic column, the concrete table top structure is modeled using frame elements and the top slab is modeled using shell elements. Thus, the seismic analysis can be done based on equation (1). The seismic response of the structure is calculated using the substructure method, which allows the reactor structure and foundation portions to be calculated separately. The base shear and overturning moments for different conditions are investigated as are deflections of structure.

Soil conditions and pile foundation

The structure is in a seismically active area. The acceleration related seismic zone Z_a = 4 and velocity related to seismic zone Z_v =3 per Canadian seismic code NBC 1995. The range of peak horizontal ground acceleration is equal 0.12 g.

At the site, surface fill is overlies a sand layer, followed by a clayey silt deposit, a till layer then bedrock. The depth to bedrock is about 18 m. Soil properties vary with depth and are characterized by the shear wave velocity and unit weight, as shown in Table 1.

Table 1. Soil Properties.

Depth (m)	Unit weight (KN / m^3)	Shear wave velocity (m/s)
0 – 2	21	163
2 – 5	20	152
5 – 8	20	150
8 – 12	22	161
12 – 18	23	230

The piles are steel pipe piles filled with concrete, diameter of 0.25 m (10 inch) and length of 18 m driven to bedrock. 96 piles are fixed to the mat foundation. Each 12 piles are placed in one row with a spacing of 1.22 m (4 feet), and the spacing ratio is 4.9. The distance between the rows is 1.52 m (5 feet), and the spacing ratio is 6.1.

The stiffness and damping of the pile foundation are calculated for different cases. In the first case a nonlinear soil-pile system is assumed, and the boundary zone model is used around the piles. In this case, the parameters of the weakened zone are selected as: $G_i / G_o = 0.1$, $t_m/r_o = 1.0$, $\beta_i = 0.07$ and $\beta_o = 0.03$. In the second case, a linear soil-pile system is assumed, the soil layers are homogeneous, and there is no weakened zone. In the last case, no pile-soil-pile interaction is assumed. That is, the group effect is not account for. Both stiffness and damping are frequency dependent. Here, the stiffness and damping are calculated at frequency f = 1.0 Hz, and the results are shown in Table 2.

Table 2. Stiffness and Damping of Pile Foundation (f = 1.0 Hz)

Pile-Soil-Pile Interaction Conditions	Stiffness			Damping		
	K_x (kN/m)	K_z (kN/m)	K_φ (kN.m/ra)	C_x (kN/m/s)	C_z (kN/m/s)	C_φ (kN.m/rad/s)
Interaction + Weakened Zone (Nonlinear)	3.845×10^5	2.395×10^6	1.067×10^8	6.383×10^3	5.100×10^4	9.646×10^4
Interaction, No Weakened Zone (Linear)	7.738×10^5	3.791×10^6	1.686×10^8	1.115×10^4	7.882×10^4	7.020×10^4
No Interaction	4.119×10^6	3.092×10^7	4.184×10^8	1.357×10^4	1.007×10^5	1.353×10^6

In table 2, K_x, K_z, and K_φ are stiffness in the horizontal, vertical and rocking directions, and C_x, C_z, and C_φ are damping constants in the same directions.

From Table 2, it can be seen that both stiffness and damping in the nonlinear case are lower than the linear case. For example, the horizontal stiffness in nonlinear case is about half of that in linear case. If the group effect is not accounted for, that is, no pile-soil-pile interaction, the values of stiffness and damping are much larger than when the interaction is included. As for the horizontal stiffness, $K_x = 7.738 \times 10^5$ kN/m in linear interaction case, and $K_x = 4.119 \times 10^6$ kN/m in the "no interaction" case. The group efficiency ratio = 0.19. It should be noted that the nonlinear soil property has an effect on the stiffness and damping of the piled foundation, and the group effect is also important.

Time history analysis

A horizontal ground acceleration from the record of the San Fernando earthquake is employed for time history analysis. The peak value of acceleration is 0.11 g as shown in Fig. 3. The time step is 0.02 second in the earthquake record, and 1024 steps are input. Thus, the seismic response is shown during 20 seconds. To investigate the influence of foundation flexibility on the superstructure, the seismic analysis of the reactor structure is conducted for two different foundation conditions: rigid base and nonlinear soil-pile system.

For the case of the rigid base, the stiffness of the foundation is assumed to be infinite with no deformation occurring in the footing. Initial seismic analysis was done in this way forty years ago, when the soil-structure interaction was not considered.

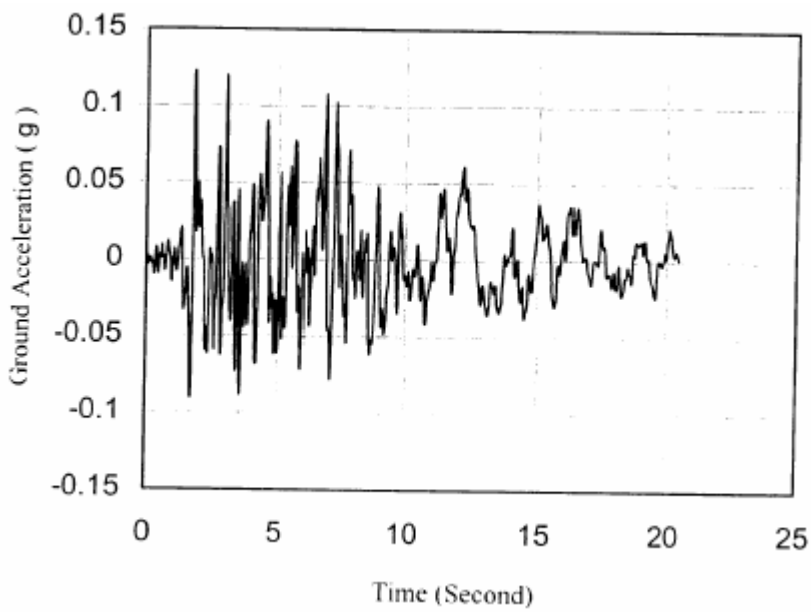


Fig. 3 Horizontal ground acceleration input to base of reactor structure.

For the nonlinear soil-pile system, the weakened and nonlinear soil properties that may occur around the piles in a seismic environment are accounted for by the boundary zone model with non-reflective interface. The stiffness and damping used in nonlinear situation are shown in Table 2.

The time history analysis is done using the finite element program SAP2000. The seismic forces and natural frequency of reactor structure are different for the two base conditions, rigid base and nonlinear pile foundation. The base shear and overturning moment are shown in Table 3 and the natural periods of reactor structure are shown in Table 4.

Table 3. Maximum Values of Seismic Response of Reactor Structure

Base Conditions	Amplitude at Top of Reactor (mm)	Base Shear of Foundation (kN)	Base Moment of Foundation (kN-m)
Flexible Base, With SSI	140	1,178	29,880
Fixed Base, No SSI	183	2,561	61,000

Table 4. Natural Periods of Reactor Structure for Different Base Conditions (Second)

Vibration Modes	Flexible Base, With SSI	Fixed Base, No SSI
1, Horizontal - X	1.473	1.345
2, Horizontal - Y	1.463	1.327
3	0.666	0.371
4	0.467	0.370
5	0.463	0.190
6	0.204	0.188
7	0.201	0.182
8, Vertical - Z	0.166	0.113

From Table 3, it can be seen that the amplitude at top of reactor is reduced to 140 mm from 183 mm when the soil-structure interaction is accounted for (flexible base). The seismic forces are different obviously due to the two base conditions. The base shear and overturning moment are reduce to about half values when the soil-structure interaction is considered. The reactor structure is slender. Thus, it can be deduced that the soil-structure interaction has an effect on the seismic response even for slender structures. From Table 4, it can be seen that the structure with a flexible base has longer natural periods than that with fixed base.

The time history of displacement at the top of reactor is shown in Fig. 4, (A) for the case with soil-structure interaction and (B) for the case without the interaction (rigid base). The base shear and overturning moment vary with time and are shown in Fig. 5 and Fig. 6, respectively, and (A) for the case with soil-structure interaction and (B) for the case without the interaction (rigid base). From these figures, it can be seen that the maximum values and time histories for the seismic forces and seismic response are very different when the foundation is considered as a fixed base or a flexible base. The theoretical prediction for structures fixed on a rigid base without soil-structure interaction does not represent the real seismic response, since the stiffness is overestimated and the damping is underestimated.

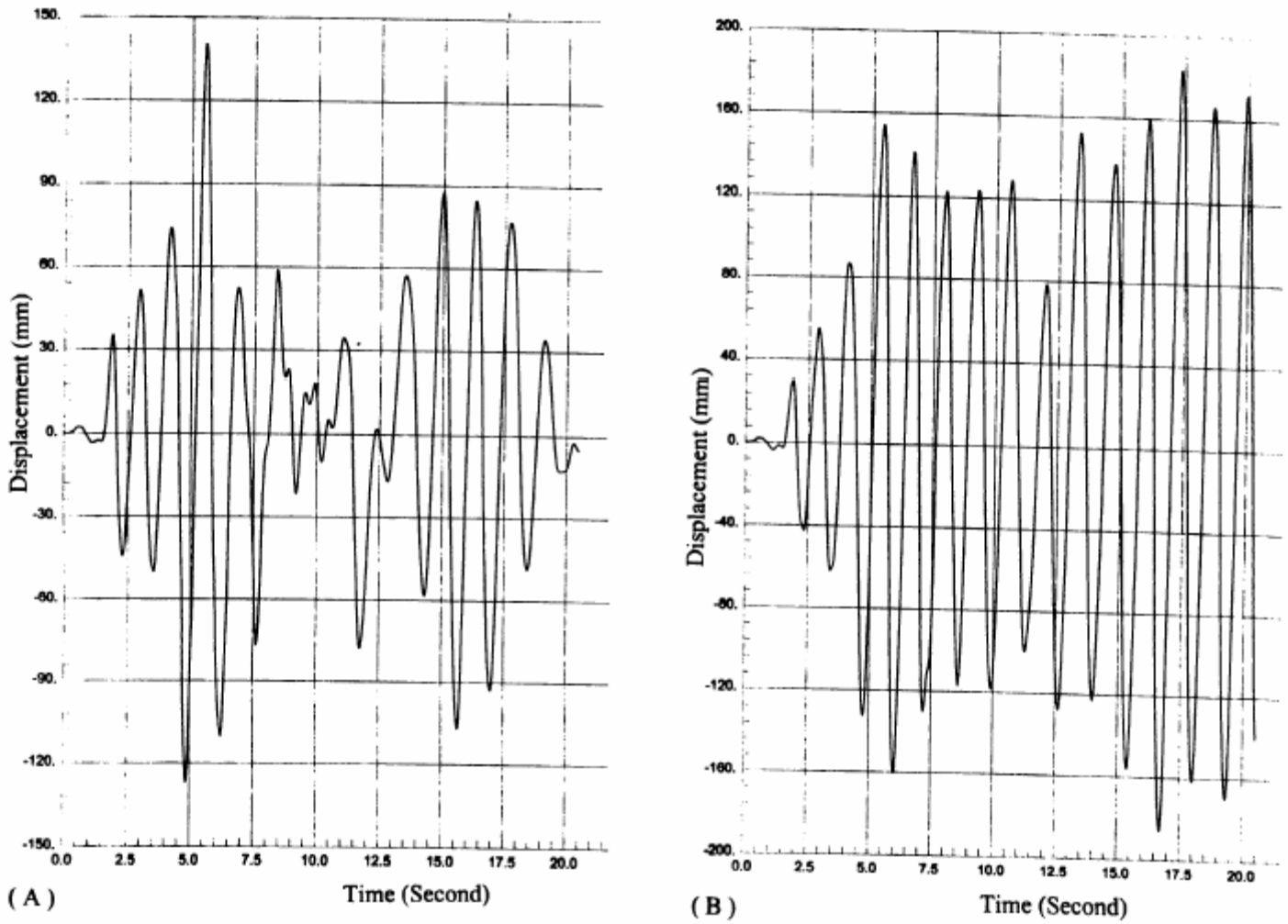


Fig. 4. Displacement at top of reactor structure (A) with soil-structure interaction (flexible base); and (B) no soil-structure interaction (rigid base).

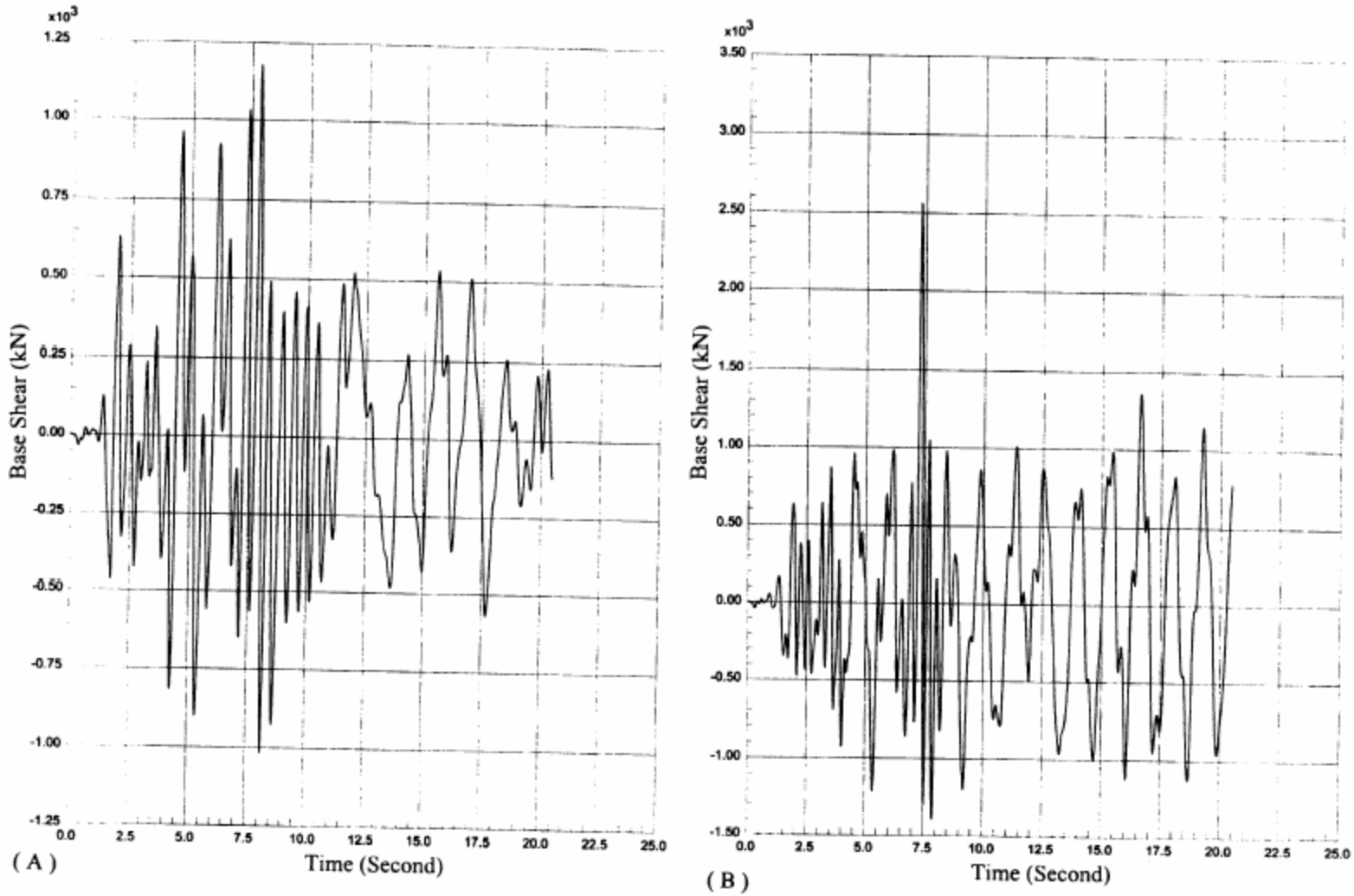


Fig. 5. Base shear of reactor structure (A) with soil-structure interaction (flexible base); and (B) no soil-structure interaction (rigid base).

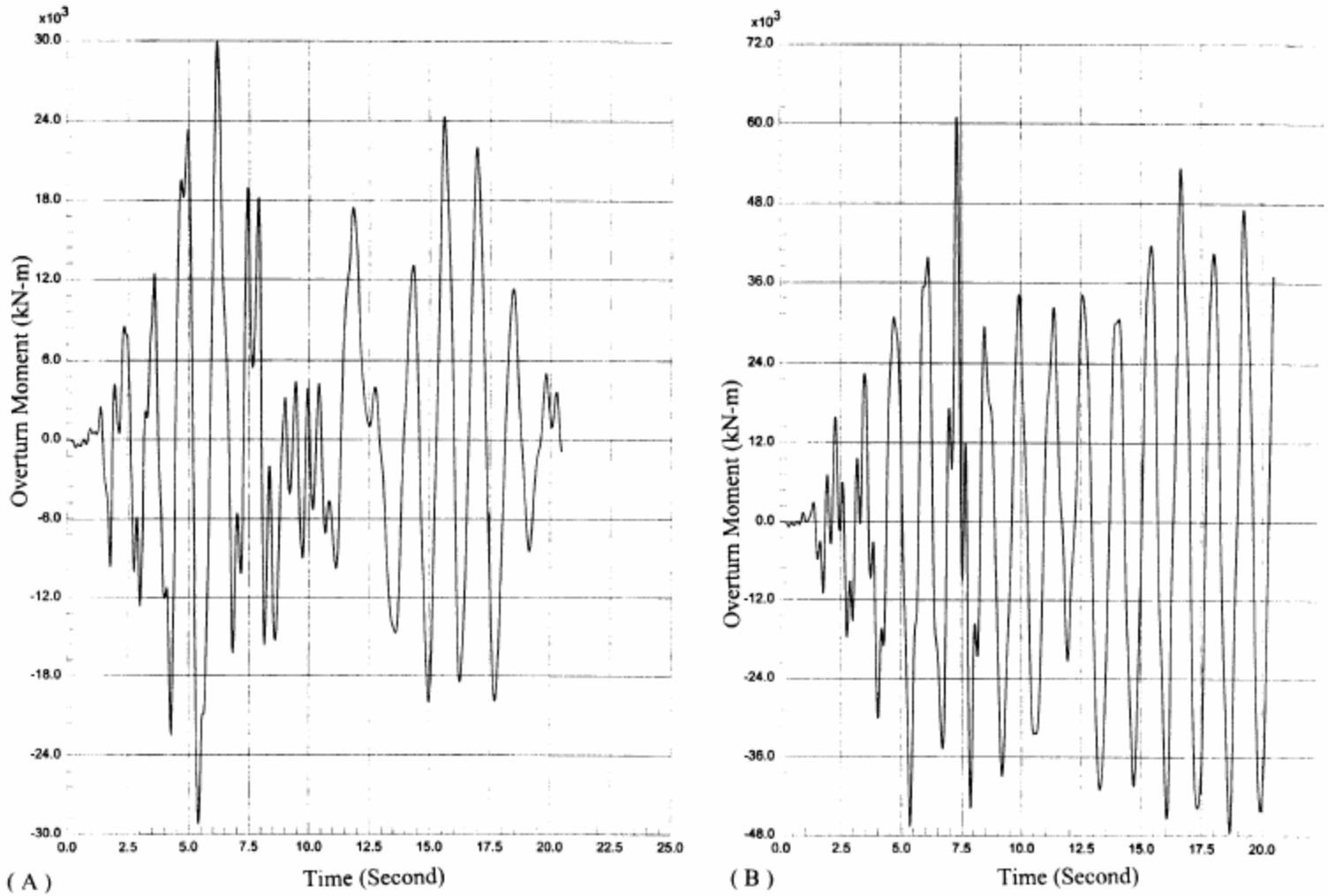


Fig. 6. Overturning moment of reactor structure (A) with soil-structure interaction (flexible base); and (B) no soil-structure interaction (rigid base).

Response spectrum analysis

An elastic dynamic analysis of a structure utilizing the peak dynamic response of all modes having a significant contribution to total structural response. Peak modal responses are calculated using the ordinates of the appropriate response spectrum curve which correspond to the modal periods. Maximum modal contributions are combined in a statistical manner to obtain an approximate total structural response.

The equivalent lateral seismic force representing elastic response, V_e, is calculated in accordance with the formula in NBC 1995.

$$V_e = v.\ S.\ I.\ F.W \tag{7}$$

where, v = 0.12, is zonal velocity ratio; S = 1.3, is seismic response factor that is relative to the fundamental period of vibration of the structure in the direction under consideration; I = 1.0, is seismic important factor of the structure; F = 1.3, foundation factor according to the local soil properties; and W is the dead load of structure.

The minimum lateral seismic force, V, is calculated in accordance with the following formula:

$$V = (V_e / R)\ U \tag{8}$$

Where R = 1.5, is the force modification factor that reflects the capability of a structure to dissipate energy through inelastic behavior; and U = 0.6, is a factor representing level of protection.

The most difficult part of the entire RSA (Response Spectrum Analysis) procedure is calculating the scaling factor. The unscaled RSA base shear is calculated using a finite element program RISA – 3D. Thus, Scale Factor = V/Unscaled RSA base shear.

The spectra are normalized using modal participation. In the calculation for scale factor, 30 vibration modes are calculated when make the modal participation to be over 90%. The response spectrum analysis is done for two base conditions, flexible base and fixed base. The seismic response and seismic forces are calculated from RISA-3D and shown in Table 5.

Table 5. Maximum Values of Seismic Response of Reactor Structure from Spectra Analysis

Base Conditions	Amplitude at Top of Reactor (mm)	Base Shear of Foundation (kN)	Base Moment of Foundation (kN-m)
Flexible Base, With SSI	205	1,944	61,000
Fixed Base, No SSI	187	2,073	65,600

From Table 5, it can be seen that the base conditions have some influence on the seismic response, but not a major amount. It is interesting to note that the seismic response and seismic forces generated from the response spectrum analysis are very close to those from the case of fixed base in the time history analysis. Thus, it can be deduced that the base conditions are not a sensitive factor for the response spectrum analysis.

CONCLUSIONS

The examination for seismic response of the reactor structure supported with different foundation conditions suggests the following conclusions:

1. The problem of soil-pile-structure interaction in a seismic environment is complex. The substructure method is a simple and realistic procedure requiring use of the commercial computer programs SAP2000 and DYNAN.

2. Nonlinear behavior of pile foundation can be simulated by means of the model of boundary zone with non-reflective interface. The validity of the model has been verified by dynamic experiments on full-scale pile foundations for both linear and nonlinear vibrations.

3. The theoretical prediction for a structure fixed on a rigid base without soil-structure interaction does not represent the real seismic response, since the stiffness is overestimated and the damping is underestimated.

4. The seismic response and seismic forces generated from the response spectrum analysis are very close to that from the time history analysis.

5. The nonlinear behavior of the soil – pile system and the group effect are two important factors which affect the stiffness and damping of foundation. A reasonable seismic analysis for industrial structures supported on pile foundations is needed to produce a safe and economic design.

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