SEISMIC REFLECTION IN AN UNSATURATED SURFACE SOIL LAYER

J. YANG\textsuperscript{1}, P.K.K. LEE\textsuperscript{1}, C.F. LEE\textsuperscript{1}

SUMMARY

Because of groundwater, soil is often modeled as a porous medium that is composed of a solid skeleton and water-filled pore spaces. In reality, however, it is a common situation that surface soils are not completely saturated. The partial saturation condition can be caused by fluctuation of water tables associated with natural or man-made processes. A detailed analysis of the 3-D borehole array records at a reclaimed site subjected to the 1995 Kobe earthquake has indicated that the saturation condition of near-surface soils may play a key role in the amplification of vertical ground motion. A great concern thus arises with further identification of the effect of saturation on seismic ground motions. In this paper, an analysis is made of a soil layer overlying rock formations subjected to inclined (SV) waves from the base. Both the soil and the bedrock are modeled as a partially saturated porous medium that is characterized by its degree of saturation, porosity and permeability. Attention of the present analysis is focused on the effect of saturation of the soil layer on the surface amplitudes and on the frequency-dependent amplification in both horizontal and vertical components. It is found that a change in degree of saturation has a significant influence on the frequency-dependent amplification in the vertical component but a negligible effect on the horizontal component. Generally, a greater amplitude ratio between the two components (denoted by $V/H$) may occur in a partially saturated soil layer.

INTRODUCTION

In geotechnical engineering soil is often modeled as a water-saturated porous material that is composed of solid skeleton and water-filled pore spaces. In reality, however, owing to fluctuating water tables associated with natural or man-made processes, it is common that surficial soils are not fully saturated. Partial saturation conditions also frequently exist in offshore sites constructed by land reclamation or in marine sediments. A detailed analysis of the 3-D borehole array observations at a reclaimed site during the Kobe earthquake has revealed that pore-water saturation of shallow soils may have a significant effect on the amplification of vertical ground motion \cite{1, 2}, raising a great interest in the importance of saturation conditions in interpreting field observations. It would be of especial value to investigate the effect of saturation on ground motions in both the horizontal and vertical components at ground surface and the relations between them. The investigation may shed light on the site-evaluation technique generally known as H/V, which is based on the interpretation of field observations on both the horizontal and

\textsuperscript{1} Department of Civil Engineering, The University of Hong Kong, Pokfulam, Hong Kong, China. E-mail: junyang@hkucc.hku.hk
vertical components of microtremors/ground motions and has been increasingly applied in engineering practice [3, 4].

In this study a problem of practical interest is analyzed, which corresponds to an inclined SV wave incident from the underlying rock formation at the base of a surface soil layer. This model has been extensively discussed in the past by assuming the soil and bedrock as an ordinary solid [5, 6]. The present analysis however assumes both the soil and bedrock as a partially water-saturated porous medium that is characterized by its degree of saturation, porosity and permeability. Attention of the analysis is focused on the effect of saturation of the soil layer on the surface amplitudes and the frequency-dependent amplification in both horizontal and vertical components.

**METHOD OF ANALYSIS**

As shown in Yang [7], the analysis is based on Biot’s theory [8] which models the interactions between the soil skeleton and pore fluid using the macroscopic laws of mechanics and on the concept of homogeneous pore fluid which assumes that the mixture of pore water and air can be approximately treated as an equivalent homogeneous fluid that completely fills the voids with a single pore pressure. In what follows the mathematical formulation is only described briefly, with a specific reference to the single-layered model. More details on the theoretical developments were given in Yang [7] where a formulation has been established for a general multi-layered system.

In a porous medium the compressional motion can be described in terms of potential functions as

\[
\left(\lambda + \alpha^2 M + 2G\right) \nabla^2 \Phi_s + \alpha M \nabla^2 \Phi_f = \rho \frac{\partial^2 \Phi_s}{\partial t^2} + \rho_f \frac{\partial^2 \Phi_f}{\partial t^2} + \rho \alpha^2 M \frac{\partial^2 \Phi}{\partial t^2} + \rho_f \alpha^2 M \frac{\partial^2 \Phi}{\partial t^2}
\]

(1)

\[
\alpha M \nabla^2 \Phi_s + M \nabla^2 \Phi_f = \rho_f \frac{\partial^2 \Phi_s}{\partial t^2} + \rho \frac{\partial^2 \Phi_f}{\partial t^2} + \frac{\eta}{k} \frac{\partial \Phi}{\partial t}
\]

(2)

and the distortional motion is described by

\[
G \nabla^2 \Psi_s = \rho \frac{\partial^2 \Psi_s}{\partial t^2} - \rho_f \frac{\partial^2 \Psi_f}{\partial t^2}
\]

(3)

\[
\rho_f \frac{\partial^2 \Psi_s}{\partial t^2} + \rho \frac{\partial^2 \Psi_f}{\partial t^2} + \frac{\eta}{k} \frac{\partial \Psi_f}{\partial t} = 0
\]

(4)

in which \( \lambda \) and \( G \) are Lamé’s constants of solid skeleton, \( \eta \) is fluid viscosity and \( k \) is permeability (with the unit \( m^2 \)). \( \alpha \) and \( M \) are parameters accounting for the compressibilities of grains and fluid and given by

\[
\alpha = 1 - \frac{K_b}{K_s}
\]

(5)

\[
M = \frac{K_s^2}{K_s \left(1 + n \frac{K_s}{K_f} - 1\right) - K_b}
\]

(6)

where \( K_s \) and \( K_b \) are bulk moduli of solid grains and skeleton, respectively; \( K_f \) is the bulk modulus of pore fluid and approximately relates to the degree of saturation, \( S_r \), bulk modulus of water, \( K_w \), and absolute pore pressure, \( p_a \), as
\[ K_f = \frac{1}{K_w + \frac{1-S_z}{p_a}} \]  

(7)

For the harmonic wave motion concerned, the potential functions for the layer (Fig. 1) can be written as

\[ \Phi_s = [A_{11} \exp(-iq_1z) + A_{21} \exp(-iq_2z) + A_{12} \exp(iq_1z) + A_{22} \exp(iq_2z)] \Omega(x,t) \]  

(8)

\[ \Phi_f = [\delta_1 A_{11} \exp(-iq_1z) + \delta_2 A_{21} \exp(-iq_2z) + \delta_1 A_{12} \exp(iq_1z) + \delta_2 A_{22} \exp(iq_2z)] \Omega(x,t) \]  

(9)

\[ \Psi_s = [B_{s1} \exp(-iq_3z) + B_{s2} \exp(iq_3z)] \Omega(x,t) \]  

(10)

\[ \Psi_f = [\delta_3 B_{s1} \exp(-iq_3z) + \delta_3 B_{s2} \exp(iq_3z)] \Omega(x,t) \]  

(11)

![Fig. 1. Model of analysis](image)

Similarly, the potentials for the half-space are given by

\[ \Psi_s = [B_x \exp(-iq_3z) + B_i \exp(iq_3z)] \Omega(x,t) \]  

(12)

\[ \Psi_f = [\delta_i B_x \exp(-iq_3z) + \delta_i B_i \exp(iq_3z)] \Omega(x,t) \]  

(13)

\[ \Phi_s = [A_{s1} \exp(-iq'_1z) + A_{s2} \exp(-iq'_2z)] \Omega(x,t) \]  

(14)

\[ \Phi_f = [\delta'_i A_{s1} \exp(-iq'_1z) + \delta'_i A_{s2} \exp(-iq'_2z)] \Omega(x,t) \]  

(15)

in which \( \Omega(x,t) = \exp[i(\omega t - px)] \), \( \delta_1, \delta_2, \delta_3, \delta'_1, \delta'_2, \) and \( \delta'_3 \) are the amplitude ratios [7].

Furthermore, the displacements and stresses in the layer and the half-space can be given in the matrix form as follows:

\[
\begin{bmatrix}
  u_x \\
  u_z \\
  w_z \\
  p_f \\
  \tau_{xz} \\
  \sigma_z
\end{bmatrix}_{-H<z<0} =
\begin{bmatrix}
  A_{11} + A_{12} \\
  A_{11} - A_{12} \\
  A_{21} + A_{22} \\
  A_{21} - A_{22} \\
  B_{s1} + B_{s2} \\
  B_{s1} - B_{s2}
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
  \alpha \\
  \beta \\
  \gamma
\end{bmatrix}_{-H<z<0}.
\]  

(16)
can be computed. Reference is made to Yang [7] for the coefficients in the matrices $[B']_{6 \times 6}$.

Now Eq. (20) can be solved for the amplitudes of the potentials and then the displacements and stresses can be computed. Reference is made to Yang [7] for the coefficients in the matrices $[P]$ and $[B]$.

**NUMERICAL RESULTS AND DISCUSSIONS**

Consider a 30-m sand layer overlying soft rock formations. The main properties of the sand and the rock are given in Table 1. In the analysis the half-space bedrock is assumed as completely saturated while the surface layer is either fully or partially saturated.

Fig. 2 shows the layer surface and base amplitudes in the horizontal and vertical components as a function of incident angle. Here the thin line denotes that the sand layer is fully saturated while the thick line denotes that the layer is partially saturated with a degree of saturation 95%. The frequency of the incident (SV) wave is taken as 1 Hz. It is observed that even a slight change in the degree of saturation may affect the surface amplitudes, especially the vertical component. In the meantime, this effect is angle dependent. Compared to the surface amplitudes, the influence of saturation is relatively small on the base amplitudes.
Table 1. Properties of soil and rock

<table>
<thead>
<tr>
<th></th>
<th>$\rho_s$ (kg/m$^3$)</th>
<th>$n$</th>
<th>$G$ (MPa)</th>
<th>$K_h$ (MPa)</th>
<th>$K_s$ (GPa)</th>
<th>$\eta$ (Ns/m$^2$)</th>
<th>$k$ (m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>2650</td>
<td>0.45</td>
<td>40</td>
<td>66.7</td>
<td>36</td>
<td>$10^{-3}$</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>Soft rock</td>
<td>2610</td>
<td>0.32</td>
<td>900</td>
<td>1500</td>
<td>36</td>
<td>$10^{-3}$</td>
<td>$10^{-14}$</td>
</tr>
</tbody>
</table>

Fig. 2. Effect of saturation on surface and base amplitudes (Thick line: unsaturated layer; Thin line: saturated layer)

In Fig. 3 the amplitude ratios between the vertical and horizontal components (denoted by V/H) at the surface are presented against the angle of incidence. It is clear that the effect of saturation on (V/H) is significant, especially at large angles of incidence. For incident angles less than about 20 deg, the ratio (V/H) is larger in a partially saturated layer than in a fully saturated layer. For example, at an incident angle of 5 degrees, the ratio (V/H) in the case of partially saturated layer is one and a half times that in the case of fully saturated layer.
Fig. 3 Effect of saturation on amplitude ratios (V/H) at surface

Fig. 4 shows the amplification in the horizontal and vertical components as a function of frequency. The frequency-dependent amplitude ratio (V/H) at surface is shown in Fig. 5. Here the incident angle of the (SV) wave is assumed as 10 degrees. It is of interest to note that the effect of saturation on the vertical amplification is large but is negligible on the horizontal amplification. The amplitude ratios (V/H) at the surface are also significantly affected by the saturation condition. In general, a greater (V/H) will occur in a partially saturated layer than in a fully saturated layer. This result is in good agreement with the field observations from the Kobe case [1].

Fig. 4. Frequency-dependent amplification in vertical and horizontal components

The stiffness of the soil layer is one of key factors influencing the site response. In Fig. 6 the frequency-dependent ratio (V/H) at the surface is presented for two cases of soil stiffness. The broken line represents
the case of $G=10$ MPa while the solid line the case of $G=120$ MPa. In the computation the surface layer is assumed as fully saturated and the incident angle is taken as 10 degrees. Fig. 6 indicates a reasonable feature that the peak in the (V/H) response shifts to low frequency end when the stiffness of the surface layer is decreased.

Fig. 5. Frequency-dependent amplitude ratios (V/H) at surface

Fig. 6. Influence of soil stiffness on the amplitude ratios (V/H)
CONCLUSIONS

A study on the saturation effect of a surface soil layer on site response to inclined (SV) waves has been presented. The model of analysis is a surface layer overlying an infinite rock formation, for which the rock is assumed as fully saturated by water while the surface soil is assumed as either fully or partially saturated. Numerical results indicate that the saturation conditions may produce a significant influence on ground motion amplification in the vertical component but a small influence on the amplification in the horizontal component. In general, a greater amplitude ratio (V/H) at the surface will occur if the surface soil layer is not fully saturated. The analysis indicates the importance of identifying the saturation conditions in the interpretation of field observations.

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REFERENCES