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A THEORETICAL MODEL FOR SITE COEFFICIENTS IN BUILDING CODE PROVISIONS

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SUMMARY

Theoretical solutions for wave propagation in layered anelastic media provide the theoretical basis for seismic wave propagation in layered soil and rock deposits. These exact theoretical solutions predict that physical characteristics of two and three dimensional wave fields in layered anelastic media are distinct from those predicted by elastic models or one dimensional anelastic models. They predict that the velocity, damping, particle motion, direction of energy flow, kinetic energy, potential energy and energy flux for anelastic waves vary with angle of incidence. These variations are most significant for anelastic waves in soils with large amounts of damping or for anelastic waves incident at the edge of a basin or near critical refraction angles.

Application of these results to the problem of site response characterization for purposes of building code provisions shows that the theoretical solution for a viscoelastic soil layer over rock provides a simple theoretical model for site coefficients, Fa and Fv, in current building code provisions (e.g. NEHRP, IBC, ICC, and ASCE 7 provisions). Parametric analyses conducted from the theoretical solutions with Mathematica show that average site response characteristics for vertically incident Type-II S waves are in good agreement with those specified in current code provisions as a function of site class (shear velocity) and base acceleration. With the exception of an inhomogeneous wave field incident at a shallow angle of incidence on a basin margin, the parametric analyses suggest that Fa and Fv in code provisions conservatively account for the average short- and long-period characteristics of the dynamic response of an anelastic soil layer.

INTRODUCTION

The theory developed to account for 2D and 3D wave fields in a layered viscoelastic media predicts several physical characteristics for P, Type-I S and Type-II S body waves and Rayleigh type surface waves not predicted for corresponding waves in elastic media. In addition to the well known characteristics of damping and dispersion predicted for 1D viscoelastic wave fields, the theory predicts that 2D and 3D waves in layered anelastic media are predominantly inhomogeneous with the amount of inhomogeneity in each layer dependent on angle of incidence.

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PHYSICAL CHARACTERISTICS OF INHOMOGENEOUS WAVES

Inhomogeneous wave fields in anelastic media result whenever 2D and 3D wave fields propagate at non-normal angles across boundaries between materials with different amounts of damping or Q^{-1} as illustrated in Figure 1. For a soil-rock boundary, the increased amount of damping in the soil results in amplitudes decreasing along surfaces of constant phase as illustrated in Figure 1. As a result the refracted wave field is inhomogeneous with the rate at which the amplitudes vary dependent on the angle of incidence. This inhomogeneity of anelastic wave fields implies that refracted P and S body waves and Rayleigh-type surface waves have velocities, damping, directions of energy flow, kinetic and potential energies, and energy flow due to interaction of inhomogeneous stress and velocity fields distinct from those for corresponding 1D anelastic wave fields. The theory predicts elliptical particle motions for inhomogeneous P waves, Type-I S waves, and Rayleigh type surface waves, but transverse motions for inhomogeneous Type-II S waves and Love waves (Borcherdt [1], [2], [3], [4], [5], Silva [6]).

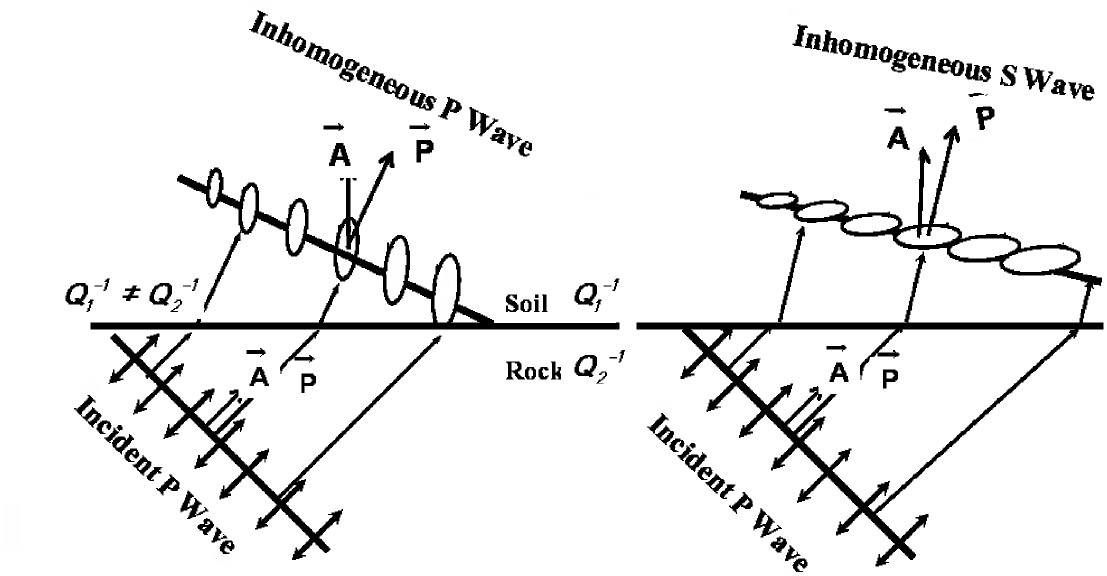

Figure 1. Diagram illustrating that refracted 2D waves in layered anelastic media are in general inhomogeneous with amplitudes varying along surfaces of constant phase. As a result, the physical characteristics of the wave fields, such as velocity, attenuation, particle motion, and energy flux vary with the amount of inhomogeneity and in turn the angle of incidence.

Empirical and numerical results show that the unique characteristics of anelastic P and S waves are most distinctive in media or soils with significant amounts of damping and in wave propagation situations in which the degree of inhomogeneity of the wave fields is largest as might occur for wave fields incident at the edge of a basin or near critical refraction angles (Borcherdt [7], [8]). Numerical results for a linear S wave refracted and reflected at a boundary between two sediment layers with large amounts of damping are shown in Figure 2. The results are calculated as a function of angle of incidence for a linear S wave with degree of inhomogeneity fixed at 60 degrees (thick solid lines, Figure 2). For comparison results are calculated using the incorrect assumption that the refracted wave is homogenous for angles greater than the elastic critical angle. The results show that the phase velocity, energy velocity, amplitude reflection and transmission coefficients, and the energy reflection and transmission coefficients differ significantly from those that would be incorrectly predicted using elastic theory.

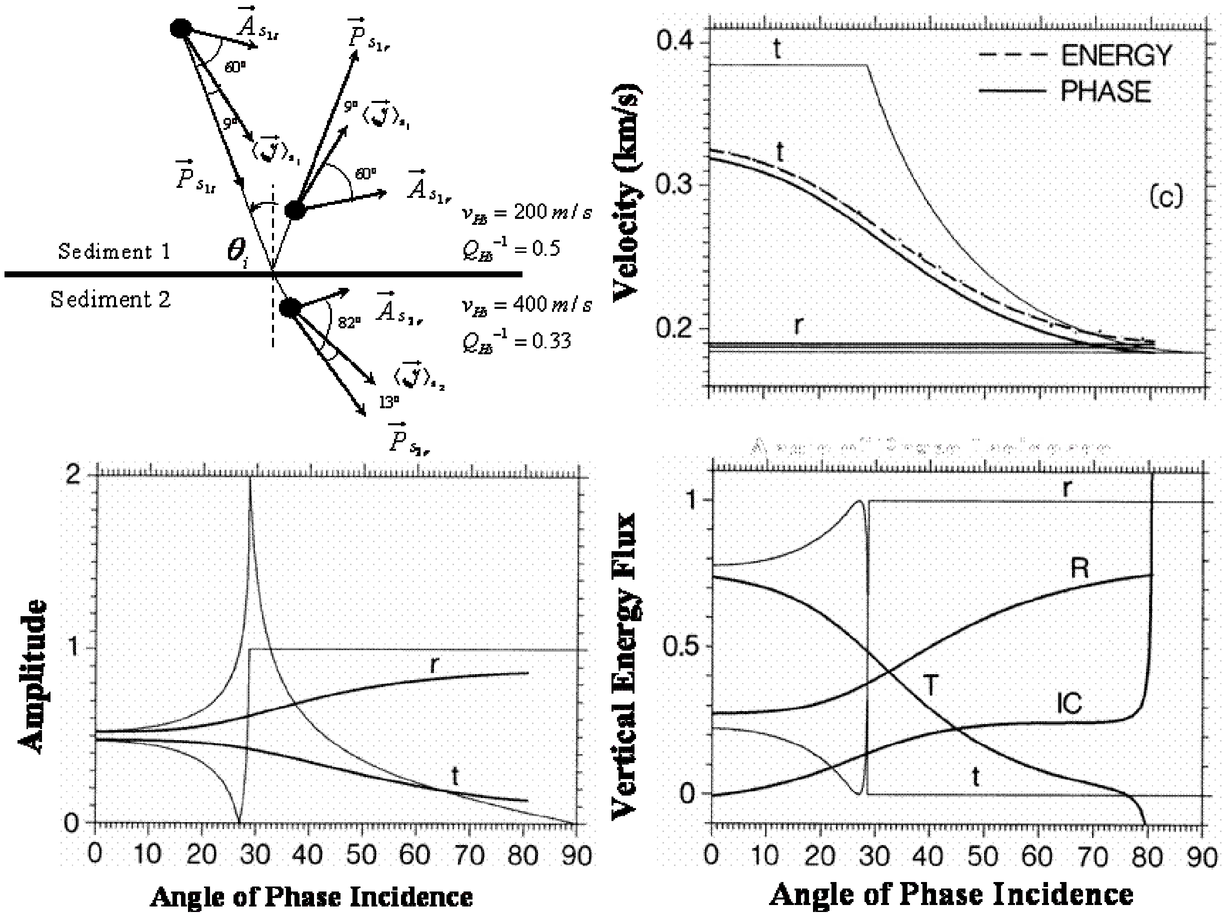

Figure 2. Physical characteristics calculated for an inhomogeneous linear S wave incident on a boundary between two soil layers with large damping ratios. The numerical results show that the characteristics of the refracted inhomogeneous linear S wave differ significantly from those that would be calculated by incorrectly assuming that the refracted wave was homogeneous for angles less than critical.

Laboratory experiments and analysis of seismic wave fields provide empirical confirmation of the distinct characteristics for body and surface waves in layered anelastic media (Becker [9], [10], Borcherdt [11], [12], [13]). The theory provides exact solutions for reflection and refraction coefficients that account for previously unexplained textbook results concerning the reflection coefficients for a water-stainless steel boundary (Brekhovskikh [14]). Numerical and empirical results for a water stainless steel boundary are shown in Figure 3. The excellent agreement between the results as calculated and measured for frequencies of 10 and 16 MHz provides evidence that helps confirm the theory. The significant dip in the reflection coefficient at an angle beyond the elastic critical angle is strongly dependent on the damping ratio or Q^{-1} . This characteristic of the reflection coefficient is used commercially for quality assurance purposes to locate impurities in high quality stainless steel needed for certain applications (Becker [9]).

A THEORETICAL MODEL FOR SITE COEFFICIENTS IN BUILDING CODE PROVISIONS

The general theory for 2D and 3D wave propagation in layered viscoelastic media provides a theoretical basis to account for site coefficients as proposed in current IBC [15] and ICC [16] code provisions. A simple theoretical model is that of a Type-II S wave incident at the base of a viscoelastic soil layer. Closed form solutions for the problem permit the response to be calculated quickly using *Mathematica* as a function of material parameters of the soil and underlying rock and parameters of the incident wave.

The model, if only evaluated for a normally incident homogeneous Type-II S wave, provides results that correspond to the 1D response that might be calculated for an incident S wave using the SHAKE program (Schnabel [17]) with material damping ratios chosen to correspond to the appropriate input ground motion level. Layer responses for this case, calculated as a function of period as normalized by the fundamental period of the site and the ratio of the soil and rock shear velocities for damping ratios of 5 % and 20 %, are shown for an incident homogeneous Type-II S wave in Figure 4. Approximate site-class boundaries and site coefficients Fa and Fv as specified in the IBC are superimposed.

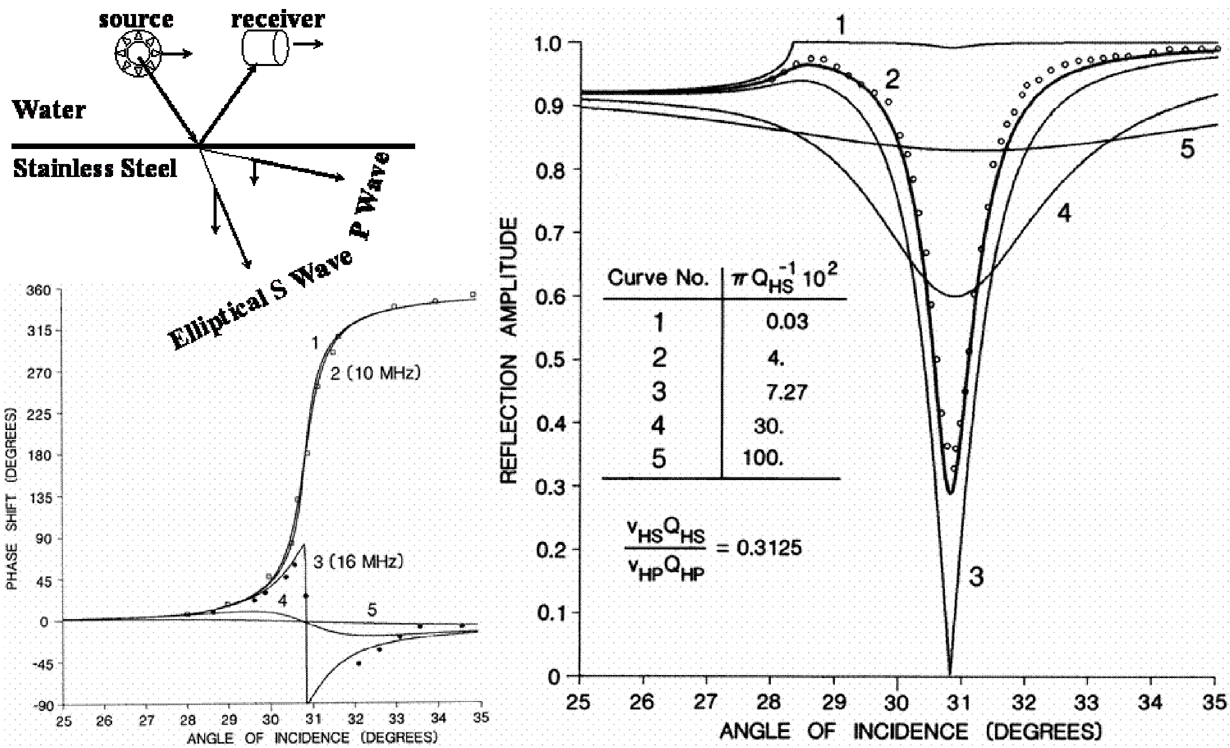

Figure 3. Comparison of theoretical predictions of the amplitude and phase calculated for a harmonic acoustic line source in water overlying a sheet of stainless steel. The excellent agreement between the numerical predictions of amplitude and phase for frequencies of 10 and 16 MHz provides excellent confirmation of the theory developed for 2D and 3D waves in layered viscoelastic media (from Borcherdt [8]).

The theoretical results for Fa and Fv in Figure 4 show dependencies on shear velocity and base acceleration or damping ratio that are in good agreement with those in the IBC as summarized in Figure 5 and in the commentary of the NEHRP recommended provisions. The agreement confirms values in the IBC previously based on empirical results derived from the Loma Prieta earthquake (Borcherdt [18]) and numerical SHAKE results (Seed [19]). The theoretical results suggest that further refinement in the values might be afforded for site class E and D sites using amplitude response as provided by the calculated spectrum.

The theoretical developments for 2D and 3D wave propagation in viscoelastic media raise important questions concerning the role of inhomogeneity of the wave fields on the response of layered soil deposits and corresponding IBC site coefficients. The response of a site class E soil layer with a damping ratio of

10 % is calculated as a function of angle of incidence for incident homogeneous and inhomogeneous Type-II S waves (Figure 6). The results show that inhomogeneity of the wave field causes the response of the surface layer to increase as the angle of incidence of the wave field approaches grazing incidence. This situation is most likely to occur when pseudo surface waves are being generated near edges of basin margins. Otherwise, the model results for incident homogeneous and inhomogeneous waves suggest that site coefficients implied by the simple vertically incident model provide conservative estimates of Fa and Fv for purposes of code provisions. The closed form solutions as parameterized herein for a viscoelastic layer are proposed as a general theoretical model to account for the principle characteristics of site coefficients in present editions of the IBC and the 2001 edition of the ICC

Recent empirical estimates of Fa and Fv, as derived from the Northridge strong-motion recordings and v_{S30} measurements are consistent at the 95 percent confidence level with the IBC coefficients for base acceleration levels greater than about 0.25 g (Borcherdt [20]). The results provide empirical confirmation of the dependence on base acceleration level and further establish the need for both a short- and a long-period site coefficient. Each of these results is confirmed by the theoretical responses shown in Figures 4 and 5. The agreement between these recent empirical results and the theoretical results provide additional evidence in support of the site coefficients specified in the 2000 IBC [15] and the 2001 ICC [16] edition.

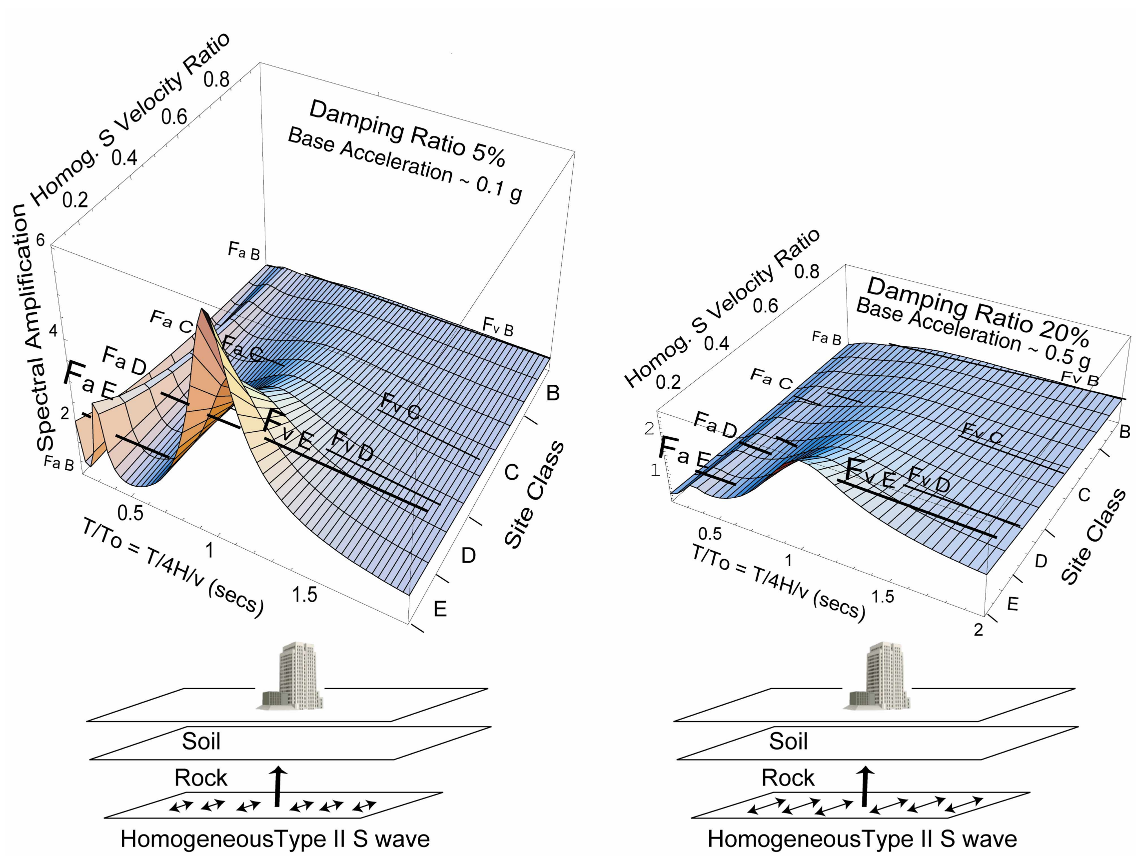

Figure 4.Theoretical amplitude response of a soil layer to a vertically incident homogeneous Type-II S wave calculated as a function of normalized period, homogeneous S velocity ratio, and damping ratio. The average short- and long-period theoretical response variations are consistent with variations in the Fa and Fv site coefficients in the IBC as shown in Figure 5.

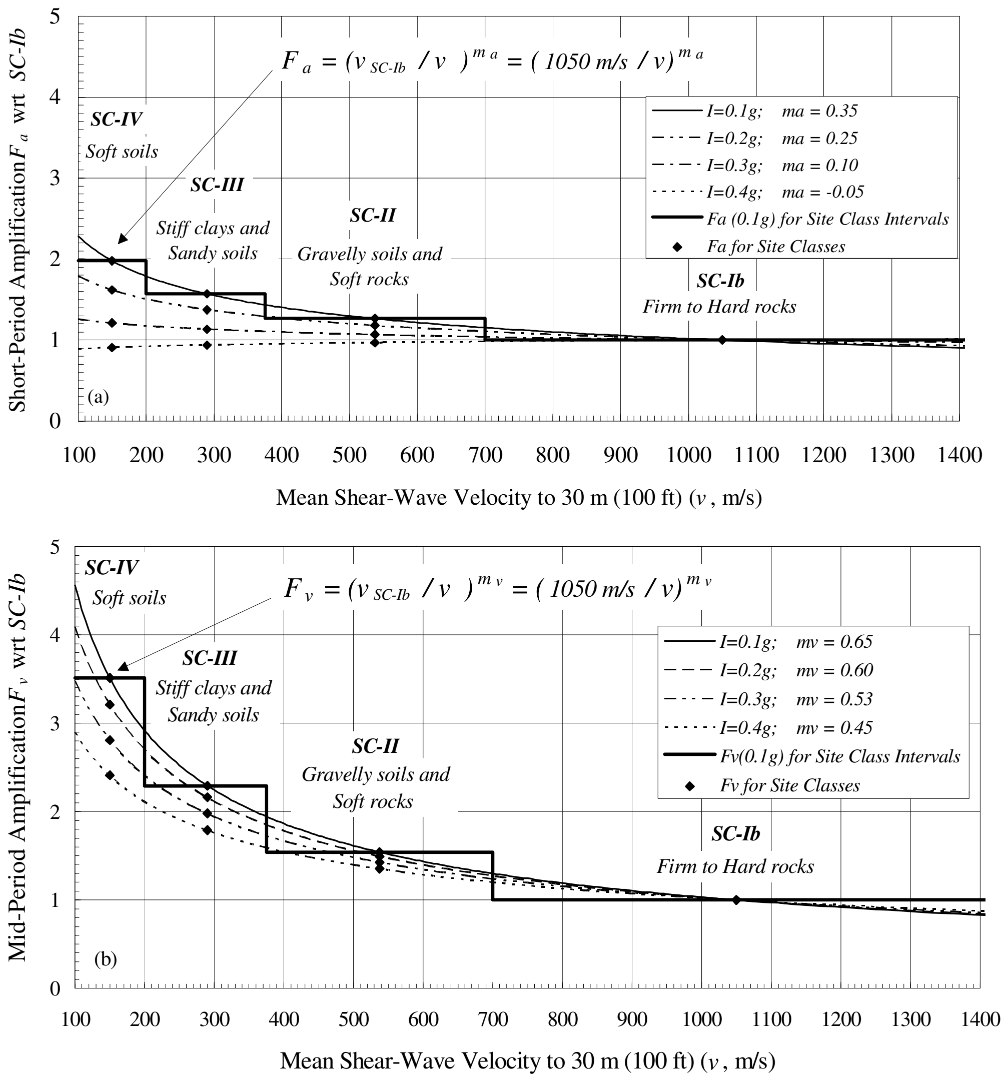

Figure 5. Short-period and long-period site coefficients Fa (a) and Fv (b) expressed as a function of shear velocity to 30 meters and input base acceleration as adopted in recent code provisions (from Borcherdt [18]).

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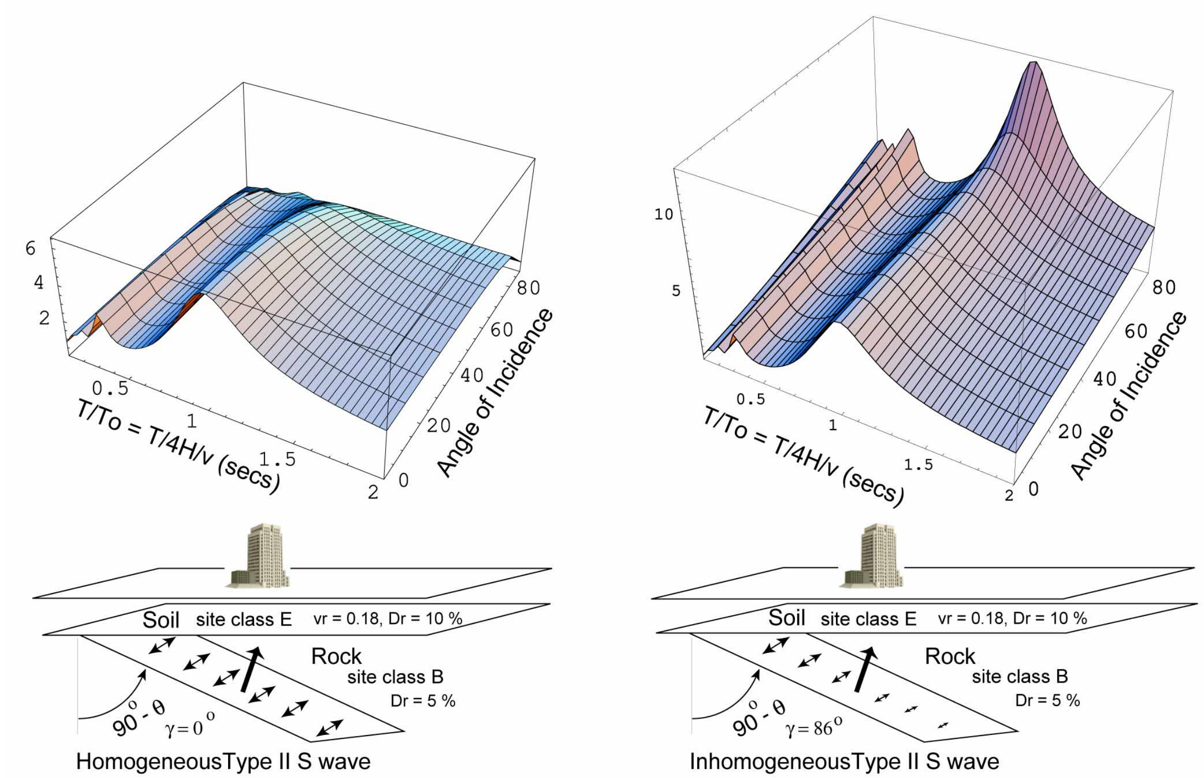

Figure 6. Theoretical amplitude response of a viscoelastic soil layer (site class E, 180 m/s) to homogeneous and inhomogeneous Type-II S waves as a function of angle of incidence. The responses show that inhomogeneity of the wave field causes the response of the surface layer to increase as the angle of incidence increases. The closed form solutions parameterized herein are proposed as a simple theoretical model to account for the dominant characteristics of site coefficients in the 2000 edition of the IBC [15] and the 2001 edition of the ICC [16].

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