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MINIMUM WEIGHT ASEISMIC DESIGN OF STEEL FRAMES CONSIDERING THE COLLAPSE MECHANISM AND CUMULATIVE DAMAGE CONSTRAINTS

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SUMMARY

This paper describes the minimum weight design problem of steel frames considering the philosophy of strong column-weak beam and cumulative damage index. Following three constraints are considered in this approach. The first is the strong column-weak beam requirement under the earthquake load determined by response spectrum method. The second is the condition to form the plastic hinges in each story simultaneously to prevent the local concentration of the damage. The final is the prescribed cumulative damage level of each plastic hinge. The optimization procedure is the sequential linear programming (SLP) using the sensitive analysis and the simplex method. This approach is applied to 5-story, 3-bay and 8-story, 5-bay steel frames, and the optimum sections are obtained. The dynamic elastic plastic analyses of minimum weight frames are performed and the achievement of the prescribed cumulative damage of each plastic hinge is checked.

INTRODUCTION

In the seismic design of building structures, it is desirable to prevent the damage concentration at the specific member and story. The reason is that the damage concentration often causes the local collapse of the story and degradation of the member. This paper presents the minimum weight building design method, which prevents the local collapse of the specific story and the local damage concentration of the specific member. Following three constraints are considered in this approach. The first is the strong column-weak beam requirement under the earthquake load determined by response spectrum method. The philosophy of strong column-weak beam is generally recommended in the seismic design, and many studies on this philosophy have been presented [1,2,3,4]. Here, the sufficient condition to form the desirable

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collapse mechanism [7] is used. The second is the condition to form the plastic hinges in each story simultaneously. It is expected that this constraint approximately equally distribute the damage of each member for earthquakes. The final is the member cumulative damage level of each plastic hinge. The cumulative damage is the important index that shows the limit state condition of the frame. However there are few researches on the minimum weight design considering this index as a constraint, although there are some researches on the ductility-constrained design [4].

The optimization problems considering above-mentioned three constraints are solved by the SLP method, and the effectiveness of this design approach is verified by time history analysis.

FORMULATION

Seismic Design Spectrum and Earthquake Loadings for Buildings

The following seismic design spectrum is employed for multi-story buildings in this study.

$$S_A(T,h)=\begin{cases}(1+6T)\cdot\alpha(h)\cdot A & (0\le T\le 0.25)\\ 2.5\cdot\alpha(h)\cdot A & (0.25\le T\le 0.5)\\ \dfrac{1.25}{T}\cdot\alpha(h)\cdot A & (0.5\le T)\end{cases} \qquad (1\sim3)$$

where T is natural period of the building, h is damping ratio, S_A is the acceleration design spectrum. This design spectrum is the simplified version of that shown in "Recommendations for Loads on Buildings" of Architectural Institute of Japan [5].

$\alpha(h)$ is the response reduction factor caused by damping, which is shown in the following equation [6].

$$\alpha(h)=\frac{R(h)}{R(0.02)}\quad ,R(h)=\sqrt{\frac{1-e^{-4\pi h\tau}}{4\pi h\tau}\left[0.424+\ln\{4\pi h\tau+1.78\}\right]} \qquad (4)$$

where τ is the ratio of earthquake duration time to natural period of the building. In this paper, τ =10.

The lateral design shear force at the i-th story Q_i is determined by SRSS method. Q_i is computed from the following equation.

$$Q_i=\sqrt{\sum_{k=1}^{n}\left[\left(\sum_{j=1}^{n}m_j\beta_k u_{jk}\right)\cdot S_A(T_k,h_k)\right]^2} \qquad (5)$$

where n is the number of stories, β_k is the participation factor for the k-th mode, m_j is the mass of the j-th story, u_{jk} is the j-th mode shape, T_k is the k-th mode period, h_k is the k-th mode damping ratio.

The First Constraints (Strong Column-Weak Beam Requirement)

The sufficient condition to form the desirable strong column-weak beam mechanism is that the function V_E shown in the following equation is positive at all the column-ends except for the base of columns on the first story [7].

$$V_{Ei}=Z_{Pi}s_{y_i}-\left|m_{Xi}\right|\ge 0 \quad (i=1,2,...,N_E) \qquad (6)$$

where Z_{pi} and s_{yi} are the plastic section modulus and the yield strength for the i-th prescribed column-end, N_E is the number of the prescribed column-ends, m_{Xi} is the maximum bending moment, which can be computed from the push over analysis assuming that the prescribed column-ends are elastic.

The Second Constraints (Simultaneous Yielding)

It is expected that the earthquake damages are approximately equal in all stories in case of the structures whose shear-yield-strength distribution is equal to that of story-shear-force response for elastic system [8]. Here, the following constraints are employed for the beams on all stories to be yielding simultaneously under the proportional loading based on Eq.(5).

$$\frac{m_{PBi}/m_{\max Bi}}{\left(m_{PB}/m_{\max B}\right)_M} \geq 1-\varepsilon_B \qquad (i=1,2,....,N_{DB}) \tag{7}$$

where m_{PBi} is the plastic moment of the i-th beam group of equal cross section, m_{maxBi} is the maximum bending moment of the i-th beam group under the static load based on Eq.(5), N_{DB} is the number of beam groups, $\varepsilon_B = 0.001$.

$(m_{PB}/m_{maxB})_M$ is defined by the following equation.

$$\left(m_{PB}/m_{\max B}\right)_M = \frac{\sum_{i=1}^{N_{DB}}\left(m_{PBi}/m_{\max Bi}\right)}{N_{DB}} \tag{8}$$

In addition to the above-mentioned constraints, the following constraints are employed to avoid the damage concentration of the bottom of the first story columns.

$$\frac{m_{PCi}/m_{\max Ci}}{\left(m_{CB}/m_{\max B}\right)_M} \geq 1 \qquad (i=1,2,....,N_{DC}) \tag{9}$$

where m_{PCi} is the plastic moment of the i-th column group of equal cross section, m_{maxCi} is the maximum bending moment of i-th column group under the static load based on Eq.(5), N_{DC} is the number of column groups. m_{maxBi} and m_{maxCi} can be computed from elastic structural analysis. It is expected that these constraints equalize the earthquake damage in each story.

The Third Constraint (Cumulative Damage Level)

Here, η_S, the cumulative damage index for the strong column-weak beam structures is defined as the following equation.

$$\eta_S = \frac{E_p}{\sum_{j=1}^{NHB} m_{PBj}\theta_{yBj} + \sum_{k=1}^{NHC} m_{PCk}\theta_{yCk}} \tag{10}$$

where N_{HB} , N_{HC} are the number of all beam ends and the number of all column bases respectively, θ_{yBj} , θ_{yCk} are the elastic limit of the member-end rotation for the j-th beam ends and the k-th column-base respectively. η_S defined by Eq.(10) represents approximately the average of cumulative damage of column-ends and beam-ends. E_p in Eq.(10) denotes the plastic energy dissipation of the building structures. Housner [9] predicted the energy input responsible for the damage in the elastic-plastic system by using the velocity response spectra in the elastic system. E_p is computed approximately from the following equation.

$$E_P = \frac{1}{2}M\{S_{V\,MAX}\}^2 \tag{11}$$

where M is the mass of the building, S_{VMAX} is the maximum value of the velocity design spectrum.

The cumulative damage constraint is shown in the following equation.

$$\eta_S \le \eta_0 \tag{12}$$

where η_0 is the constraint value.

Optimum Design Problem

Assuming that the member cross-sections are similar in shape, the plastic section modulus *Zp* and the moment of inertia *I* can be described as the following equations.

$$Z_p = \alpha_{ZP} \cdot A^{1.5},\ I = \alpha_I \cdot A^2 \tag{13),(14}$$

The optimum design problem can be stated as follows :

Find the design variables $A_1, A_2, ..., A_{ND}$ which minimize

$$W = \rho \sum_{i=1}^{ND} A_i L_i \tag{15}$$

subjected to

$$G_j \le 1 \quad (j = 1,.., N_M) \tag{16}$$

$$A_{Li} \le A_i \le A_{Ui} \quad (i = 1,..., N_D) \tag{17}$$

where L_i is the total length of the members corresponding to the design variable A_i, ρ is the specific gravity of their materials, N_D is the number of design variables, N_M is the number of constraints, G_j is the constraint function computed from Eq.(6),(7),(9),(12), A_{Li},A_{Ui} are the upper and lower bounds, respectively, for design variable A_i.

OPTIMIZATION ALGORITHM

The above-mentioned optimum design problem can be solved by the sequential linear programming method (SLP)[10]. The optimization algorithm is shown as follows.

[Step1] Input the constraint value η_0 and the initial design variables $\{A_I\}$.

[Step2] $\{A^a\}=\{A_I\}$, $\alpha_M = 0.1$.

[Step3] Compute the natural periods and mode shapes from the eigen analysis, and compute the earthquake load from Eq.(5).

[Step4] Calculate the design sensitivities for the design variable $\{A^a\}$, $\left[\nabla G^a\right]$ by finite differences.

[Step5] Set up the approximate linear programming problem shown as follows :

Find $\{A\}=(A_1,A_2,\ldots,A_{ND})^T$ which minimize the structural weight W subjected to

$$\left[\nabla G^a\right]\left[\{A\} - \{A^a\}\right] \le \{1\} \tag{18}$$

$$(1 - \alpha_M)\{A^a\} \le \{A\} \le (1 + \alpha_M)\{A^a\} \tag{19}$$

$$\{A_L\} \le \{A\} \le \{A_U\} \tag{20}$$

[Step6] Solve the linear programming problem with the simplex method, and get the solution $\{A_{LP}\}$.

[Step7] Check the convergence for the structural weight. If the convergence condition is satisfied, $\{A^a\}=\{A_{LP}\}$ and go to Step8, otherwise $\{A^a\}=\{A_{LP}\}$ and go to Step4.

[Step8] Check the convergence for the natural periods. If the convergence condition is satisfied, the calculation shall be finished, otherwise $\{A^a\}=\{A_{LP}\}$ and go to Step3.

DESIGN EXAMPLES

5-story,3-bay and 8-story,5-bay steel frames shown in Fig.1(A),(B) are designed by this presented method. 35 members of the 5-story structure are categorized into 7 design variable groups as indicated in

Fig.1(A). 88 members of the 8-story structure are categorized into 11 design variable groups as indicated in Fig.1(B). The design constants are shown in Table 1. The design conditions are shown in Table 2. The optimum solutions are shown in Table 3(A),(B). A,Mp,W,T_1 and NL in Table3(A),(B) are the cross sectional areas of members, the plastic moments of members, the structural weight, the first mode period and the number of iterations of SLP, respectively. The initial section areas for column members are 200 cm^2, and those of beam members are 100 cm^2 in these examples. It is confirmed that this SLP algorithm provides good convergence within 20 iterations. Table 4 shows the solutions for frame I-C obtained from 20 initial design points based on random numbers. It is observed that there are few differences among these solutions except for one solution.

TIME HISTORY ANALYSISES

The time history analyses of minimum weight frames obtained by the prescribed optimization algorithm are performed. Input earthquake motions are 10 different artificial acceleration records compatible with the prescribed seismic design spectrum shown in Eqs.(1),(2),(3). These earthquake records are non-stationary waves based on the envelope curve given by the following equations according to Jennings model.

$$E(t) = \begin{cases} (t/T_b)^2 & (0 \le t \le T_b) \\ 1 & (T_b \le t \le T_c) \\ e^{-a(t-Tc)} & (T_c \le t \le T_d) \end{cases} \tag{21}$$

where t is the time, $E(t)$ is the envelope function, T_b=3.8(sec.),T_c=19.4(sec.),T_d=40.96(sec.), a =0.11. One of these acceleration records is shown in Fig.2. The time history analysis program [11] uses Newmark-Beta method (Beta=1/4) for the time integration. The time interval for numerical integration is 0.005 second. The plastic stiffness matrix is generated by generalized plastic hinge theory considering Prager's strain hardening rule. It is assumed that the stress-strain relationship is bilinear and the post-yielding stiffness is 0.01E. This analysis uses the damping matrix proportional to the stiffness matrix.

The cumulative damages shown in Fig.3(A),(B) are the ratios of cumulative rotations of plastic hinges to the elastic limits of member-ends rotations. The response values are averages for prescribed 10 earthquakes. It is confirmed from Fig.3(A),(B) that cumulative damages are quite small at column-ends except for the bottom of the first story columns. These results show that Eqs.(6) function well not only under static loadings but also under seismic loadings. It is also confirmed that cumulative damages for frames I-B,I-C,I-D,II-B,II-C and II-D with constraints(7),(9) are approximately equal in each story although those for frames I-A,II-A without constraints(7),(9) vary greatly in each story. These results show that Eqs.(7) and (9) equalize the earthquake damages in each story. It is also confirmed that cumulative damages for frames I-B,I-C,I-D,II-B,II-C and II-D are close to the limiting value η_0.

CONCLUSIONS

Minimum weight aseismic design of steel frames considering the collapse mechanism and cumulative damage constraints has been presented. The optimum solutions considering three constraints on the collapse mechanism and cumulative damage constraints have been obtained by the SLP method. It has been confirmed that the optimization algorithm based on the SLP method provides good convergence. It has been confirmed from the time history analysis of optimum frames that three constraints presented in this paper function well not only under static loadings but also under seismic loadings.

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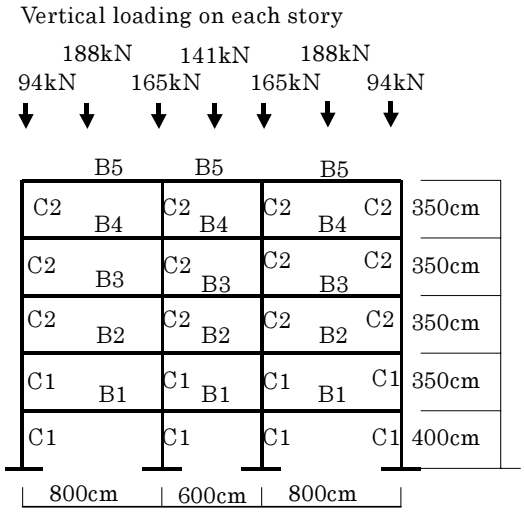

Fig.1(A) Frame I

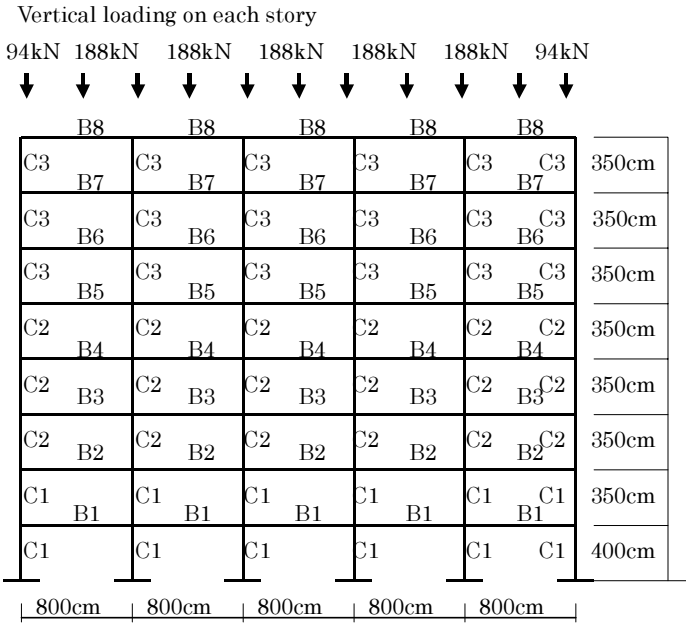

Fig.1(B) Frame II

Table1 Design constants

α_{ZP} (box column)	0.949
α_{ZP} (H-beam)	1.783
α_{I} (box column)	1.076
α_{I} (H-beam)	3.648
Yield Strength Fy	323(N/mm^2)
Young's Modulus E	205800(N/mm^2)
η_0	3.0,6.0,9.0

Table2 Design conditions

	story,bay	η_0	contraints
I-A	5-story 3-bay	3.0	Eqs.(6),(12)
I-B	5-story 3-bay	3.0	Eqs.(6),(7),(9),(12)
I-C	5-story 3-bay	6.0	Eqs.(6),(7),(9),(12)
I-D	5-story 3-bay	9.0	Eqs.(6),(7),(9),(12)
II-A	8-story 5-bay	3.0	Eqs.(6),(12)
II-B	8-story 5-bay	3.0	Eqs.(6),(7),(9),(12)
II-C	8-story 5-bay	6.0	Eqs.(6),(7),(9),(12)
II-D	8-story 5-bay	9.0	Eqs.(6),(7),(9),(12)

Table3(A) Optimum solutions (Frame I)

	I-A		I-B		I-C		I-D	
	A	Mp	A	Mp	A	Mp	A	Mp
C1	432.4	2759.7	478.2	3210.5	252.8	1233.8	207.2	915.7
C2	375.8	2236.4	378.9	2264.8	207.5	917.7	189.5	800.9
B1	264.7	2483.3	252.2	2309.9	127.2	827.7	100.2	578.4
B2	256.9	2375.5	257.4	2381.4	131.7	871.5	105.1	621.5
B3	207.7	1725.8	224.6	1941.4	113.7	698.8	88.7	481.9
B4	210.7	1763.0	189.2	1500.4	86.1	460.6	75.5	378.4
B5	75.5	378.4	75.5	378.3	75.5	378.4	75.5	378.4
Unit	cm^2	kNm	cm^2	kNm	cm^2	kNm	cm^2	kNm
W(t)	40.3		41.2		22.0		19.0	
T_1(sec.)	0.651		0.638		1.201		1.453	
NL	15		13		13		13	

Table3(B) Optimum solutions (Frame II)

	II-A		II-B		II-C		II-D	
	A	Mp	A	Mp	A	Mp	A	Mp
C1	467.1	3098.8	537.2	3822.0	325.9	1806.1	278.6	1427.9
C2	436.9	2803.8	442.0	2852.8	267.6	1343.6	224.8	1034.9
C3	329.8	1839.5	325.4	1802.2	217.0	982.0	219.4	997.6
B1	275.9	2643.1	248.8	2263.8	126.6	821.0	80.6	416.9
B2	273.9	2614.6	281.3	2720.5	157.1	1135.8	128.3	837.9
B3	251.4	2299.1	261.4	2437.3	143.9	995.7	107.2	639.9
B4	267.2	2519.6	237.8	2114.8	130.6	860.4	102.2	595.9
B5	195.4	1574.9	215.0	1817.9	118.8	746.7	89.0	483.8
B6	172.4	1305.4	194.7	1567.0	104.8	619.0	75.7	380.0
B7	172.3	1304.4	148.1	1039.8	75.7	380.0	75.7	380.0
B8	75.7	380.0	75.5	378.2	75.7	380.0	75.7	380.0
Unit	cm^2	kNm	cm^2	kNm	cm^2	kNm	cm^2	kNm
W(t)	107.3		110.7		64.8		54.0	
T_1(sec.)	1.076		1.051		1.833		2.304	
NL	18		16		15		19	

Table 4 Solutions from 20 initial design points based on random numbers(Frame I-C)

No.	Weight	C1	C2	B1	B2	B3	B4	B5
1	22.019	252.781	207.508	127.228	131.678	113.653	86.071	75.498
2	22.019	252.781	207.508	127.228	131.678	113.653	86.071	75.498
3	22.019	252.781	207.508	127.228	131.678	113.653	86.070	75.498
4	22.019	252.782	207.509	127.228	131.678	113.653	86.070	75.498
5	22.019	252.781	207.506	127.227	131.677	113.653	86.072	75.498
6	22.019	252.781	207.507	127.228	131.677	113.653	86.071	75.498
7	22.019	252.780	207.505	127.227	131.676	113.653	86.074	75.498
8	22.019	252.782	207.509	127.228	131.678	113.653	86.070	75.498
9	49.260	640.231	640.231	115.951	282.899	75.537	207.091	75.537
10	22.017	252.744	207.457	127.206	131.658	113.654	86.126	75.498
11	22.019	252.780	207.505	127.227	131.676	113.653	86.074	75.498
12	22.019	252.781	207.507	127.228	131.677	113.653	86.072	75.498
13	22.019	252.781	207.508	127.228	131.678	113.653	86.071	75.498
14	22.019	252.781	207.508	127.228	131.678	113.653	86.071	75.498
15	22.019	252.781	207.508	127.228	131.678	113.653	86.071	75.498
16	22.019	252.782	207.509	127.228	131.678	113.653	86.070	75.498
17	22.019	252.784	207.512	127.229	131.679	113.654	86.067	75.498
18	22.019	252.781	207.508	127.228	131.677	113.653	86.071	75.498
19	22.019	252.781	207.508	127.228	131.678	113.653	86.070	75.498
20	22.019	252.781	207.508	127.228	131.678	113.653	86.071	75.498
unit	t	cm^2	cm^2	cm^2	cm^2	cm^2	cm^2	cm^2

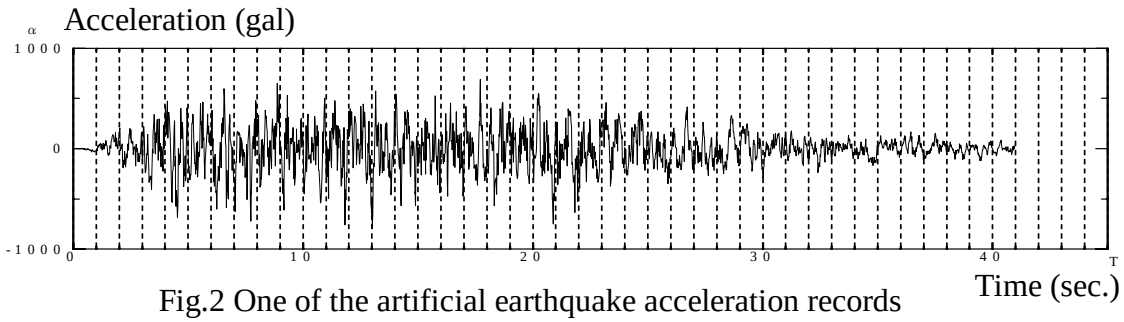

Fig.2 One of the artificial earthquake acceleration records

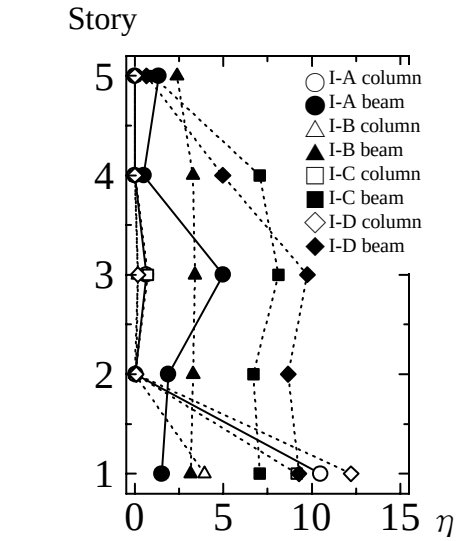

Fig.3(A) Cumulative damage(Frame I)

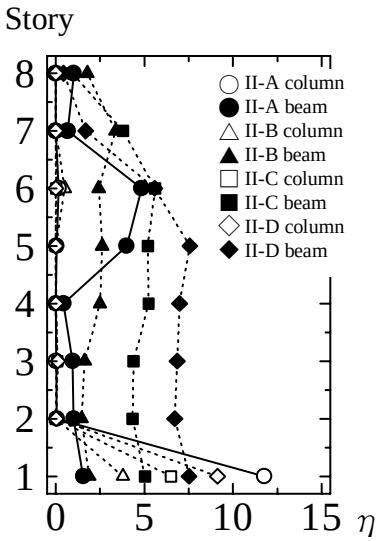

Fig.3(B) Cumulative damage(Frame II)