



NOVEL SEISMIC CORRECTION APPROACHES WITHOUT INSTRUMENT DATA, USING ADAPTIVE METHODS AND DE-NOISING.

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SUMMARY

This paper compares two adaptive methods of de-convolving the instrument responses from seismic events against the standard single-degree-of-freedom (SDOF) method. The Least Mean Squares (LMS) algorithm and the square root, Recursive Least Squares (RLS) algorithm are considered and investigated using three seismic events. Both adaptive methods do not assume any knowledge of instrument data, but use seismic readouts from which to estimate the inverse instrument response. The paper shows that in the absence of instrument data, adaptive methods provide reasonably consistent acceleration response spectra and power spectral densities. In addition the square root, RLS provides results more comparable with theoretical trends and should be the inverse filter of first preference if instrument data is not available.

INTRODUCTION

In earthquake engineering analysis and particularly the dynamic behaviour of structures, the importance of credible ground motion time series cannot be underestimated. Reliable and extensive sets of ground motion time-series, recorded from actual earthquakes, are essential. In most cases however seismic data sets have insufficient information regarding the type of recording instrument used, furthermore in a lot of cases information on the instrument is simply not available and researchers clearly state that instrument correction is not applied to the data.

Typical correction techniques Alexander [1], Converse [2], Sunder [6] used are necessary to (i) digitize, that is equi-sample the data, (ii) correct for instrument characteristics (iii) de-trend, (iv) de-noise using the wavelet transform or band-pass filter using digital Butterworth, Chebyshev filters or digital Finite Impulse Response (FIR) filters (v) resample to an appropriate sampling rate. The sequence of the component (ii) to (v) and exact algorithms used in these correction techniques vary significantly, as can the resulting recaptured original ground motion itself.

An ideal seismic recording instrument, from a signal processing point of view, should yield an instrument response, which up to about 25Hz gives 0dB and zero-phase. Most corrected seismic data however assume a 2nd order, single-degree-of-freedom (SDOF) instrument function with which to de-convolve the

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instrument response from the ground motion. This paper discusses and compares inverse filtering implementation of the Least Mean Squares (LMS) algorithm and the QR-recursive least squares (RLS) algorithm. The resulting inverse filters are applied to the data in order to de-convolve the instrument response. The advantage of this scheme is that it does not require any information regarding the instrument, it only requires the data, which the instrument has provided, from which to determine an estimate of the inverse of the instrument response. The paper then discusses the implementation of the translation invariant wavelet transform in order to de-noise rather than filter the resulting seismic data with a band-pass response.

2nd ORDER INSTRUMENT CORRECTION

Most seismic correction methods apply a 2nd order, single-degree-of-freedom (SDOF) instrument function with which to inverse filter or de-convolve the accelerometer response. To obtain estimates of the ground acceleration from the recorded relative displacement response, the SDOF instrument correction is applied as follows:

$$a_g(t) = -\ddot{x}(t) - 2\gamma\omega\dot{x}(t) - \omega^2x(t) \quad (1)$$

where γ is the accelerometer viscous damping ratio, ω is accelerometer natural frequency and $a_g(t)$ is the ground acceleration.

The above expression (1) can be used to de-convolve the recorded motion from the ground acceleration in either the time Sunder [3] or frequency domain Kumar [4], Khemici [5].

Unfortunately in a large number of seismic time-histories the important instrument parameters are either not given or are not known. Therefore researchers use an estimate for the instrument parameters in order to de-convolve an instrument response, or they do not instrument de-convolve at all.

Typically equation (1) can be solved in the time-domain using for example central difference, as in (2)

$$a_g = \frac{1}{-4\omega^2T^2}[(1 + 4\gamma\omega T + 4\omega^2T^2)a_i - (2 + 4\gamma\omega T)a_{i-2} + a_{i-4}] \quad (2)$$

where T is the sampling rate; or it may be solved in the frequency domain as in (3) and (4)

$$\ddot{X}_g(f) = -H(f)A(f) \quad (3)$$

$$H(f) = \left\{ \left(1 - \frac{f}{f_i} \right) + i \left(2\gamma \frac{f}{f_i} \right) \right\} \quad (4)$$

where the approximate acceleration output of the instrument is $A(f) = \omega^2 X(f)$. The ground acceleration in time can therefore be recovered from the inverse Fourier transform of the ground acceleration $\ddot{X}_g(f)$.

Over a limited range figure 1 shows the responses in the time and frequency domain are almost the same with the central difference demonstrating a flat response up to approximately 10Hz therefore over this range it can be inferred that the acceleration is approximately equal to the ground acceleration. At higher frequencies further corrections must be applied. However using frequency domain de-convolution the response is approximately flat up to 20Hz.

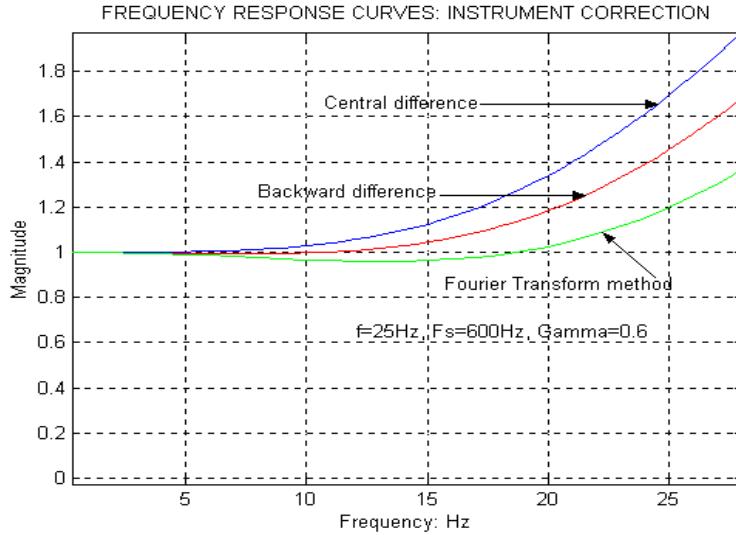


Figure 1, Comparison of frequency response curves.

DE-CONVOLUTION WITHOUT INSTRUMENT CORRECTION

The above estimates assume a 2nd order instrument response, which may not be the case and assume a set of parameters, which may be quite different for particular accelerometers where these are not known. Moreover in the latter case, the coefficient estimates describing the de-convolution filters is sub-optimal in that they do not take account any variation in time. They are 1st approximations and apply to signals, which are assumed stationary rather than non-stationary.

Therefore methods of instrument de-convolution are examined where the accelerometer characteristics are not known. In particular we examine two algorithms for de-convolution or inverse filtering. These are the (i) recursive least squares (RLS) Haykin [6] and (ii) least mean squares (LMS) Stearns [7], Hayes [8] algorithms. The RLS operates by minimizing the least square error as in (5) the LMS operates by minimizing the least *mean-square* error as in (6).

$$\varepsilon(n) = \sum_{i=0}^n |e(i)|^2 \quad (5)$$

$$\varepsilon(n) = E\{|e(n)|^2\} \quad (6)$$

Where the error $e(n) = d(n) - h^T x(n)$, $d(n)$ = the desired signal and $h^T x(n)$ = estimate of $d(n)$.

Minimizing the mean square error requires statistical information such as the expectation E , of the auto and cross-correlation; where these are unknown then the statistics are estimated. In The LMS adaptive filter for example, ensemble averages are estimated using instantaneous values, however the convergence rates may be too slow for some applications. Moreover sequences of data with the same or similar statistics will produce the same coefficients since these will depend on the ensemble averages. The RLS on the other hand minimizes the least square error, which doesn't require or make any statistical assumptions regarding the data. The coefficients will be different for different sets of data. However the coefficients will be optimal for the different data sets, even if the statistics for the data will be the same or similar.

The inverse filter algorithm used is shown in figure 2. It is a modification on the usual diagram because the delay branch is taken from the signal $s(n)$, where normally the delay branch would be taken from $x(n)$. However, in this case the input $x(n)$ (the ground motion) is not available, therefore as an approximation the convolved signal $s(n)$ (the output from the accelerometer), is delayed and used as the desired or training signal. The algorithm in this sense is therefore still supervised. Usually $x(n)$ is the reference input signal, which is passed through the unknown system whose estimated inverse transfer function is desired. In this case however the input signal has already been convolved with the instrument response information. However it is possible to derive a reference signal by assuming $x(n) \approx s(n)$ and delaying the process

$$x(n) \approx s(n) = d(n) + w(n) \quad (7)$$

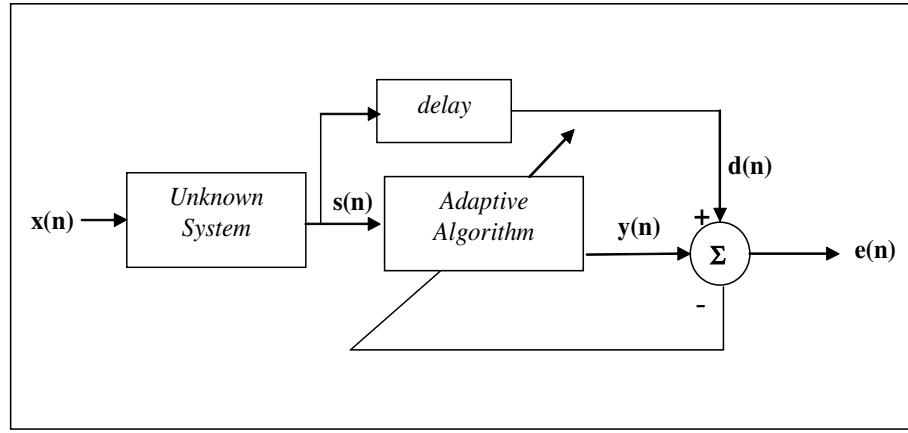


Figure 2, Modified Inverse filter diagram

If we assume that $d(n)$ is a narrow-band process and that $w(n)$ is a broadband process with

$$E\{w(n) w(n-k)\} = 0 \quad |k| > k_0 \quad (8)$$

If $d(n)$ and $w(n)$ are uncorrelated, then

$$E\{w(n) s(n-k)\} = E\{w(n) d(n-k)\} + E\{w(n) w(n-k)\} \quad (9)$$

If therefore $n_0 > k_0$ then the delayed process $s(n-n_0)$ will be uncorrelated with the noise $w(n)$ and correlated with $d(n)$, where we assume that $s(n)$ is a narrow-band process. Therefore $s(n-n_0)$ can be used as a reference signal to estimate $x(n)$.

The approach is shown in figure 2 and the plots shown below in figure 3 and figure 4 demonstrate the utility of this approach. White noise is passed through a low-pass, finite impulse response (FIR) filter, Trifunac [9] and is then delayed by the length of the adaptive filter and used as the desired (training) signal in the adaptive algorithm. The original filtered, random signal is used as the data signal, which carries the 'unknown' filter-response information. The plots show the original and inverse magnitude response with their respective linear phase responses. The number of taps used to describe the inverse filter is small, typically 11-21 FIR coefficients. The figures do show however that the RLS demonstrates a better recovery response than the LMS algorithm. Though computationally the latter is more efficient.

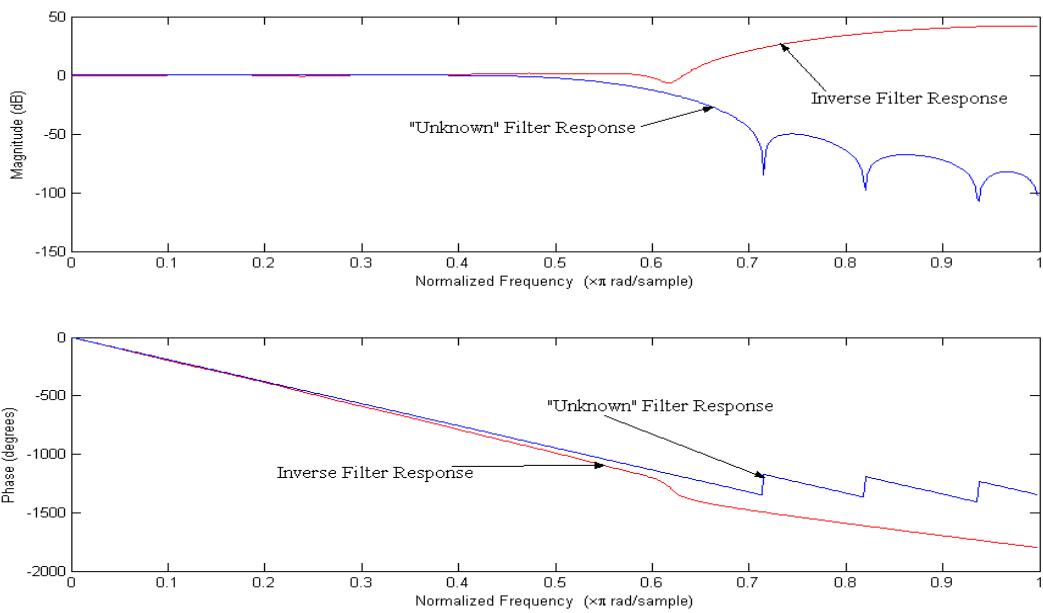


Figure 3, Comparison of RLS recovered filter and original "unknown" FIR test filter

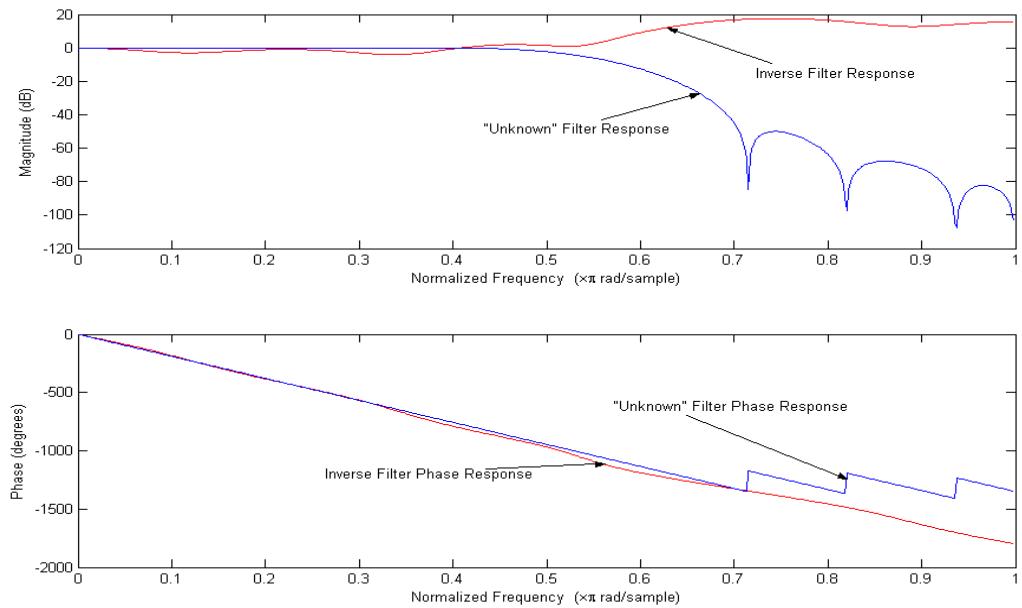


Figure 4, Comparison of LMS recovered filter and original "unknown" FIR test filter

THE LEAST MEAN SQUARES ALGORITHM

The LMS algorithm Haykin [6], Hayes [7] develops from Wiener filter theory and the steepest descent algorithm giving an adaptive filter which has a coefficient update given by:

$$h_{n+1} = h_n + \mu E\{e(n)x^*(n)\}$$

Where μ is the step size, which determines the rate at which the weight vector converges and $e(n)$ is the error. However, generally the expectation is not known and therefore it must be replaced with an estimate such as the sample mean given by:

$$\hat{E}\{e(n)x^*(n)\} = \frac{\mu}{L} \sum_{l=0}^{L-1} e(n-l)x^*(n-l) \quad (11)$$

Hence the coefficient update becomes:

$$h_{n+1} = h_n + \frac{\mu}{L} \sum_{l=0}^{L-1} e(n-l)x^*(n-l) \quad (12)$$

If we consider a one-point sampler ie $L = 1$, the coefficient update expression simplifies to:

$$h_{n+1} = h_n + \mu e(n)x^*(n) \quad (13)$$

Equation (13) expresses the LMS algorithm. Unfortunately the selection of the step size makes it difficult to design with the algorithm. Moreover if $x(n)$ is large the LMS experiences noise amplification. Therefore one way around the problem is to bound the step size for mean-square convergence, where the step size takes on the form as follows:

$$\mu = \frac{\alpha}{\|x(n)\|^2} \quad (14)$$

where α is a normalized step size with $0 < \alpha < 2$. This leads to the normalized LMS (nLMS) algorithm given by equation (15).

$$h_{n+1} = h_n + \frac{\alpha e(n)x^*(n)}{\|x(n)\|^2 + \varepsilon} \quad (15)$$

In (15) the noise amplification problem is reduced, however a similar problem occurs if $x(n)$ becomes too small, therefore we modify the expression by including a small positive number ε into the expression.

THE SQUARE ROOT RECURSIVE LEAST SQUARES ALGORITHM

The RLS algorithm Haykin [6], Stearns [8] can be considered in terms of a least squares solution Chong [10] of the system of linear equations $Ah = d$, where rank A is n , the number of unknowns. The objective is to find the vector (or vectors) h of filter coefficients which will satisfy equation (16). This has the well-known solution equations (17) and (18).

$$\text{minimise } \left\{ \|Ah - d\|^2 \right\} \quad (16)$$

$$h = (A^T A)^{-1} A^T d \quad (17)$$

$$h = (G)^{-1} A^T d = P A^T d \quad (18)$$

However in order to obviate the need of evaluating explicitly the inverse autocorrelation matrix P , the RLS algorithm provides an efficient method of updating the least squares estimate of the inverse filter coefficients as new data arrive. This is shown in the expression (19).

$$h_k = h_{k-1} + P_k u_k (d_k - u_k h_{k-1}) \quad (19)$$

Where the matrix A is replaced by a single data row, u , and d_k forms the desired signal. The updated value of the filter coefficient h_k is obtained by adding to the previous value, the 2nd term on the right, which can be considered as a “correction term”. The term in brackets is the *a priori* estimation error defined by (20)

$$\epsilon_k = d_k - u_k^T h_{k-1} = d_k - h_{k-1}^T u_k \quad (20)$$

The 2nd term on the right of equation (5) represents an estimate of the desired signal, based on the previous least squares estimate of the filter coefficient.

The inverse autocorrelation matrix P_k can be evaluated using Woodbury’s identity, which provides an efficient method of updating the matrix, once initialized with an arbitrary value. The update is given in (21) where the forgetting factor is λ .

$$P_k = \lambda^{-1} P_{k-1} - \frac{\lambda^{-2} P_{k-1} u_k u_k^T P_{k-1}}{1 + \lambda^{-1} u_k^T P_{k-1} u_k} \quad (21)$$

Equations (19), (20) and (21) form the basis of the RLS algorithm used in order to obtain the inverse filter coefficients with which to deconvolve the instrument response.

The algorithm requires an estimate of desired data, which is derived from actual seismic data. The approach used is to delay the accelerogram data by the filter order required and use this new vector as the desired training data in the algorithm, with the actual data as the data carrying the unknown instrument response. This is consistent in this application as that of an inverse filter. It is relatively straightforward to apply because ultimately only the forgetting factor needs to be adjusted. Typically this is between 0.9-0.99 and in this case was assigned a value of 0.9. A decrease in the value of the forgetting factor results in an increase in the attenuation of error data occurring further back in time.

The RLS algorithm can however become numerically unstable, therefore a variant of the RLS algorithm is used in this paper which, reduces the dynamic range and almost guarantees stable solutions. This is the QR decomposition-based RLS algorithm deduced from the square-root Kalman filter counterpart Haykin

[6], Sayed [11]. The ‘square-root’ is in fact a Cholesky factorization of the inverse correlation matrix. The derivation of this algorithm depends on the use of an orthogonal triangulation process known as QR decomposition.

$$QA = \begin{bmatrix} R \\ 0 \end{bmatrix} \quad (22)$$

Where 0 is the null matrix, R is upper triangular and Q is a unitary matrix. The QR decomposition of a matrix requires that certain elements of a vector be reduced to zero. The unitary matrix used in the algorithm is based in this case on a Givens rotation Haykin [6], Sayed [11] which zero’s out the elements of the input data vector and modifies the square root of the inverse correlation matrix. The QR-RLS is as follows in equation (23) below.

$$\begin{bmatrix} 1 & \lambda^{-1/2} u^H P^{1/2}(n-1) \\ 0 & \lambda^{-1/2} P^{1/2}(n-1) \end{bmatrix} U(n) = \begin{bmatrix} \gamma^{-1/2}(n) & 0 \\ k(n)\gamma^{-1/2}(n) & P^{1/2}(n) \end{bmatrix} \quad (23)$$

Where P = the inverse correlation matrix, λ = *forgetting* factor, γ = a scalar and the gain vector is determined from the 1st column of the post-array. $U(n)$ is a unitary transformation which operates on the elements of $\lambda^{1/2} u^H(n) P^{1/2}(n-1)$ in the pre-array zeroing out each one to give a zero-block entry in the post-array. The filter coefficients are then updated commencing with equation (24), which is the gain vector. This is followed by equation (25) the a priori estimation error.

$$k(n) = [k(n)\gamma^{-1/2}(n) / \gamma^{-1/2}(n)] \quad (24)$$

$$\varepsilon(n) = d(n) - h^H(n-1)u(n) \quad (25)$$

$$h(n) = h(n-1) + k(n)\varepsilon(n) \quad (26)$$

This in turn, leads to the updating of the least-squares weight vector, $h(n)$, in equation (26). These inverse-filter weights are then convolved with the original seismic data in order to obtain an estimate of the true ground motion.

DISCUSSION OF FREQUENCY-RESPONSES DERIVED FROM SEISMIC EVENTS

The frequency responses show that the LMS based de-convolution algorithm does not seem to perform well when compared with the frequency responses to that of the theoretical, 2nd order response. The LMS algorithm demonstrates reasonable inverse filter characteristics in simulation with white noise. However when applied to the Garvey, Sitka and El-Centro seismic Ambraseys [12] events the general trend of the frequency responses is different to that of the theoretical curves for a range of frequencies. The linear portions of the theoretical curves show a linear instrument characteristic (≈ 0 dB) up to 8Hz and 20Hz in Figure 1 and figure 5. The linear portions of the responses derived from the seismic events show linear (≈ 0 dB) regions between 7Hz and 17Hz and continue to follow the trend of theoretical response until approximately 40Hz and 10dB. However at higher frequencies the trend is substantially different as shown in figure 5 showing more of a low-pass characteristic.

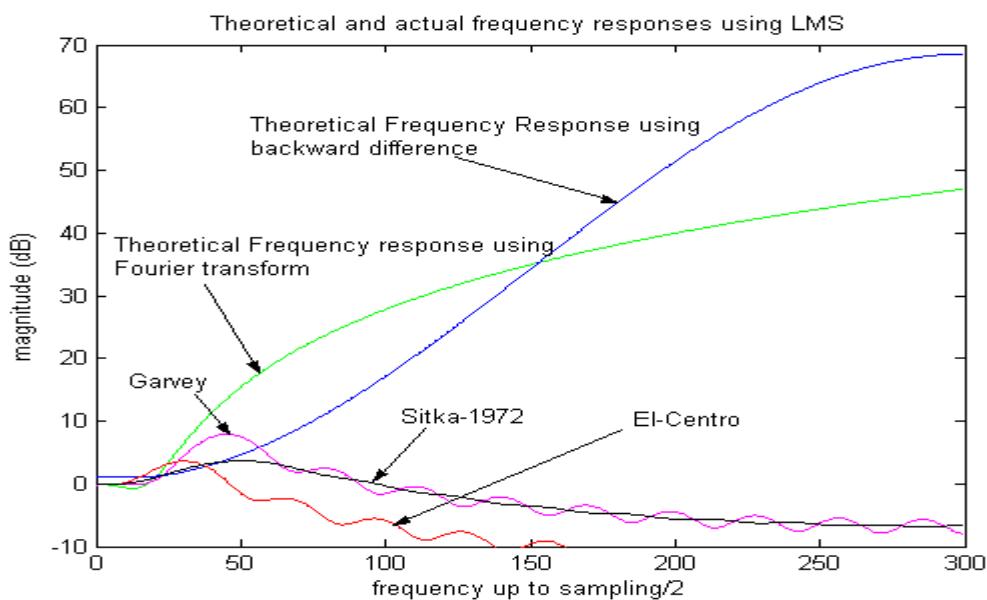


Figure 5, Comparison of theoretical and actual inverse filters using the LMS algorithm

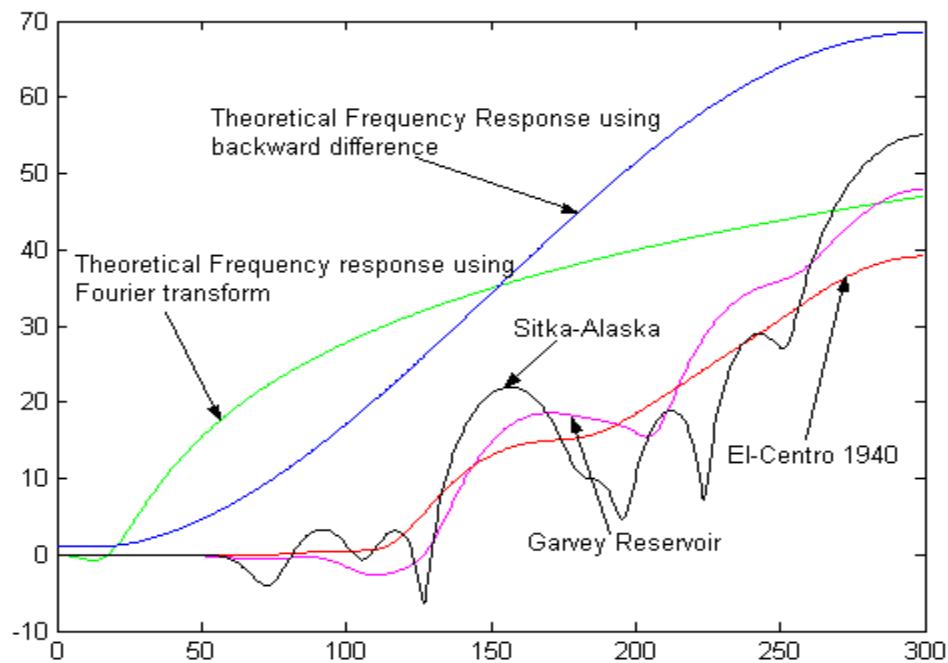


Figure 6, Comparison of theoretical and actual inverse filters using the square root RMS algorithm

The implementation using the square root RLS algorithm shows better overall response characteristics when compared with the theoretical responses. The 0dB regions extend to approximately 20Hz and the overall trend is high-pass more in line with theoretical expectations. In simulation with white noise the square root RLS exhibits better performance in recovering the inverse filter characteristics and this performance extends to the derived frequency responses from actual seismic events.

Both the LMS and the RLS always produces a linear phase response because the adaptive model is of the FIR type, while the SDOF model has its own non-linear phase response. A linear phase response is equivalent to a time shift of the time-series. Thus, for the case of most dynamic structural analyses, the RLS and the LMS algorithms can be regarded neutral with respect to phase, they do not adjust the phase. The SDOF model phase response is almost linear for low frequencies and thus can be thought to be fairly neutral for these frequencies. If the SDOF model is used it may incorrectly adjust the phase content of the mid- to high frequency range. This could be due to the instrument not actual being a SDOF system and errors in its parameters.

DE-NOISING OF SEISMIC DATA

In order to remove noise from the seismic signal, it is band-pass filtered using either FIR filters, or IIR filters such as Butterworth or Elliptic filters Rabiner [13], Oppenheim [14]. However de-noising using the discrete wavelet transform (DWT) Ingrid Daubechies [15] is a better alternative since it is less invasive and is not frequency selective. It simply removes signals below a certain threshold Donoho [16], Walker [17]. This is the fundamental difference between using a band-pass filter and wavelet thresholding. The band-pass filtering does not consider the energy content of the signal and noise. Hence the removed ‘noise’ may or may not have a high-energy content. De-noising also obviates the need to adjust filter cut-off frequencies to fit particular seismic events and is computationally efficient. The paper uses the shift-invariant wavelet transform (or the stationary wavelet transform, SWT) Donoho and Johnstone [18], Coifman and Donoho [19] with which to de-noise the seismic data prior to de-convolving the instrument response.

The coefficients of the discrete wavelet transform (DWT) Burrus [20] do not shift with a signal, this means that the signal is no longer orthogonal to most of the basis functions. Many more coefficients would be necessary to describe the signal and the coefficient dimensions would also be much smaller reducing the effectiveness of any de-noising scheme.

The problems with shift-invariance are connected with the alignment between features in the signal and features of the wavelet basis. In particular at discontinuities where Gibbs like phenomena can occur with unwanted oscillations. An approach to surmount this problem is described in which it is suggested to forcibly shift the signal, so that its features change positions in relation to the wavelet basis. Then to un-shift and retrieve the de-noised signal hopefully without any unwanted noise or spurious oscillation.

Following the arguments of Coifman and Donoho [19] we introduce a circulant shift operator such that

$$(S_h x)_t = x_{(t+h) \bmod n}$$

where the operator S_h denotes the circulant shift by h . The operator has an inverse $(S_h)^{-1}$, therefore whole process is described as follows: For an analysis technique A , calculate the time-shifted version of A , \tilde{A} therefore,

$$\tilde{A}(x : S_h) = (S_h)^{-1}(A(S_h(x))) \quad (27)$$

However a problem does occur if a signal contains many discontinuities, because in this case a best shift for one discontinuity may not be that for another. Therefore Coifmann and Donoho [19] propose to apply a range of shifts H and then to average over several such shifts obtained.

Therefore equation (19) is modified to the following

$$\tilde{A}(x:(S_h)_{h \in H}) = \text{Average}_{h \in H} (S_h^{-1}(A(S_h(x)))) \quad (28)$$

or

$$\text{Average} [\text{ Shift} \rightarrow \text{Denoise} \rightarrow \text{UnShift}] \quad (29)$$

This is the method employed when processing the data described in this paper.

DISCUSSION OF RESULTS

A comparison of results for seismic data de-convolved using both the LMS and square root RLS for the El-Centro 1940 event, the Sitka 1972 Alaskan event and the Garvey Reservoir event does not show a marked difference in the results, both at low frequencies. The acceleration response spectra are virtually identical for both the adaptive inverse filters as are the power spectral densities. This would be in line with low-frequency behavior for both the LMS and RLS algorithms, which showed that trends were reasonably consistent up to about 20Hz. The frequency response showed that for higher frequencies the responses were substantially different, however this is not reflected in the acceleration response spectra or in the estimates of power spectral densities. Amplitudes are too small at the higher frequencies in order to observe any measurable effect. However, comparison with standard correction methods using band-pass filtering and 2nd order instrument correction does indeed show significant differences and is shown in Figure 7 for the Sitka seismic event.

In this case instrument data was available and given as 0.049 for the period and damping 0.570. This was applied to the correction procedure using the standard 2nd order SDOF correction and compared with the results obtained from the QR-RLS and LMS correction. The % increases in the acceleration response spectrum at some structural frequencies is shown below in table 1

Structural Frequency [Hz]	% increase in Total Acceleration
0.08	9%
0.1	7%
0.4	4%
0.8	10%
7.0	13%
10.0	14%

Table 1, Percentage increase in total acceleration due to RMS/LMS algorithm

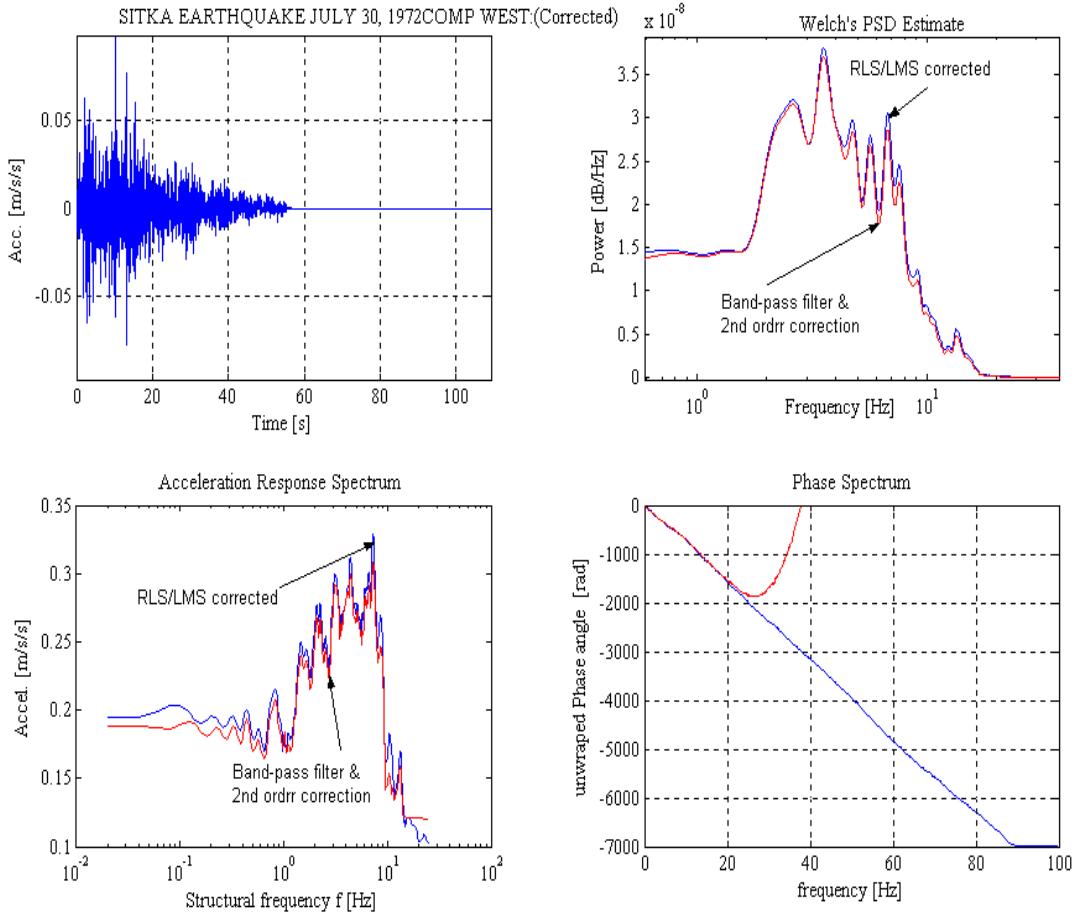


Figure 7, Sitka event, Alaska, 1972: Power, Phase and Response Spectra

SUMMARY

The paper shows that inverse filtering using the LMS or RLS algorithm provides acceptable results when compared to the standard 2nd order SDOF type de-convolution in either the time or frequency domain. Analysis of frequency responses for both the LMS and RLS corrected data show however that the LMS algorithm used as a de-convolver does not provide a reasonable inverse filter response over the complete frequency range. In most cases seismic data sets have insufficient information regarding the type of recording instrument used, furthermore in a lot of cases information on the instrument is not available and so instrument correction is not applied to the data. The RLS algorithm however provides a reasonable solution to the problem and is a better indication of the actual instrument response than the LMS. The latter seems to provide a solution identical to that of the RLS, but its frequency response departs markedly from theoretical predictions. This may be a function of the fact that the statistical assumptions inherent in the LMS simply do not apply to these seismic events. The RLS algorithm demonstrates a consistency in its frequency responses as well, though in this case there isn't a dependence on any statistical assumptions. Nevertheless the consistency in frequency response behaviour suggests that seismic events may be rather stationary.

An important point to note is that the RLS does not require any information regarding the instrument, it only requires the data, which the instrument has provided and from which it determines an estimate of the inverse of the instrument response. An ideal instrument should yield an instrument response, which gives 0dB and zero or linear-phase up to about 25Hz. The RLS estimate of the inverse of the instrument characteristics showed a magnitude response better than the ideal and a linear phase response. This is in both the El Centro, Sitka and Garvey Reservoir seismic events. This contrasts with the 2nd order differential equation estimate of the inverse of the instrument characteristics, which showed 0dB up to approximately 15Hz and a non-linear phase change over the sampling interval [27], suggesting that the 2nd order instrument correction does distort the phase. It also suggests that the instrument's behaviour is much better than that predicted by the 2nd order differential equation. In conclusion, standard 2nd order de-convolution is not necessarily a good reflection of instrument performance, but is probably better than not performing a de-convolution at all. However the paper shows that there are suitable alternatives such as the RLS algorithm which provides results, which depend on the actual data and are a better reflection of instrument performance.

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