



COMBINED DETERMINISTIC-STOCHASTIC ANALYSIS OF LOCAL SITE RESPONSE

Ronaldo I. BORJA,¹ José E. ANDRADE,¹ and Richard J. ARMSTRONG¹

SUMMARY

We investigate the sensitivity of two deterministic local site response models to variations in the input material parameters. The associated computer codes are SHAKE, a program that employs an equivalent linear analysis algorithm, and SPECTRA, a fully nonlinear finite element program with a constitutive model based on bounding surface plasticity. We use a stochastic approach to study the statistics of the model predictions as functions of the input parameters treated as random variables. Sensitivity is quantified in terms of the Arias intensity for both SHAKE and SPECTRA, as well as through the maximum permanent relative displacement for SPECTRA (SHAKE is an equivalent linear analysis code and thus predicts no plastic deformation). In conducting the combined deterministic-stochastic studies, we assume that the input excitation is given and use the stochastic approach to construct empirical cumulative distribution functions for the response variables. The input excitation utilized in the studies is the M6.2 seismic event that occurred in Lotung, Taiwan on July 30, 1986, better known in the literature as the LSST12 event.

INTRODUCTION

It is generally recognized that soils exhibit nonlinear behavior under shear loading conditions. Shear modulus decreases with increasing strain accompanied by an increase in material damping (Hardin and Drnevich 1982). In local site response analysis, these changes in material properties control the amplitude and frequency content of computed ground motions and must be captured by the mathematical model to obtain accurate and realistic predictions. In general, two approaches are commonly used to model nonlinear cyclic soil stress-strain behavior: equivalent linear and truly nonlinear. The computer program SHAKE (Schnabel et al. 1972) is an example of an equivalent linear program. It has the advantage of mathematical simplicity but has the disadvantage of giving poor predictions at large strains, as well as inability to model plastic deformation and/or failure. The computer code SPECTRA (Borja et al. 2002) is an example of a truly nonlinear program. Like other nonlinear models, it has the advantage of capturing plastic deformation and/or failure, and can exhibit superior accuracy provided the material parameters have been 'calibrated' properly. Such calibration process, however, can become unwieldy and tricky

¹ Department of Civil and Environmental Engineering, Stanford University, Stanford, California, 94305-4020, USA.

particularly if a model is sensitive to variations in the material parameters. Mathematical models that exhibit strong sensitivity to such variations are unlikely to be reliable and useful in practice, particularly if the input parameters are known to exhibit significant statistical variations. The primary goals of this paper are thus to advance a systematic methodology for quantifying the sensitivity of any mathematical model to inherent variations in the material parameters, and to apply this methodology to the two site response models described previously. Such studies are useful for quantifying the level of confidence placed on the predictions of the mathematical model considering the statistical variations in the input material parameters.

GENERAL METHODOLOGY

Sensitivity of a given deterministic model to input material parameters can be quantified in general terms by choosing a relevant response function (usually a scalar variable) and investigating the statistical variation of this response function in terms of the statistical variation of the input parameters. Our goal here is not to develop fragility curves, and so we assume that the forcing function is given and focus the study on the uncertainties in material properties. For problems where a closed-form relation exists between the response function and the input parameters, an analytical expression describing the propagation of statistical variations is possible. However, such closed-form solutions are generally not available for more complex problems, and thus the uncertainties are best propagated to the response variables numerically. The latter can be carried out sequentially by first generating random variables and determining the parameters derived from such random variables; these parameters are then input into the deterministic model to determine the value of the desired response function. By performing a multitude of simulations (say, of the Monte Carlo type), the statistics of the response variable can be quantified in terms of an empirical cumulative distribution function (ECDF).

More specifically, for the problem at hand, we choose the Arias intensity (Arias 1970) and relative permanent displacement as the desired response functions. Both are ‘cumulative’ in nature and thus reflect response measures that develop over a significant time window. In contrast, the peak ground acceleration (PGA) occurs over a very narrow time window and thus may not serve as a desirable response measure for statistical analysis (since it is known that the PGA is inherently sensitive to the value of damping ratio anyway). Thus, for a given seismic excitation the sensitivity of a given deterministic model to input material parameters can be quantified numerically by first determining the probability density function (PDF) of the soil parameters from partial descriptors such as mean and standard deviation. Values of the random variables are then generated from these distributions, and are subsequently input into the deterministic models (i.e., SHAKE and SPECTRA) to determine the corresponding values of Arias intensity and permanent deformation for the given seismic excitation. The procedure is repeated hundreds of times to calculate the corresponding ECDFs.

MATERIAL PARAMETERS FOR SHAKE AND SPECTRA

For the program SHAKE the relevant material parameters exhibiting significant statistical variations include the elastic shear modulus G^e and the modulus reduction and damping ratio values at each shear strain level. We shall then treat these parameters as random variables. SHAKE uses the modulus reduction and damping ratio versus shear strain curves directly in the algorithm to calculate the effective shear strain in each layer. SPECTRA uses the modulus reduction curve indirectly to determine the specific model parameters as described below; however, it does not need the entire damping ratio versus shear strain curve but only the asymptotic value of this curve at zero shear strain. Because the two codes rely

essentially on the same material information to fully characterize the soil deposit, a comparison of their relative sensitivities to statistical variations of soil properties is meaningful.

Borja and Amies (1994) presented the bounding surface plasticity model used in the present study. The bounding surface is a right circular cylinder centered about the hydrostatic axis; the yield surface is also a cylinder inside the bounding surface, and has its axis parallel to that of the bounding surface. However, the radius of the yield surface is zero in order to effectively capture plastic deformation at the onset of loading. Loading/unloading criteria are formulated for a general three-dimensional loading condition and have been incorporated into the stress-point integration algorithm using the midpoint rule. A pictorial representation of the bounding and yield surfaces on the deviatoric plane is shown in Fig. 1.

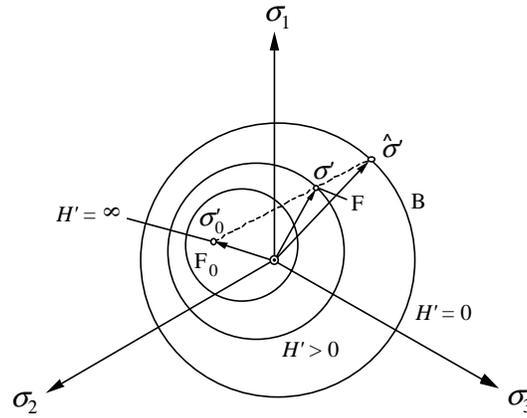


Figure 1. Unloading point, stress point, and image point on deviatoric plane for the bounding surface plasticity model [after Borja and Amies (1994)].

The radius of the bounding surface is given by the expression

$$R = \sqrt{2} \tau_{\max}, \quad (1)$$

where τ_{\max} is the maximum shear stress obtained from simple shear tests. Assuming a hyperbolic stress-strain relation (Hardin and Drnevich 1972),

$$\tau_{\max} = \frac{\theta_0 G^e \gamma_0}{1 - \theta_0}. \quad (2)$$

In the above equation, the pair γ_0 (= shear strain) and θ_0 (ratio of secant shear modulus to elastic shear modulus) define a point on the modulus reduction curve at large values of γ . SPECTRA interpolates the value of the plastic modulus H inside the bounding surface using an exponential hardening function, expressed analytically in terms of a coefficient h and an exponent m . Borja and Amies (1994) also presented analytical expressions for these two parameters in terms of the modulus reduction curve. A final parameter used by SPECTRA is the damping ratio χ used to construct the damping matrix. This parameter takes the form

$$\chi = 2\xi_0 / \omega, \quad (3)$$

where ξ_0 is the asymptotic value of damping ratio at zero shear strain, and ω is the angular (dominant) frequency of the input motion. All of these parameters can be defined as random variables whose characters depend on the statistical information available for the relevant soil properties.

DESCRIPTION OF LSST SITE AND INPUT MOTION

Lotung is a seismically active region in the northeastern part of Taiwan, and was the site of a strong motion Large-Scale Seismic Test (LSST) instrumentation system installed by the Electric Power Research Institute, in cooperation with Taiwan Power Company, for soil-structure interaction research. One of the instrumentation arrays consisted of a set of downhole three-component accelerometers extending to a depth of 47 m, known as the DHB array. The role of the DHB array is to monitor free-field responses resulting from seismic activities at the LSST site. On 30 July 1986 a M6.2 earthquake shook the test site (denoted as the LSST12 event). The earthquake had an epicentral distance of 6 km, a focal depth of 1.6 km, and PGA accelerations of 0.155g (EW), 0.190g (NS), and 0.195 (UD). This seismic event has been investigated extensively by a number of researchers, and the recordings of the DHB array have been analyzed and simulated many times using different site response analysis codes (Borja et al. 2002). The objective of this paper is not to repeat these previous analyses, but rather, to focus on the propagation of uncertainties in the soil properties to the response functions represented by Arias intensity and the total plastic shear strain.

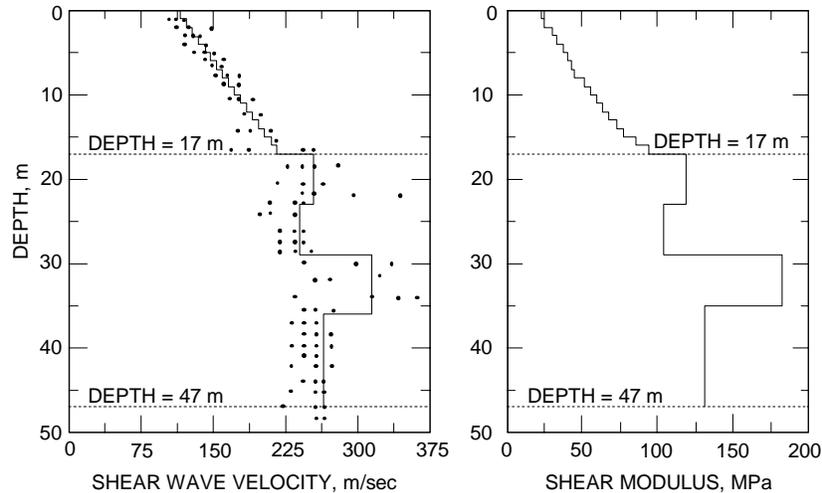


Figure 2. Shear wave velocity and shear modulus for the LSST site: solid dots are data points and solid line is average.

From extensive studies conducted at the LSST site, we have determined the soil parameters needed to run the programs SHAKE and SPECTRA, including probability density functions based on the method of moments. Figure 2 shows shear wave velocity measurements at the test site, along with average values and corresponding average shear modulus (Borja et al. 1999). These data are useful for quantifying the soil

response at very low strains. Figure 3 shows the moduli reduction and damping ratio curves as functions of shear strains (Zeghal et al. 1995). In order to cover the entire 47 m of soil, the curves reported by Zeghal et al. (1995) at 6, 11, and 17 meters are taken to be representative of the soil in 0-6m, 6-11m, and 11-17m, respectively. Furthermore, the curves at 17m are also assumed to be representative of the soil in 17-47m. These curves are used by SHAKE to determine the equivalent effective strain on which to conduct the equivalent linear analysis, and by SPECTRA to characterize the evolution of the plastic variables of the bounding surface model.

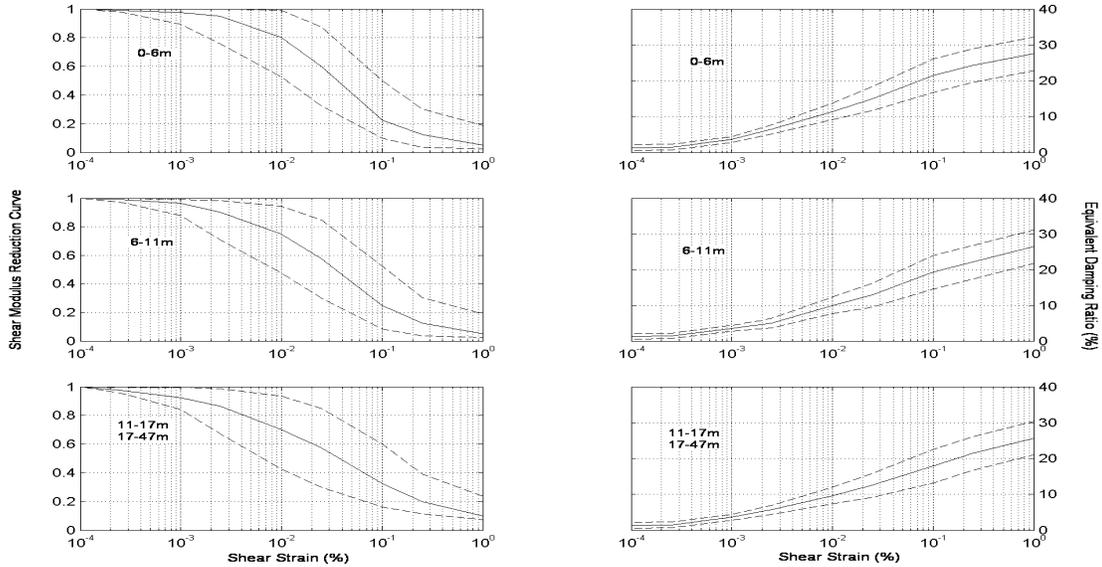


Figure 3. Moduli reduction and equivalent damping ratio at various levels of shear strain: solid lines are mean values, and dashed lines are upper and lower bounds.

RESULTS AND DISCUSSION

Figure 4 shows the result produced from the combined stochastic-deterministic site response analyses after 500 simulations. The upper portion of Fig. 4 shows the ECDF for Arias intensity I_a generated by SPECTRA ($I_{a,sp}$) and SHAKE ($I_{a,sh}$), where

$$I_a = \frac{\pi}{2g} \int_0^{\infty} [a(t)]^2 dt. \quad (4)$$

In the lower portion, the median and 90% confidence intervals of the ECDFs, obtained from bootstrapping the original set of simulations, are shown. The mean and standard deviation of $I_{a,sp}$ are 3.515E-03 g-sec and 1.175E-03 g-sec, respectively, while the mean and standard deviation of $I_{a,sh}$ are 3.298E-03 g-sec and 8.013E-04 g-sec, respectively. These results can be used to quantify the sensitivity of the responses predicted by SPECTRA and SHAKE.

It is clear that the variability in $I_{a,sp}$ is greater than that in $I_{a,sh}$. Furthermore, the coefficient of variation (C.O.V) of $I_{a,sp}$ is 0.334 while that of $I_{a,sh}$ is 0.243. Nevertheless, the bootstrapped 90% confidence intervals suggest that the difference between the two models is only significant in the upper tail of the distribution. Between the values of 20 and 60 percentile, statistically speaking, there is no significant difference between the two models due to the fact that the confidence intervals are entangled.

If the value of $I_{a,r} = 0.005$ g-sec, calculated from the recorded acceleration history at the surface, is taken as the “true” value of I_a , the upper plot in Fig. 4 may suggest that the median of $I_{a,sp}$ (0.003412 g-sec) is close to this value that the median of $I_{a,sh}$ (0.003294 g-sec). This suggests that SPECTRA predicts more accurate values for I_a than does SHAKE.

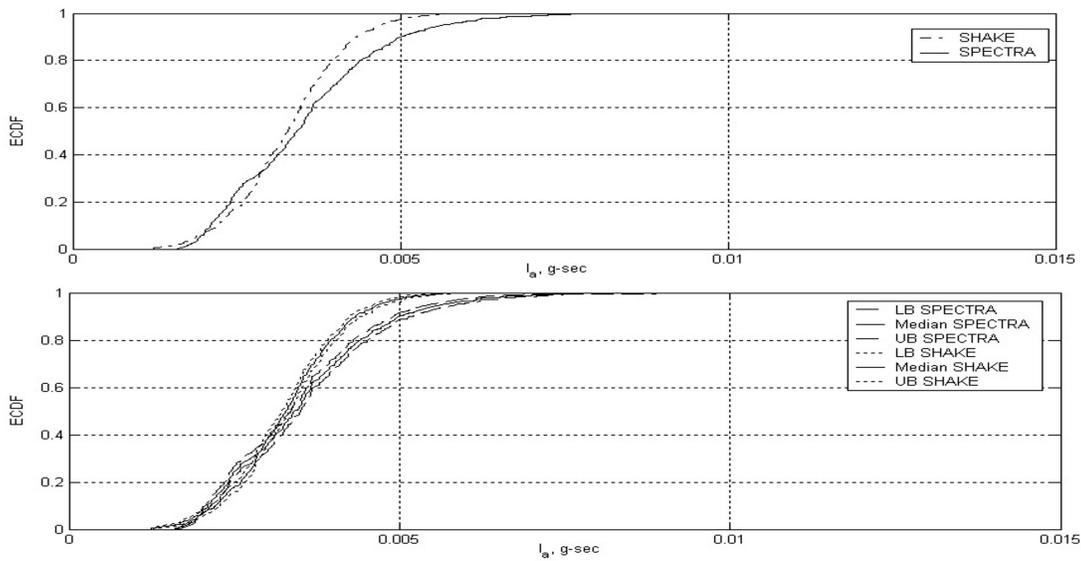


Figure 4. Median empirical cumulative distribution functions for Arias intensity using SPECTRA and SHAKE (upper plot). Median and 90% confidence intervals (lower plot).

In addition to I_a , we also calculated the ECDF for the final (permanent) relative displacement between the top of the soil column and the point at a depth of 17 m, and the results are summarized in Fig. 5. Like the Arias intensity, permanent relative displacement is a ‘cumulative’ response that is amenable to statistical analysis. The upper portion of Fig. 5 shows the ECDF predicted by SPECTRA, and the lower portion shows the median and 90% confidence intervals of the ECDF. The mean, standard deviation, and coefficient of variation are 0.00477 m, 0.00248 m and 0.517, respectively. From the confidence intervals depicted in the low portion of Fig. 5, it is seen that there is a 90% probability that the median of the relative displacement lies within 0.00401 and 0.0043 m. Finally, there is 70% probability that the predicted values fall between one plus or minus one standard deviation from the distribution mean. As noted earlier, SHAKE is an equivalent linear analysis program and thus predicts zero permanent displacement.

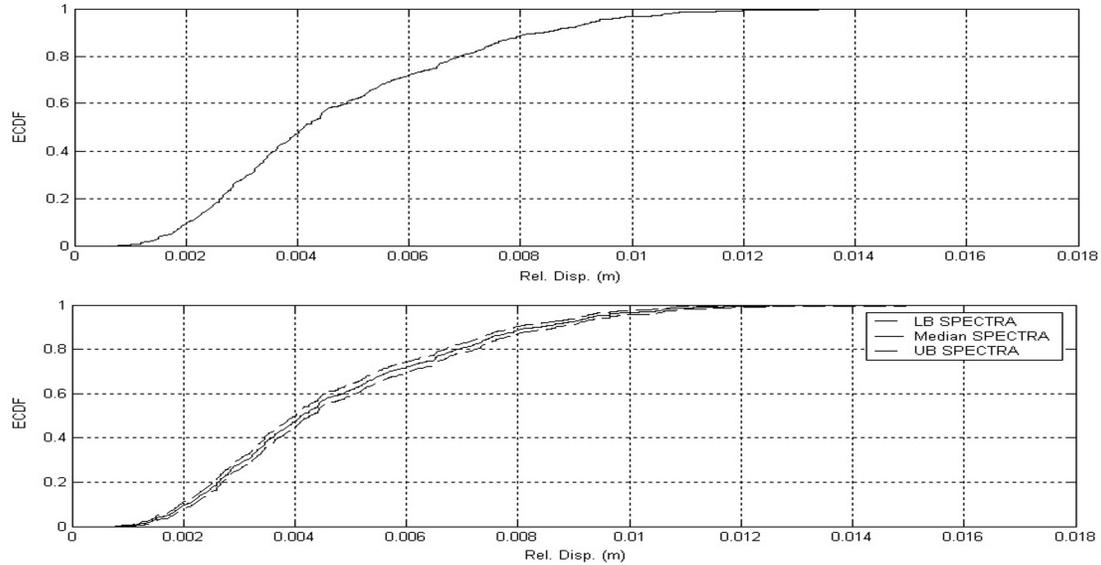


Figure 5. Median empirical cumulative distribution functions for permanent relative displacement using SPECTRA (upper plot). Median and 90% confidence intervals (lower plot).

SUMMARY AND CONCLUSIONS

We have used a combined deterministic-stochastic analysis to study the relative sensitivities of SHAKE and SPECTRA to variations in the input soil properties, using the LSST12 event in Lotung, Taiwan, as the given input motion. From the results obtained above, we observed that the greatest variability is experienced in the extreme values of the distribution of $I_{a,sp}$. However, between 20 and 60 percentile, the 90% confidence intervals suggest that there is no clear difference between the distributions of Arias intensity predicted by the two models. Thus, it can be inferred that if the variability in the model parameters is not large, the two models will generate equivalent responses. We have also generated the ECDF of the relative permanent displacement predicted by SPECTRA. No such ECDF is possible for SHAKE because this program predicts no permanent ground deformation. The results presented in this study imply that all the advantages of using nonlinear models can be gained without compromising the quality of the results due to variations in the model parameters. This equivalent variability may stem from the fact that although the model used by SPECTRA is nonlinear, it is based on the same parent parameters that are used by its linear equivalent counterpart.

ACKNOWLEDGMENTS

This work is supported by National Science Foundation, Grant No. CMS-0201317-001, under the direction of Dr. C.S. Astill. The authors are grateful to Dr. H.T. Tang and Electric Power Research Institute for making the digitized data for the Lotung site available. The second author acknowledges a Shah Family Fellowship through the Department of Civil and Environmental Engineering at Stanford. The third author acknowledges an NSERC scholarship from the government of Canada and a fellowship from the Department of Civil and Environmental Engineering at Stanford.

REFERENCES

1. Arias A. "A measure of earthquake intensity." In: RJ Hansen (Ed.), *Seismic Design for Nuclear Power Plants*, MIT, Cambridge, Mass. 1970; 438-483.
2. Borja RI, Duvernay BG, Lin CH. "Ground response in Lotung: Total stress analyses and parametric studies." *Journal of Geotechnical and GeoEnvironmental Engineering*, ASCE 2002; 128(1): 54-63.
3. Borja RI, Chao HY, Montáns FJ, Lin CH. "Nonlinear ground response at Lotung LSST site." *Journal of Geotechnical Engineering*, ASCE 1999; 125:187-197.
4. Borja RI, Amies AP. "Multiaxial cyclic plasticity model for clays." *Journal of Geotechnical Engineering*, ASCE 1994; 120(6): 1051-1070.
5. Hardin BO, Drnevich VP. "Shear modulus and damping in soils: design equations and curves." *Journal of the Soil Mechanics and Foundations Division*, ASCE 1972; 98(7): 667-692.
6. Schnabel PB, Lysmer J, Seed HB. "SHAKE---A computer program for earthquake response analyses of horizontally layered sites." Report No. EERC 72-12, Univ. of California, Berkeley.
7. Zeghal M, Elgamal AW, Tang HT, Stepp JC. "Lotung downhole array. II: Evaluation of soil nonlinear properties." *Journal of Geotechnical Engineering*, ASCE 1995; 121(4):363-378.