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EVALUATION OF IMPACT MODELS FOR SEISMIC POUNDING

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SUMMARY

This paper investigates the efficacy of various impact models such as the stereomechanical and the contact force-based linear spring, Kelvin and Hertz models in capturing the seismic pounding response of adjacent structures. A Hertz contact model with nonlinear damping (Hertzdamp model) is also introduced for pounding simulation. Parameter studies conducted using two degree-of-freedom linear oscillators indicate that the system displacements from the stereomechanical, Kelvin and Hertzdamp models are similar, despite using different methodologies, for a given restitution coefficient. Impact models accounting for energy dissipation are best suited for pounding simulation, with the Hertzdamp model being an effective contact force-based approach.

INTRODUCTION

During an earthquake, adjacent structures having different dynamic characteristics can vibrate out-of-phase resulting in impact or *pounding* if the at-rest separation between the adjacent structures is insufficient to accommodate the relative displacements. Seismic pounding generates high magnitude and short duration acceleration pulses that can induce structural damage. Furthermore, pounding can amplify the global response of participating structural systems. The highly congested building system in many metropolitan cities constitutes a major concern for seismic pounding damage. In the case of bridges, the multiple-frame and multi-span simply supported bridges are most susceptible to pounding damage due to numerous independent components and lack of continuity in the structure.

Recent earthquakes have illustrated several instances of pounding damage in both building and bridge structures. Pounding of adjacent unreinforced masonry buildings resulting in shear failure of the brickwork leading to partial collapse of the wall was observed during the 1989 Loma Prieta earthquake (Benuska [1]). The 1994 Northridge earthquake revealed substantial impact damage at the expansion hinges and abutments of connectors at the I-5/SR-14 Interchange, which was located at close proximity to the epicenter (Hall [2]). Impact between a six-story building and two-story building in Golcuk, Turkey during the 1999 Kocaeli earthquake contributed to column failure above the third floor slab in the taller building, and shear failure of two second-floor piers in the smaller building (Youd [3]). The 1999 Chi-Chi

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earthquake in Taiwan revealed hammering at the expansion joints in some bridges which resulted in damage to shear keys, bearings and anchor bolts (Usarski [4]). Failure of girder ends and bearing damage due to pounding of adjacent simply supported spans were reported after the 2001 Bhuj earthquake in Gujarat, India (Jain [5]). Based on the observations from past earthquakes, closely spaced buildings can experience infill wall damage, column shear failure and possible collapse due to pounding. Pounding in bridges can lead to local crushing and spalling of concrete, result in damage to column bents, abutments, shear keys, bearing pads and restrainers, and possibly contribute to the collapse of deck spans.

Pounding is a highly nonlinear phenomenon and two analytical techniques are available for modeling - the contact element method and the stereomechanical approach. In the former approach, a contact element is activated when the structures come into contact. A spring with high stiffness is used to avoid overlapping between adjacent segments, sometimes in conjunction with a damper. The contact elements used in the past include the linear spring (Maison [6]), the energy-dissipating Kelvin-Voigt (Anagnostopoulos [7], Jankowski [8]) and the nonlinear Hertz contact element (Pantelides [9], Chau [10]). The contact element approach has its limitations, with the exact value of spring stiffness to be used being unclear. Moreover, using a spring of very high stiffness can result in unrealistically high impact forces and also lead to numerical convergence problems. The stereomechanical approach assumes instantaneous impact and uses momentum balance and the coefficient of restitution to modify velocities of the colliding bodies after impact. This approach has been used by Papadrakakis [11], Athanassiadou [12] and Malhotra [13]. However, the stereomechanical approach is no longer valid if the impact duration is large enough so that significant changes occur in the configuration of the system. Furthermore, it cannot be easily implemented into existing commercial software.

With several available analytical models, it is imperative to compare their impact performances to determine their applicability and efficacy in capturing the pounding phenomenon during earthquakes. In particular, it is important to compare the stereomechanical and contact force-based approaches, to ascertain the effect of impact modeling methodology on the response of participating systems. Furthermore, the effect of energy loss during pounding on the responses of participating systems needs to be examined.

This paper examines the efficacy of various analytical models in representing the pounding response of closely spaced adjacent structures. The adjacent structures are represented by a simplified two degree-of-freedom (DOF) model. Only elastic system response is considered. Existing impact models such as the stereomechanical approach, and the contact force-based linear spring, Kelvin and Hertz models are implemented. In addition, a Hertz contact model with a nonlinear damper is also introduced for pounding simulation. The effects of energy loss are studied by selecting two values for the coefficient of restitution (e) where applicable, e = 1.0 (no energy loss) and e = 0.6 (some energy loss). Parameter studies are then conducted with the two-DOF system subject to a suite of ground motion records to assess the effect of impact model type, ground motion characteristics, and impact energy loss on the global responses of the participating structural systems.

SIMPLIFIED MODEL FOR POUNDING INVESTIGATIONS

A simplified two degree-of-freedom (DOF) model is developed, as shown in Figure 1, to investigate seismic pounding between adjacent structures. The adjacent structures can be closely spaced buildings or bridge frames. Each DOF is characterized by mass m_i, initial stiffness k_i and viscous damping coefficient c_i and is assumed to behave elastically. Using a force-based approach to model impact, the equations of motion for the two-DOF system subjected to horizontal ground motion $\ddot{u}_g$ can be written as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix}+\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix}+\begin{Bmatrix} R_1(u_1) \\ R_2(u_2) \end{Bmatrix}+\begin{Bmatrix} F_c(u_1-u_2-g_p) \\ -F_c(u_1-u_2-g_p) \end{Bmatrix}=-\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\ddot{u}_g \qquad (1)$$

where $\ddot{u}_i$, $\dot{u}_i$, u_i are the acceleration, velocity and displacement relative to the ground, and dot denotes differentiation with respect to time; R_i is the system restoring force and F_c is contact force due to pounding. Impact occurs when the gap between the two bodies closes, i.e., $u_1 - u_2 - g_p > 0$. Several studies have shown that the damping of participating systems is not a significant factor affecting the pounding response (Anagnostopoulos [7], Athanassiadou [12]). Hence, modal damping of 5% is assigned to each DOF. The solution of Equation (1) is obtained numerically using the 4th order Runge-Kutta method (Kreyzig [14]).

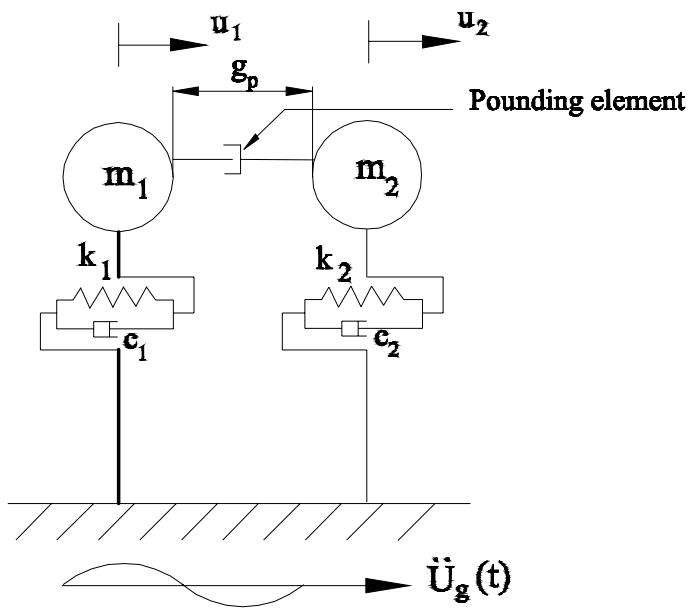

Figure 1 – Two DOF model idealization of adjacent structures

ANALYTICAL IMPACT MODELS

Typically, pounding is modeled using either a continuous force model or via a stereomechanical (coefficient of restitution) approach, as described earlier. Several of the existing impact models are considered in this study. In addition, a contact model based on the Hertz law and using a nonlinear hysteresis damper for energy dissipation is also introduced. The analytical formulations of the various impact models are outlined below:

Stereomechanical model

This approach uses the momentum conservation principle and the coefficient of restitution to model impact. The duration of impact is neglected. The coefficient of restitution (e) is defined as the ratio of separation velocities of the bodies after impact to their approaching velocities before impact (Goldsmith [15]). Since this is not a force-based approach the impact force under this approach is zero ($F_c = 0$). However, the velocities of the colliding bodies are adjusted after impact, as shown in Equation 2.

$$v_1' = v_1 - (1+e)\frac{m_2(v_1 - v_2)}{m_1 + m_2} \tag{2a}$$

$$v_2' = v_2 + (1+e)\frac{m_1(v_1 - v_2)}{m_1 + m_2} \tag{2b}$$

where v_1', v_2' are the velocities of the colliding masses (m_1, m_2) after impact, v_1, v_2 are the velocities before impact and e is the coefficient of restitution.

Linear spring model

A linear spring of high stiffness (k_l) can be used to simulate impact once the gap between adjacent bodies closes, as shown in Figure 2. The contact force during impact is taken as:

$$F_c = k_l\left(u_1 - u_2 - g_p\right) \;;\; u_1 - u_2 - g_p \geq 0 \tag{3a}$$

$$F_c = 0 \;;\; u_1 - u_2 - g_p < 0 \tag{3b}$$

This contact force-based approach is relatively straight forward, and can be easily implemented in commercial software. However, energy loss during impact cannot be modeled.

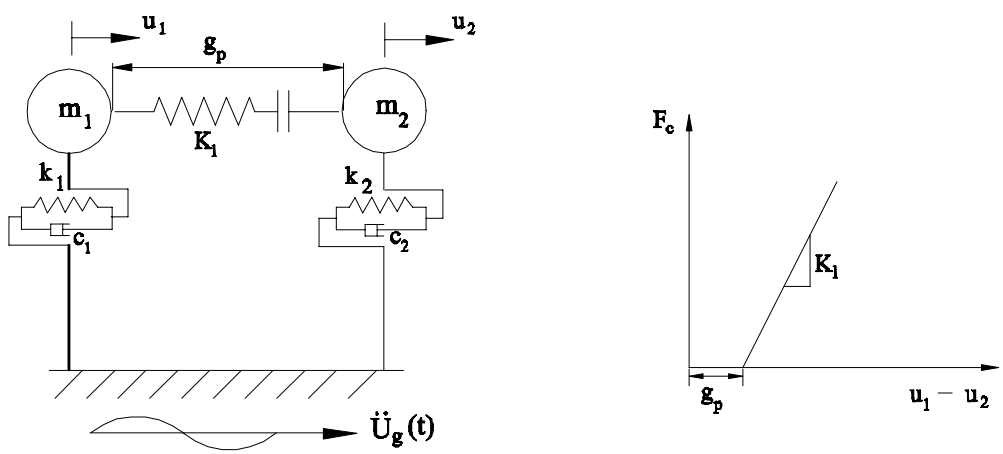

Figure 2 – Linear spring model and contact force relation

Kelvin model

A linear spring of stiffness (k_k) is used in conjunction with a damper element (c_k), as shown in Figure 3. This model is capable of modeling energy dissipation during impact and the impact force representation is:

$$F_c = k_k\left(u_1 - u_2 - g_p\right) + c_k\left(\dot{u}_1 - \dot{u}_2\right) \;;\; u_1 - u_2 - g_p \geq 0 \tag{4a}$$

$$F_c = 0 \;;\; u_1 - u_2 - g_p < 0 \tag{4b}$$

The damping coefficient c_k can be related to the coefficient of restitution (e), by equating the energy losses during impact.

$$c_k = 2\xi\sqrt{k_k\left(\frac{m_1 m_2}{m_1+m_2}\right)} \;\; ; \;\; \xi = -\frac{\ln e}{\sqrt{\pi^2+(\ln e)^2}} \tag{5}$$

where m_1, m_2 are the masses of the colliding bodies.

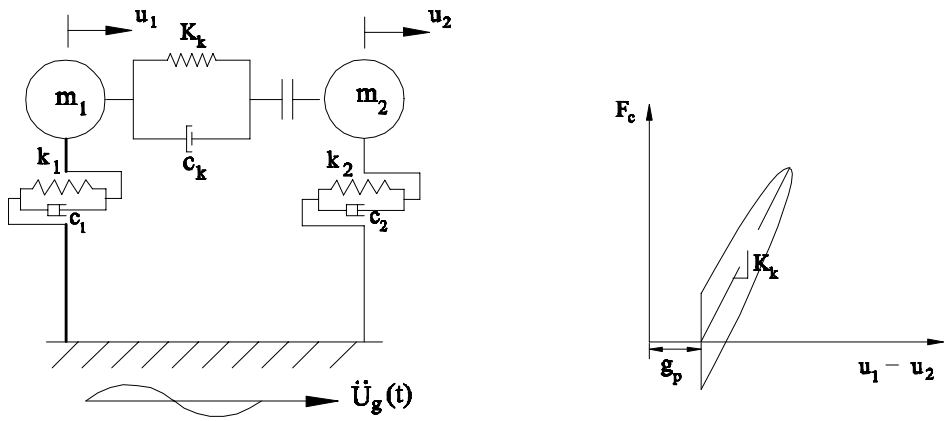

Figure 3 – Kelvin model and its contact force relation

Hertz model

Another popular contact force model for representing pounding is the Hertz model, which uses a nonlinear spring of stiffness (k_h), as illustrated in Figure 4. The impact force representation is:

$$F_c = k_h\left(u_1 - u_2 - g_p\right)^{3/2} \;;\; u_1 - u_2 - g_p \geq 0 \tag{6a}$$

$$F_c = 0 \qquad\qquad ;\; u_1 - u_2 - g_p < 0 \tag{6b}$$

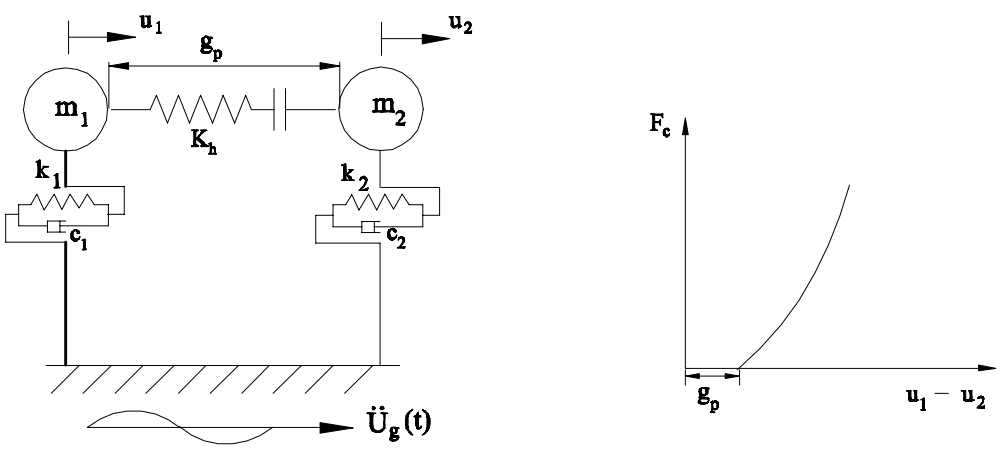

Figure 4 – Hertz model and its contact force relation

Hertz model with nonlinear damper (Hertzdamp model)

The Hertz model suffers from the limitation that it cannot represent the energy dissipated during impact. Hence, an improved version of the Hertz model is considered herein, whereby a nonlinear damper is used in conjunction with the Hertz spring. Similar models have been used in other areas such as robotics, and multi-body systems (Hunt [16], Lankarani [17]). However, its efficacy in structural engineering has not been considered. The contact force can be expressed as:

$$F_c = k_h\left(u_1 - u_2 - g_p\right)^{3/2} + c_h\left(\dot{u}_1 - \dot{u}_2\right) \;;\; u_1 - u_2 - g_p \geq 0 \tag{7a}$$

$$F_c = 0 \qquad\qquad ;\; u_1 - u_2 - g_p < 0 \tag{7b}$$

where c_h is the damping coefficient, $u_1 - u_2 - g_p$ is the relative penetration and $\dot{u}_1 - \dot{u}_2$ is the penetration velocity. A nonlinear damping coefficient (c_h) is proposed so that the hysteresis loop matches the expected loop due to a compressive load that is applied to and removed from a body within its elastic range at a slow rate.

$$c_h = \zeta\delta^n \tag{8}$$

where ζ is the damping constant, and δ is the relative penetration ($u_1 - u_2 - g_p$).

Equating the energy loss during stereomechanical impact to the energy dissipated by the damper, the value of ζ can be related to the spring constant, k_h, the coefficient of restitution, e, and the relative velocity of the bodies at the instant of impact, $v_1 - v_2$, as shown below.

$$\zeta = \frac{3k_h\left(1 - e^2\right)}{4\left(v_1 - v_2\right)} \tag{9}$$

Hence, the force during contact in (7a) can be expressed as:

$$F_c = k_h\left(u_1 - u_2 - g_p\right)^{3/2}\left[1 + \frac{3(1 - e^2)}{4\left(v_1 - v_2\right)}\left(\dot{u}_1 - \dot{u}_2\right)\right] \;;u_1 - u_2 - g_p \geq 0 \tag{10}$$

The Hertz model with nonlinear damper shall be referred to as the Hertzdamp model throughout the rest of this paper. The performance of the Hertzdamp model is compared with the responses of the other impact models in the following section.

HERTZDAMP MODEL RESPONSE AND COMPARISON

The elastic, two-DOF system shown in Figure 1 is subjected to the 1940 El Centro record having a Peak Ground Acceleration (PGA) of 0.35g. The periods of DOF_1 and DOF_2 are 0.25 and 0.50 seconds, respectively. Two cases are evaluated; Case 1, where the separation between the models (g_p) is very large so as to preclude pounding and Case 2, where g_p is small enough so that pounding can occur. The gap between the models in Case 2 is taken as:

$$g_{p_{Case2}} = \chi D_{np} = \chi \max\left\{u_1(t) - u_2(t)\right\}_{Case1} \tag{11}$$

where χ is the gap ratio parameter and D_{np} is the maximum relative displacement from Case 1. Assuming $\chi = 0.5$, results in a gap of 0.85 inches for Case 2. Pounding is implemented using all the impact models discussed earlier. The coefficient of restitution is assumed as 0.6, where applicable. The stiffness parameters of the various impact models are assumed to be the same for consistency (25000 units). Figure 5 presents the system responses when pounding is analyzed using the Hertzdamp model.

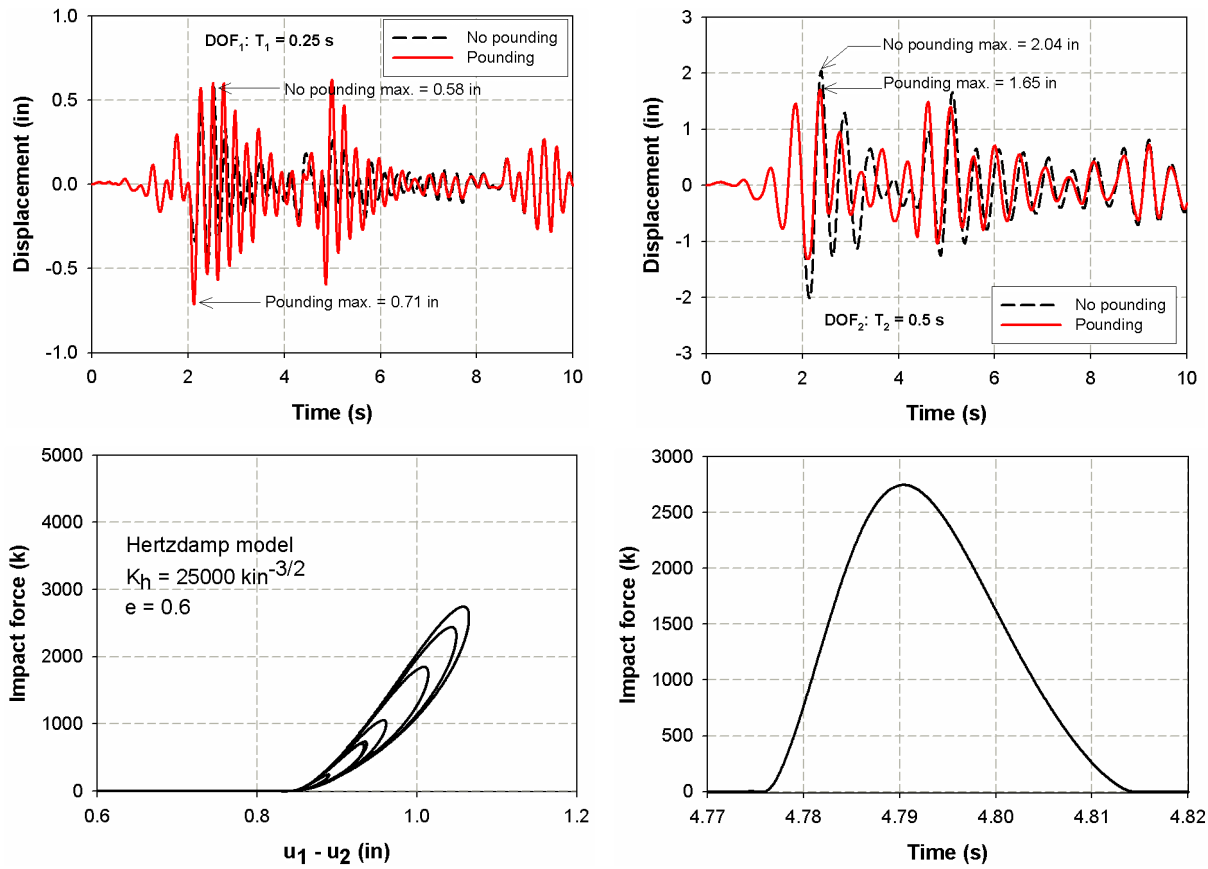

Figure 5 – System responses using the Hertzdamp model for pounding simulation; El Centro record (10 seconds); clockwise from top – time history of displacements (DOF_1), time history of displacements (DOF_2), Hertzdamp impact force vs. time, Hertzdamp impact force vs. relative displacement.

The time history of displacements for the pounding and no pounding cases shows that pounding increases the maximum displacement of DOF_1 (stiffer system) from 0.58 in to 0.71 in. Conversely, for DOF_2 (flexible system), impact reduces the maximum displacement from 2.04 in to 1.65 in. The nonlinearity and energy loss associated with impact are illustrated by the impact force vs. relative displacement plot. For completeness, the variation of impact force as a function of time is also presented for one instance of impact. To compare the performance of the Hertzdamp model, the maximum pounding responses of the two-DOF system using the various impact models are normalized with respect to the no-pounding response. The displacement and acceleration amplifications due to pounding for DOF_1, and the maximum impact force from the various models, for the El Centro record are presented in Figure 6.

Clearly, the models which cannot represent energy loss (linear spring and hertz models) overestimate the displacement amplification due to pounding. The displacement amplifications from the Hertzdamp and stereomechanical models are very similar. The displacement amplification from the Kelvin model is the smallest. This can be attributed to a larger hysteretic loop and the presence of some impact force even as the bodies just touch each other (relative displacement = g_{pCase2}). The amplification in the acceleration response of DOF_1 and the maximum impact force are much higher for models based on a linear spring, such as the Kelvin and linear spring models. The Hertzdamp model provides the lowest impact force among force-based models. The stereomechanical model is not a force based model. Hence, there is no impact force and consequently, no amplification in the acceleration response.

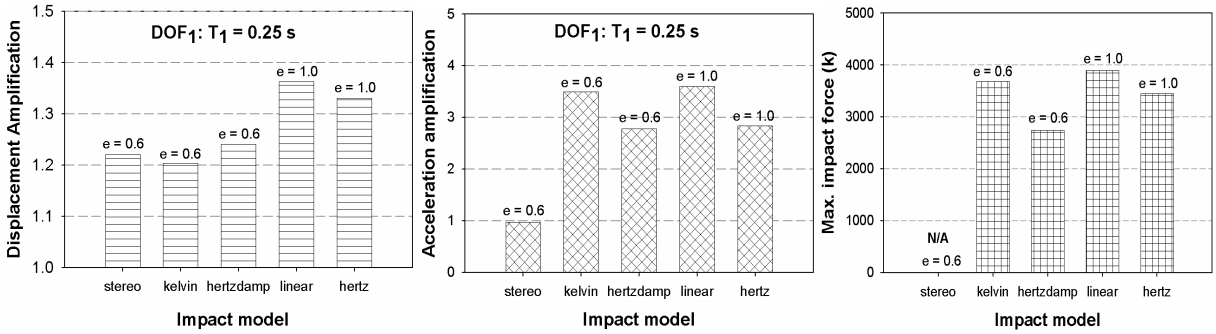

Figure 6 – Comparison between various impact models for two-DOF linear system with $T_1/T_2 = 0.5$; El Centro record; L-R displacement amplification due to pounding – DOF_1, acceleration amplification due to pounding – DOF_1, maximum impact force

This case study has illustrated the effects of energy loss, nonlinearity in impact stiffness and compared the performance of the various impact models, for one ground motion record. In the following section, a parameter study investigating the effects of model type and energy loss during impact is presented, for a suite of ground motion records.

PARAMETER STUDY TO ASSESS THE PERFORMANCE OF VARIOUS IMPACT MODELS

The cogency of various impact models in predicting the pounding response of adjacent structures is investigated using the two DOF model shown in Figure 1. Equal masses of 7.8 kip-s^2/in are selected. The system is assumed to be elastic and highly out-of-phase, with a period ratio, $T_1/T_2 = 0.3$. A suite of twenty seven ground motion records from thirteen different earthquakes is selected, as listed in Table 1. The ground motion records are grouped into three levels depending on the peak ground acceleration (PGA) as, low ($0.1g \leq PGA \leq 0.3g$), moderate ($0.4g \leq PGA \leq 0.6g$) and high ($0.7g \leq PGA \leq 0.9g$). The records are chosen such that the ground motion period ratio (T_2/T_g = flexible system period over the ground motion characteristic period) is less than one (Zone I response) (DesRoches [18]).

Pounding is simulated using all the impact models discussed earlier. The stiffness of the linear contact spring (k_l) is assumed as 25,000 kip/in. The stiffness parameters of the other models are assumed the same for consistency. To study the effect of energy loss during impact, two values of the coefficient of restitution are selected, e = 1.0 (no energy loss) and e = 0.6 (some energy loss). It should be noted that at e = 1.0, the Kelvin model reduces to the linear spring and the Hertzdamp model reduces to the Hertz model. The effect of pounding is expressed in terms of response amplification, which is the ratio of the maximum response when pounding occurs to the maximum response when there is no pounding. The gap between adjacent structures is set very large for the no-pounding analysis and assumed as ½ inch for the pounding analysis. Figures 7 and 8 present the mean values of displacement and acceleration amplification due to pounding for the various impact models. Since pounding amplifies the stiff system response in Zone I, only the stiff system amplifications are presented.

Effect of model type

The stereomechanical and contact force-based models (Kelvin, Hertzdamp) predict similar displacement responses, despite using different impact methodologies, for a given coefficient of restitution (e). The differences in displacement amplification between the various models are within 12% of each other, at all levels of PGA, for a particular value of e. However, the contact force-based models predict higher accelerations due to pounding. The system acceleration responses from the stereomechanical model are smaller than those from the contact models, and follow the corresponding displacement trends. Generally,

the Hertzdamp model predicts lower acceleration amplifications than the Kelvin model, at all ground motion levels. This trend is observed at both values of e.

Table 1 - Suite of twenty seven earthquake ground motion records used in parameter study

PGA Level	PGA (g)	Earthquake	M_w	Station	Φ°	EPD (km)	T_g (s)
LOW	0.11	Northridge, 1994	6.7	Wonderland Ave	095	22.7	0.80
	0.10	Imperial Valley, 1979	5.2	5054 Bonds Corner	230	15.6	0.75
	0.09	San Fernando, 1971	6.6	Pasadena	000	25.7	0.85
	0.19	Loma Prieta, 1989	6.9	Fremont	090	43.4	0.70
	0.19	Morgan Hill, 1984	6.2	Gilroy Array #3	000	14.6	1.10
	0.21	N. Palm Springs, 1986	6.0	Morongo Valley	135	10.1	1.90
	0.30	Whittier Narrows, 1987	6.0	E Grand Ave	180	9.0	0.70
	0.28	Landers, 1992	7.3	Joshua Tree	090	11.3	0.70
	0.29	Morgan Hill, 1984	6.2	Gilroy Array #6	090	11.8	1.20
MODERATE	0.37	Loma Prieta, 1989	6.9	WAHO	000	16.9	0.85
	0.42	Northridge, 1994	6.7	Mulhol	009	19.6	0.85
	0.39	Cape Mendocino, 1992	7.1	Rio Dell Overpass	270	12.3	0.65
	0.51	Northridge, 1994	6.7	Old Ridge Route	360	22.6	0.95
	0.48	Loma Prieta, 1989	6.9	Coyote Lake Dam	285	21.8	0.65
	0.48	Northridge, 1994	6.7	W Lost Canyon	270	12.2	0.70
	0.51	Loma Prieta, 1989	6.9	Saratoga – Aloha Ave	000	11.7	1.80
	0.59	N Palm Springs, 1986	6.0	5070 N Palm Springs	210	8.2	1.10
	0.59	Cape Mendocino, 1992	7.1	Petrolia	000	9.5	0.75
HIGH	0.61	Loma Prieta, 1989	6.9	16 LGPC	090	6.1	0.80
	0.60	Coalinga, 1983	5.8	Pleasant Valley P.P.	045	17.4	0.65
	0.57	Northridge, 1994	6.7	Old Ridge Route	090	22.6	0.80
	0.66	Cape Mendocino, 1992	7.1	Petrolia	090	9.5	0.70
	0.82	Duzce, 1999	7.1	Bolu	090	17.6	0.90
	0.84	Coalinga, 1983	5.8	Transmitter Hill	270	9.2	0.75
	0.84	Northridge, 1994	6.7	Rinaldi	228	7.1	1.05
	0.89	Superstition Hills, 1987	6.7	286 Superstition Mtn	135	4.3	0.70
	1.04	Cape Mendocino, 1992	7.1	Cape Mendocino	090	8.5	2.00

PGA - Peak Ground Acceleration; M_w – Moment magnitude; Φ° - Component; EPD – Epicentral distance; T_g – Characteristic period of ground motion record

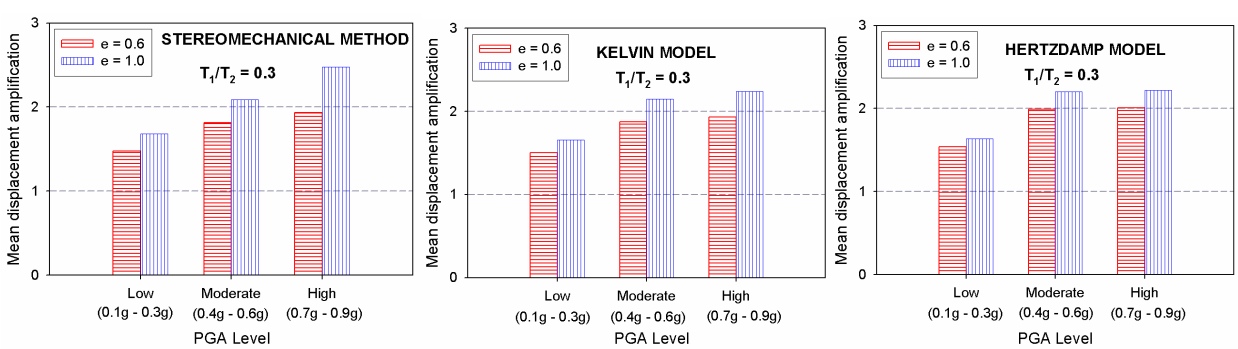

Figure 7 – Mean displacement amplifications due to pounding (DOF_1) – elastic systems; T_1/T_2 = 0.3; e = 1.0, 0.6; Nine ground motion records used at each PGA level

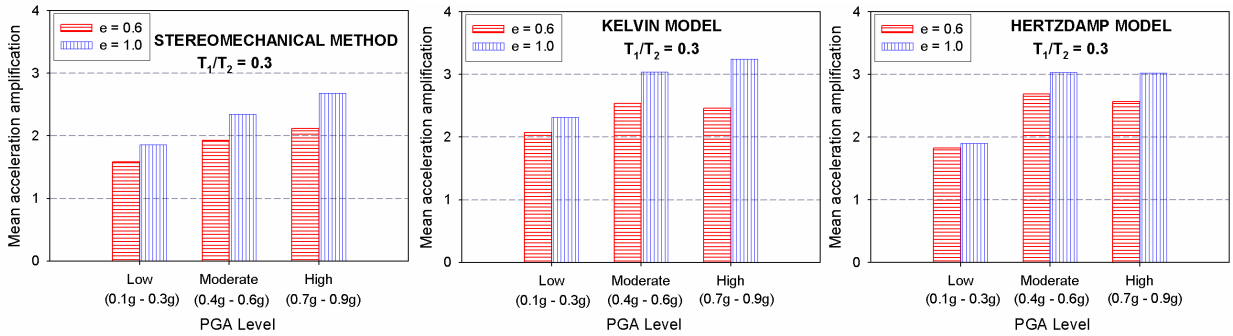

Figure 8 – Mean acceleration amplifications due to pounding (DOF_1) – elastic systems; $T_1/T_2 = 0.3$; e = 1.0, 0.6; Nine ground motion records used at each PGA level

Effect of impact energy loss

Neglecting energy dissipation due to impact (e = 1.0) overestimates both the displacement and acceleration responses of the stiff system. Energy loss is more significant at high PGA levels. On the average, energy loss during impact reduces the stiff system displacements by 28%, 16%, and 10% for the stereomechanical, Kelvin and Hertzdamp models, respectively, when subjected to high levels of PGA. The corresponding reductions in the stiff system accelerations are 27%, 32%, and 18%. Among all the impact models, the Hertzdamp model exhibits the least variation in system response with respect to changes in e.

CONCLUSIONS

Recent earthquakes have clearly shown that closely spaced buildings, adjacent frames and girder ends in bridges are vulnerable to seismic pounding damage. This study investigates the efficacy of various analytical models used for representing pounding between adjacent structures. Existing impact models such as the contact force-based linear spring, Kelvin, and Hertz models and the coefficient of restitution based stereomechanical approach are evaluated. In addition, a contact model based on the Hertz law and using a nonlinear hysteresis damper (Hertzdamp model) is also introduced for pounding simulation.

Parameter studies are conducted using two degree-of-freedom elastic oscillators having a system period ratio, $T_1/T_2 = 0.3$, subjected to different levels of ground motion (low, moderate and high). To examine the effects of energy loss during impact, two values of the coefficient of restitution are chosen, e = 1.0 (no energy loss) and e = 0.6 (some energy loss). The results indicate that the displacement responses from the stereomechanical and contact force-based models are similar, although they use different methodologies, provided the same restitution coefficient is used in all models. For a given value of e, the differences in stiff system displacements between various impact models remain small (< 12%) at all ground excitation levels. Greater amplifications in the stiff system response due to pounding are observed at higher levels of PGA ($0.7g \leq PGA \leq 0.9g$). These trends are observed for all the impact models. On the average, neglecting energy loss during impact overestimates the stiff system displacement by 28%, 16%, and 10% for the stereomechanical, Kelvin and Hertzdamp models, respectively, when subjected to high levels of PGA. The corresponding reductions in the stiff system acceleration responses are 27%, 32%, and 18%.

Based on the results of this study, pounding models that account for energy loss during impact are best suited to simulate pounding. The Hertzdamp model appears to be an effective contact based approach, as it can model energy loss and also shows least variation with respect to changes in the coefficient of restitution (e). However, a simplified model based on the Hertzdamp model needs to be developed for easy implementation into existing commercial software.

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