



## **SYSTEM IDENTIFICATION OF STRUCTURES FROM SEISMIC RESPONSE DATA VIA WAVELET PACKET METHOD**

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### **SUMMARY**

This paper presents a procedure of establishing the discrete equations of motion for a structure via wavelet packet method from its measured responses and inputs. Then, the natural frequencies, damping ratios and mode shapes of the structure can be directly determined. The proposed procedure is applied to process the acceleration responses of a five-story steel frame, subjected to 8%, 20%, 40%, 52% and 60% of the strength of the Kobe earthquake, in shaking table tests. The steel frame responded nonlinearly when it subjected to 60% Kobe earthquake, while it responded linearly when it subjected to the other strengths of the Kobe earthquake. This work also investigates the differences in modal parameters identified from the responses to the inputs with different strengths of the Kobe earthquake. The accuracy of the present results is confirmed by comparing the present results with the published obtained from the responses to the 8% Kobe earthquake input.

### **INTRODUCTION**

Monitoring of an existing structure is increasingly important in current efforts to evaluate the safety of the structure when its material deteriorates or when the structure has been subjected to severe loading like a strong earthquake. It is desirable to use the measured responses of a structure to conform the construction quality, to validate or improve analytical finite element structural models, or to determine whether a structure is damaged and further, the nature of any such damage.

The dynamic characteristics of a structure are frequently extracted from its measured seismic responses, and the damage of a structure is conventionally assessed from observed dynamic responses by detecting changes in the modal parameters of the structure. The concept underlying such an approach is that damage

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to a structure reduces its natural frequencies, increases the modal damping, and changes the modal shapes. Conventional techniques in the time domain, such as the time series approach associated with the ARX or ARMAX model [1-3] and the subspace approach [4, 5], have been often applied to determining the dynamic characteristics of a structure from its seismic responses. Hearn and Testa [6] applied a perturbation method to process measured dynamic responses of a steel frame in a laboratory, and found that changes in natural frequencies and modal damping are good indices for damage. However, from dynamic tests on bridges, Alampalli and Fu [7] and Salawu and Williams [8] concluded that the change in natural frequencies is not sufficiently sensitive to detect local damage in the structure.

In recent years, a new and powerful mathematical tool, called wavelet transformation, has been developed, the history of whose development can be found in introductory papers [9, 10] and books [11, 12]. Unlike a Fourier transform, which expresses a signal in terms of frequency components, the wavelet transform decomposes a signal into frequency components that are functions of time. The advantages of the wavelet transformation over the Fourier transformation have been addressed in several papers and books [10-12]. The wavelet transformation has been successfully applied in mathematics, physics and engineering, especially for signal processing and solving nonlinear problems.

Wavelet transformations have also caught the attention of researchers in the field of system identification, for use in determining the dynamic characteristics of a time invariant linear system. Schoenwald [13] identified the parameters in the equation of motion for a system with single degree of freedom by applying a continuous wavelet transform to the equation of motion. Ruzzene *et al.* [14] applied a discrete wavelet transform and the Hilbert transform technique to determine the natural frequencies and damping of a structure system from its free vibration responses. Robertson *et al.* [15, 16] developed a procedure for extracting impulse response data from the dynamic responses of a structure and used an eigensystem realization algorithm to identify the dynamic characteristics of the structure. Gouttebroze and Lardies [17] developed a wavelet identification approach in the time-frequency domain for elucidating the natural frequencies and damping of a structure from free vibration responses. Their approach cannot directly determine the mode shapes. Lardies and Gouttebroze [18] further applied their wavelet identification technique [17] to process the measured ambient vibrations of a TV tower, by first extracting a free vibration signal from the measured ambient vibration responses, using the conventional random decrement technique.

These existing methodologies, involving wavelet transformation for identifying the dynamic characteristics of a structure, use the wavelet transform either to extract impulse response functions or to determine natural frequencies and damping from free vibration responses. This work proposes a new procedure for applying a wavelet packet transformation to the earthquake responses of a structure, to determine its natural frequencies, damping ratios and mode shapes. This procedure applies a wavelet packet transformation to discrete equations of motion of a structure. Then, the parameters of the equations of motion are determined by a least-squares approach, and are directly used to determine the dynamic characteristics of the structure. This procedure is applied to the measured dynamic responses of a five-story steel frame in a shaking table test to demonstrate the feasibility of applying the proposed procedure to real data. The modal parameters of the frame subjected to 8%, 20%, 40%, 52% and 60% of the strength of the Kobe earthquake are identified from the measured acceleration responses. The steel frame responded nonlinearly when it subjected to 60% Kobe earthquake. Hence, this work also investigates the differences in modal parameters identified from the responses to the inputs with different strengths of the Kobe earthquake. The accuracy of the present results is confirmed by comparing the present results with the published [5] obtained from the responses to the 8% Kobe earthquake input.

## METHODOLOGY

### Wavelet packet

As the natural extension of the wavelet transform theory, Coifman, Meyer and Wickerhauser [19-21] first proposed their generalization results as wavelet packets to design efficient schemes for representing and compressing signals and images. The idea is to construct a library of orthonormal bases for  $L^2(R)$ . Assume a measured signal,  $s(t)$ , belongs to functional space  $U_{j+1}^{(0)}$ . In the wavelet packet analysis, the space is orthogonally decomposed into as many subspaces as needed (see Fig. 1), and is expressed as,

$$U_{j+1}^{(0)} = U_j^{(0)} \oplus U_j^{(1)} = U_{j-1}^{(0)} \oplus U_{j-1}^{(1)} \oplus U_{j-1}^{(2)} \oplus U_{j-1}^{(3)} = \dots = U_{j-k-1}^{(0)} \oplus U_{j-k-1}^{(1)} \oplus \dots \oplus U_{j-k-1}^{(2^{k+1}-2)} \oplus U_{j-k-1}^{(2^{k+1}-1)} \quad (1)$$

where each space  $U_{j+1}^{(m)}$  is decomposed into two orthogonal subspaces  $U_j^{(2m)}$  and  $U_j^{(2m+1)}$ . That is  $U_{j+1}^{(m)} = U_j^{(2m)} \oplus U_j^{(2m+1)}$ . Space  $U_j^{(m)}$  is spanned by  $(j, m)$  wavelet packet that is a set of orthonormal functions defined as

$$\{\mu_{m,j,k}(t) = 2^{j/2} \mu_m(2^j t - k), k \in Z\}. \quad (2)$$

The wavelet packets can be easily obtained using MATLAB wavelet toolbox and choosing a wanted mother wavelet function.

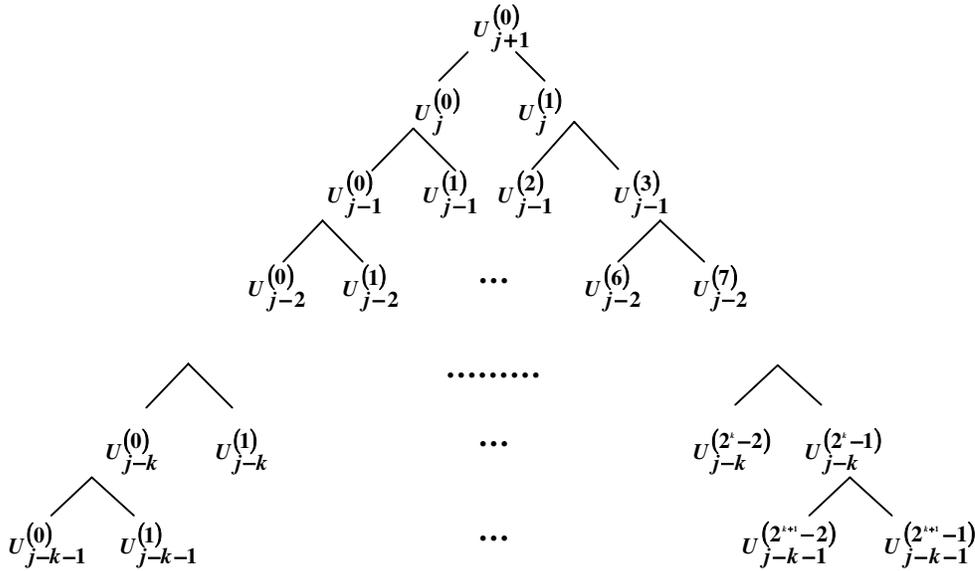


Fig. 1. Decomposition of functional space in wavelet packet method

The functions  $\mu_{m,j,k}$  satisfy the following orthonormal conditions:

$$\langle \mu_{m,j,k}, \mu_{m,j,l} \rangle = \begin{cases} 1 & \text{when } k = l \\ 0 & \text{when } k \neq l \end{cases} \quad \text{and} \quad \langle \mu_{m,j,k}, \mu_{n,j,l} \rangle = 0 \quad \text{when } m \neq n, \quad (3)$$

where the inner product operation is defined as  $\langle \mu_m, \mu_n \rangle = \int_R \mu_m(t) \mu_n^*(t) dt$  and ‘\*’ denotes the conjugate operation.

### System identification

The dynamic behaviors of a linear structure are described by the equations of motion,

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\} + [\mathbf{C}]\{\dot{\mathbf{x}}\} + [\mathbf{K}]\{\mathbf{x}\} = \{\mathbf{f}\}, \quad (4)$$

where  $[\mathbf{M}]$ ,  $[\mathbf{C}]$  and  $[\mathbf{K}]$  are the mass, damping and stiffness matrices of the structure system, respectively;  $\{\ddot{\mathbf{x}}\}$ ,  $\{\dot{\mathbf{x}}\}$  and  $\{\mathbf{x}\}$  are the acceleration, velocity and displacement response vectors of the system, and  $\{\mathbf{f}\}$  is the input force vector. Usually, not all degrees of freedom of the system are measured in a field experiment for economic reasons. Only some parts of  $\{\ddot{\mathbf{x}}\}$  or  $\{\dot{\mathbf{x}}\}$  are measured. Consequently, the measured response vector  $\{\mathbf{y}\}$ , which can be velocity or acceleration responses, satisfies the following discrete equation [22].

$$\{\mathbf{y}(t)\} = \sum_{i=1}^I [\boldsymbol{\Phi}]_i \{\mathbf{y}(t-i)\} + \sum_{j=0}^J [\boldsymbol{\theta}]_j \{\mathbf{f}(t-j)\}, \quad (5)$$

where  $\{\mathbf{y}(t-i)\}$  and  $\{\mathbf{f}(t-i)\}$  are the measured responses and forces at time  $t-i$ , respectively,  $[\boldsymbol{\Phi}]_i$  and  $[\boldsymbol{\theta}]_i$  are the parameter matrices to be determined.

Performing a wavelet packet analysis on  $\{\mathbf{y}(t-i)\}$  and  $\{\mathbf{f}(t-j)\}$  with  $i=0, 1, 2, \dots, I$  and  $j=0, 1, 2, \dots, J$  at the  $\hat{k}^{th}$  level of subspaces (i.e. at  $U_{j-\hat{k}+1}^{(m)}$  subspaces in Fig.1) yields

$$\{\mathbf{y}(t-i)\} = \sum_{m=0}^{\bar{m}} \sum_{l=0}^{\bar{l}} \{\mathbf{y}^{(i)}(m, \hat{k}, l)\} \mu_{m, \hat{k}, l}(t), \quad (6a)$$

$$\{\mathbf{f}(t-j)\} = \sum_{m=0}^{\bar{m}} \sum_{l=0}^{\bar{l}} \{\mathbf{f}^{(j)}(m, \hat{k}, l)\} \mu_{m, \hat{k}, l}(t). \quad (6b)$$

Notably,  $\bar{m}$  is set equal to  $2\hat{k}+1$  in the following analyses. For finite measured responses and input forces,  $\bar{l}$  depends on  $m$ . Substituting Eqs. (6a) and (6b) into Eq. (5); performing the inner product with respect to  $\mu_{m, \hat{k}, l}(t)$  on both sides of the resulting equation, and applying the orthonormal properties specified by Eq. (3) yields

$$\{\mathbf{y}^{(0)}(m, \hat{l}, l)\} = \sum_{i=1}^I [\boldsymbol{\Phi}]_i \{\mathbf{y}^{(i)}(m, \hat{l}, l)\} + \sum_{j=0}^J [\boldsymbol{\theta}]_j \{\mathbf{f}^{(j)}(m, \hat{l}, l)\}. \quad (7)$$

Rearranging Eq. (7) for different values of  $m$  and  $l$  yields

$$[\mathbf{Y}^{(0)}] = [\bar{\mathbf{C}}] \begin{bmatrix} [\mathbf{Y}] \\ [\mathbf{F}] \end{bmatrix}, \quad (8)$$

where

$$[\bar{\mathbf{C}}] = \begin{bmatrix} [\boldsymbol{\Phi}]_1 & [\boldsymbol{\Phi}]_2 & \dots & [\boldsymbol{\Phi}]_I & [\boldsymbol{\theta}]_0 & [\boldsymbol{\theta}]_1 & \dots & [\boldsymbol{\theta}]_J \end{bmatrix},$$

$$[\mathbf{Y}] = \begin{bmatrix} [\mathbf{Y}^{(1)}]^T & [\mathbf{Y}^{(2)}]^T & \dots & [\mathbf{Y}^{(I)}]^T \end{bmatrix}^T,$$

$$[\mathbf{F}] = \begin{bmatrix} [\mathbf{F}^{(0)}]^T & [\mathbf{F}^{(1)}]^T & \dots & [\mathbf{F}^{(J)}]^T \end{bmatrix}^T, \quad [\mathbf{Y}^{(i)}] = \begin{bmatrix} \mathbf{y}^{(i)}(1, \hat{l}, 1) & \mathbf{y}^{(i)}(1, \hat{l}, 2) & \dots & \mathbf{y}^{(i)}(\bar{m}, \hat{l}, \bar{l}) \end{bmatrix}$$

$$[F^{(i)}] = [f^{(i)}(1, \hat{i}, 1) \quad f^{(i)}(1, \hat{i}, 2) \quad \dots \quad f^{(i)}(\bar{m}, \hat{i}, \bar{l})]$$

Typically, Eq. (8) represents a set of over-determined linear algebraic equations. The solution for the matrix of parameters  $[\bar{C}]$  is determined by a conventional least-squares approach:

$$[\bar{C}] = [Y^{(0)}] \begin{bmatrix} [Y] \\ [F] \end{bmatrix}^T \left( \begin{bmatrix} [Y] \\ [F] \end{bmatrix} \begin{bmatrix} [Y] \\ [F] \end{bmatrix}^T \right)^{-1} \quad (9)$$

Equation (5) reveals that the modal parameters of a structure are determined from  $[\Phi]_i$  with  $i=1, 2, \dots, I$ . Following the procedure given in [23], one can determine the natural frequencies, damping ratios and mode shapes of the structure from the obtained  $[\Phi]_i$ .

## APPLICATION

Shaking table tests are often performed in a laboratory to examine the behavior of structures in earthquakes. The National Center for Research in Earthquake Engineering in Taiwan undertook a series of shaking table tests on a 3 m long, 2 m wide, and 6.5 m high steel frame [24] (Fig. 2) to generate a set of earthquake response data for this benchmark model of a five-story steel structure. Lead blocks were piled on each floor such that the mass of each floor was approximately 3664 kg. The frames were subjected to the base excitation of the Kobe earthquake, weakened by various levels. The displacement, velocity, and acceleration response histories of each floor were recorded during the shaking table tests. Additionally, some strain gauges were also installed in one of the columns and near the first floor. The sampling rate of the raw data was 1000 Hz.

Notably, it was reported [24] that the frame responded linearly when it subjected to 8%, 10%, 20%, 40%, and 52% of the strength of the Kobe earthquake. Measured strains and visual inspection revealed that 60% of the strength of the Kobe earthquake input caused the steel columns near the first floor to yield. In the following, only the responses and inputs in the long span direction are discussed.

### **Examination of the frame in the Kobe earthquake with various reduction levels**

The measured acceleration responses from  $t=4.5$  to 12.5 seconds (see Fig.3) corresponding to the base excitations with various reduction levels of the Kobe earthquake, were processed. Table 1 summarizes the identified dynamic characteristics of the frame by using the proposed procedure with setting  $I=J=70$  in eq.(5) and  $\hat{k}=3$  in eqs. (6a) and (6b). Notably, using large  $I$  and  $J$  eliminates the effects of noise and different mother wavelet functions on the identified results [25].

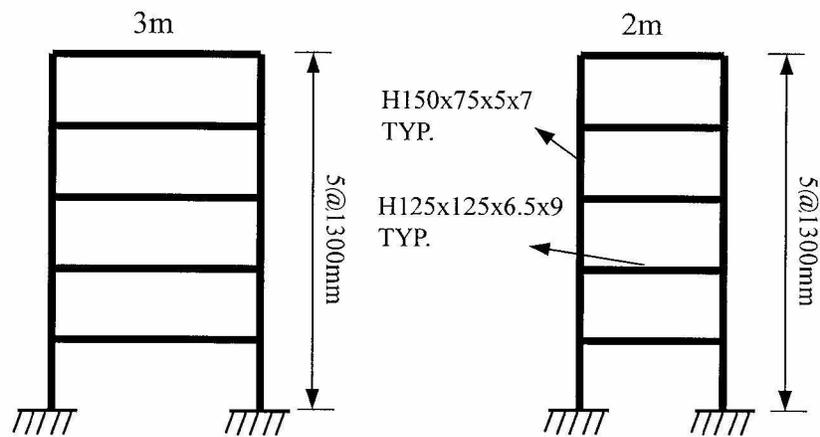


Fig. 2. A photo and simple sketch of a five-story steel frame

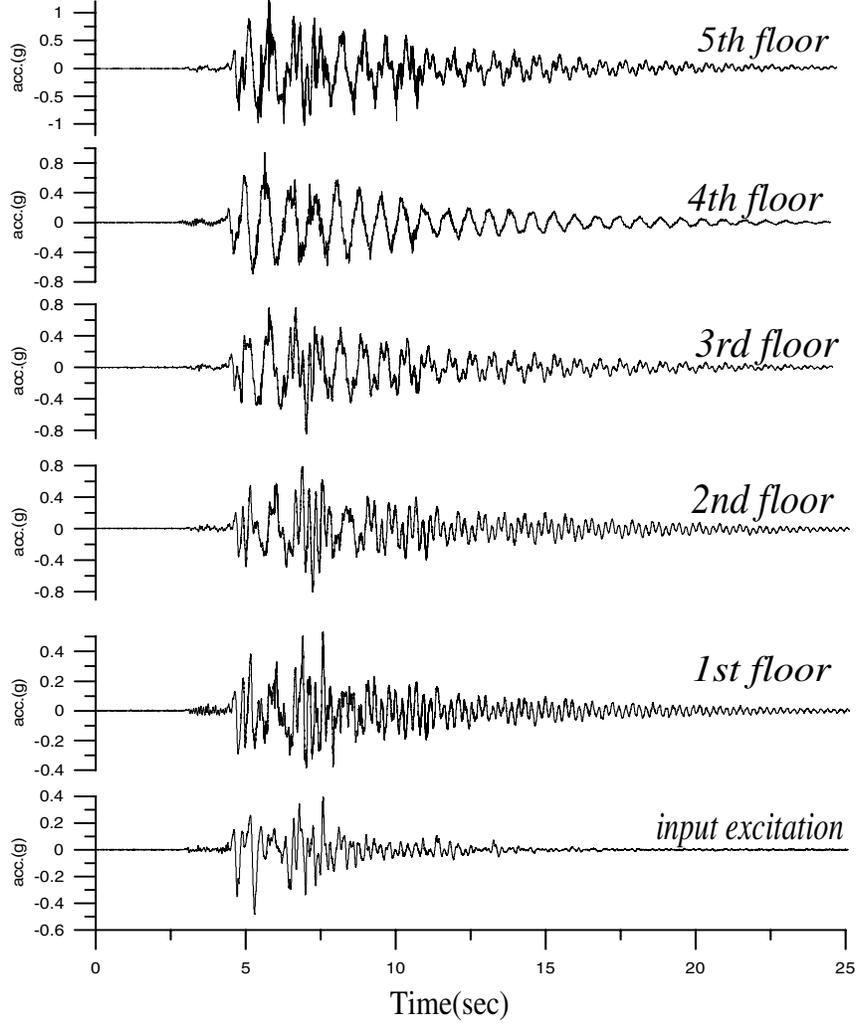


Fig. 3. Response histories for 60% Kobe earthquake input.

In Table 1,  $e$  is defined as [26]

$$e = \left( \frac{(\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^T (\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^*}{\boldsymbol{\varphi}_{iR}^T \boldsymbol{\varphi}_{iR}^*} \right)^{1/2} \quad (10)$$

where the complex constant,  $a$ , is obtained by minimizing  $(\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^T (\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^*$ , \* denotes the complex conjugate,  $\boldsymbol{\varphi}_{iR}$  and  $\boldsymbol{\varphi}_{iC}$  represent the  $i^{\text{th}}$  complex mode shapes for the reference state and the current state to which it is to be compared, respectively. When the two modal shapes are highly correlated,  $e$  is close to zero.  $e$  values are more sensitive to the changes in modal shapes than  $MAC$  (modal assurance criterion) values [27]. Here, the values of  $e$  designate the correlation between the modal shapes for an input of 8% Kobe earthquake and those for inputs with other reduction levels.

Table 1. Identified modal parameters for different inputs.

Excitation	Mode	Frequency(Hz)	Damping(%)	$e$ (%)
8%	1	1.40 (1.40)	1.48 (1.30)	(0.35)
	2	4.53 (4.53)	0.17 (0.16)	(0.76)
	3	8.24 (8.23)	0.19 (0.19)	(2.76)
	4	12.39 (12.39)	0.13 (0.13)	(0.58)
	5	16.00 (15.99)	0.12 (0.10)	(0.47)
20%	1	1.39	1.68	0.72
	2	4.53	0.25	0.88
	3	8.23	0.34	2.25
	4	12.38	0.17	0.59
	5	15.96	0.16	0.56
40%	1	1.38	2.55	1.19
	2	4.50	0.47	1.22
	3	8.19	0.25	2.96
	4	12.36	0.17	1.8
	5	15.93	0.14	1.16
52%	1	1.37	3.03	1.31
	2	4.49	0.69	1.07
	3	8.15	0.51	2.65
	4	12.33	0.10	2.12
	5	15.92	0.61	2.65
60%	1	1.36	4.23	2.65
	2	4.45	0.88	5.64
	3	8.09	0.71	9.15
	4	12.24	0.73	10.16
	5	15.88	0.36	8.35

Comparing the present results with the parenthesized results that were obtained by using a subspace method [5] confirms the accuracy of the present results. Table 1 reveals that the frequencies for each mode generally decrease as the excitation magnitude increases, but the changes in frequency are not so considerable. Generally, the modal damping values increase with excitation magnitude. The modal damping values for the 60% Kobe earthquake are much greater than those for the 8% Kobe. Interestingly, only the damping for the first mode exceeds 1% while the damping for the other modes is typically much less than 1%. Notably,  $e$  values clearly show that the modal shapes of the higher modes (3<sup>rd</sup> to 5<sup>th</sup>) for the 60% Kobe earthquake notably differ from those for the 8% Kobe earthquake, since the corresponding  $e$  values far exceed 5%. The damping and  $e$  values truly reflect the fact of possible damage of the frame under the 60% Kobe earthquake input.

In the present procedure of identifying the dynamic characteristics of a structure from its dynamic responses, the discrete equations of motion (i.e. eq. (5)) for the measured degrees of freedom were also established. It is worthwhile to investigate the errors of the one step ahead predicted output from the established discrete equations. Figures 4 and 5 show the comparisons

between the observed responses and predicted ones for the 8% and 60% Kobe earthquake inputs, respectively. Notably, the responses in these two figures were normalized according to the maximum responses in each figure, respectively. Comparing Figs. 4 and 5 finds that the discrepancies between prediction and measurement for the 60% Kobe earthquake input far exceed those for the 8% Kobe earthquake input. This observation is useful for structural damage assessment.

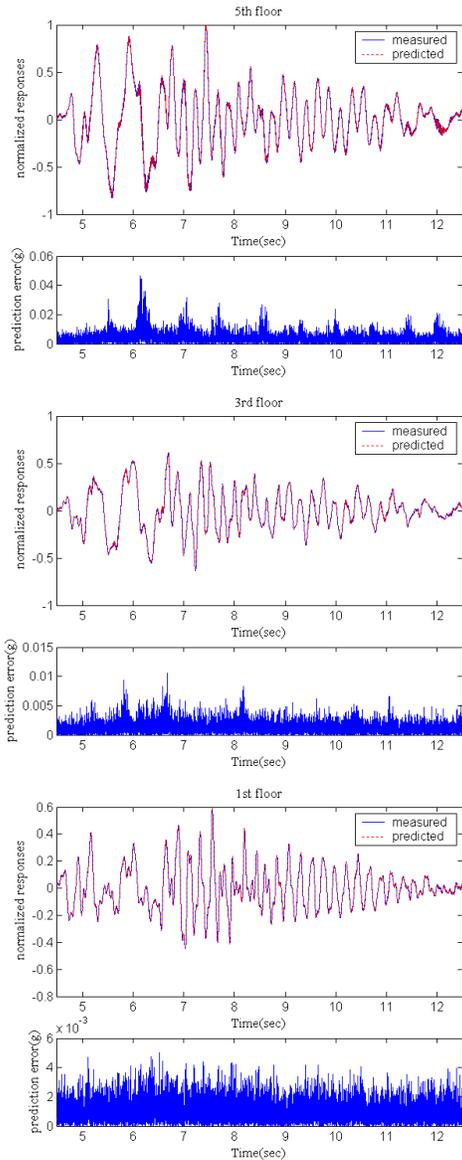


Fig. 4. Comparison between the measured and predicted responses for 8% Kobe earthquake

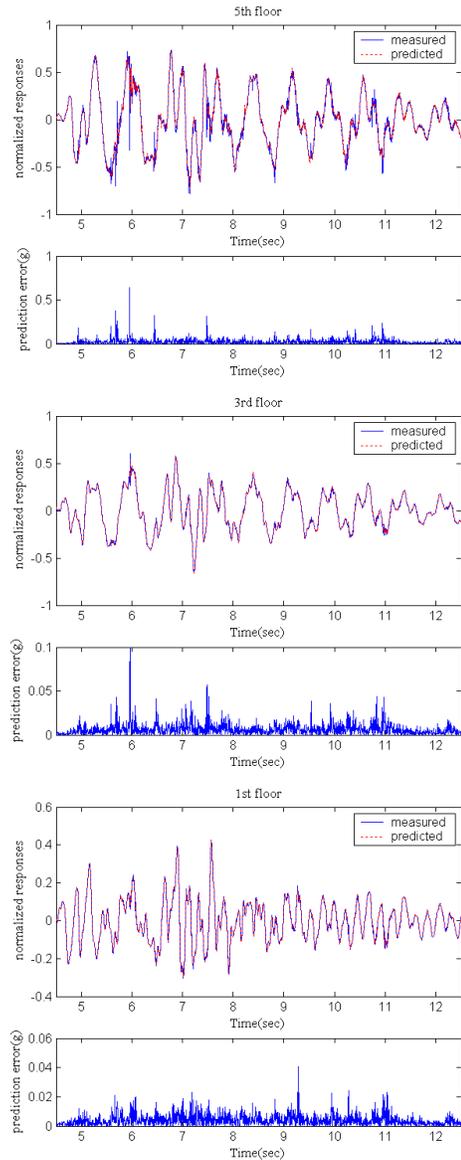


Fig. 5. Comparison between the measured and predicted responses for 60% Kobe earthquake

### Identifying dynamic characteristics from windowed responses

Since 60% of the strength of the Kobe earthquake input caused the steel columns near the first floor to yield, the frame should be considered as a time varying system. Consequently, it is interesting to study the changes in modal parameters obtained from different portions of response records. The responses from  $t=4$  to 14 seconds were divided into nine segments, each of 2 seconds, each of which overlays 1 second of the previous one. Table 2 presents the modal parameters, obtained from different segments of data for the 8% and 60% Kobe earthquakes. Notably, the  $e$  values for the first segment were computed with reference to the modal shapes obtained using the responses between  $t=4.5$  and 12.5 seconds, while the  $e$  values for other segments refer to the modal shapes for the first segment.

Table 2. Identified modal parameters from windowed responses.

Excitation	Parameters	Mode	Segment									Mean( $\mu$ )	STD( $\sigma$ )	$\sigma/\mu$	
			1	2	3	4	5	6	7	8	9				
8%	Frequency(Hz)	1	1.39	1.39	1.40	1.40	1.40	1.40	1.40	1.40	1.41	1.40	0.006	0.0043	
		2	4.53	4.53	4.53	4.53	4.53	4.53	4.53	4.53	4.54	4.53	0.0027	0.0006	
		3	8.24	8.25	8.24	8.23	8.24	8.24	8.24	8.24	8.24	8.24	0.0041	0.0005	
		4	12.38	12.37	12.38	12.39	12.39	12.39	12.39	12.39	12.39	12.38	12.39	0.0081	0.0007
		5	16.00	15.99	15.99	15.98	16.00	16.00	16.00	16.01	16.00	16.00	16.00	0.0093	0.0006
	Damping(%)	1	1.80	1.58	1.28	1.00	1.43	1.44	1.08	1.31	0.80	1.28	0.37	0.29	
		2	0.12	0.17	0.16	0.21	0.16	0.18	0.22	0.22	0.25	0.19	0.04	0.22	
		3	0.07	0.17	0.09	0.23	0.22	0.12	0.13	0.27	0.22	0.16	0.07	0.44	
		4	0.11	0.13	0.15	0.13	0.09	0.14	0.05	0.13	0.15	0.11	0.04	0.31	
		5	0.14	0.03	0.03	0.34	0.15	0.10	0.12	0.13	0.07	0.12	0.09	0.79	
	$e$ (%)	1	1.08	0.82	0.95	1.49	0.93	0.83	2.40	2.55	1.22	1.40	0.70	0.50	
		2	1.11	0.31	0.81	1.13	1.78	1.68	1.66	2.52	2.07	1.50	0.71	0.48	
		3	0.43	3.86	3.24	1.13	1.60	2.27	2.70	1.58	2.54	2.37	0.92	0.39	
		4	0.63	1.03	0.30	1.29	1.39	1.25	0.83	3.81	1.73	1.45	1.04	0.72	
		5	0.81	0.64	2.79	2.33	0.62	0.68	0.80	1.89	0.78	1.32	0.88	0.67	
60%	Frequency(Hz)	1	1.33	1.35	1.36	1.35	1.36	1.38	1.37	1.38	1.38	1.36	0.0188	0.0138	
		2	4.48	4.44	4.43	4.44	4.46	4.47	4.49	4.50	4.50	4.47	0.0264	0.0059	
		3	8.11	8.08	8.06	8.04	8.12	8.22	8.15	8.16	8.19	8.13	0.0609	0.0075	
		4	12.19	12.19	12.26	12.25	12.23	12.34	12.35	12.33	12.33	12.28	0.0659	0.0054	
		5	15.92	15.83	15.88	15.88	15.87	15.89	15.88	15.89	15.91	15.88	0.0249	0.0016	
	Damping(%)	1	6.06	4.54	3.80	3.41	3.15	2.32	2.35	2.01	1.83	3.27	1.37	0.42	
		2	0.95	1.11	0.96	0.99	0.89	0.75	0.52	0.44	0.31	0.77	0.28	0.36	
		3	0.97	0.14	0.85	1.20	0.77	0.49	0.15	0.05	0.19	0.54	0.42	0.79	
		4	0.62	0.47	1.01	1.08	0.36	0.40	0.76	0.07	0.10	0.54	0.36	0.66	
		5	0.46	1.11	0.26	0.11	0.15	0.46	0.52	0.19	0.31	0.40	0.30	0.77	
	$e$ (%)	1	0.49	0.48	0.81	0.69	0.51	0.62	0.88	0.41	0.56	0.62	0.16	0.26	
		2	1.52	1.85	1.97	1.75	1.18	0.68	0.76	0.46	0.35	1.13	0.66	0.58	
		3	2.35	1.55	3.01	3.24	3.60	3.92	5.64	2.73	4.18	3.48	1.19	0.34	
		4	2.37	3.33	2.67	2.09	1.70	7.68	5.83	2.13	2.89	3.54	2.11	0.60	
		5	0.72	6.15	1.54	1.64	2.12	0.54	1.74	2.44	2.65	2.35	1.66	0.71	

Table 2 shows that using different segments of data leads to no significant variation in the identified frequencies. However, the ratios of the mean values to standard deviations of the identified frequencies from different segments are much larger for the 60% Kobe earthquake input than those for the 8% Kobe earthquake input. The damping values do differ somewhat with different segments of data. The modal shapes obtained from different segments for the 8% Kobe earthquake input show no significant differences. Interestingly, considerable differences in higher modal shapes ( $e \geq 5\%$ ) are observed for only two segments in the 60% Kobe earthquake input.

### CONCLUDING REMARKS

This paper presented a wavelet-based approach for identifying the dynamic characteristics of a structure from its seismic responses. A wavelet packet method was applied to process the measured responses. The discrete equations of motion corresponding to the measured degrees of freedom were reconstructed using the orthonormal properties of wavelet packets. The coefficients in the discrete equations of motion were determined by a conventional least-squares approach. Then, the natural frequencies, damping ratios and mode shapes of the structure were calculated from these coefficients.

The proposed procedure was applied to process the acceleration responses of a five-story steel frame, subjected to 8%, 20%, 40%, 52% and 60% of the strength of the Kobe earthquake, in shaking table tests. The proposed method of estimating the modal parameters was verified by excellent agreement between the present results and those obtained by a subspace method, for the frame subjected to the 8% Kobe earthquake. The reported nonlinear responses to the 60% Kobe earthquake input were found to change significantly modal shapes from those for the frame subjected to the other strengths of the Kobe earthquake. Furthermore, the prediction errors of the established discrete equations of motion for the responses to 60% Kobe earthquake input were considerably larger than those for the 8% Kobe earthquake input.

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