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VALIDATION OF SINGLE STOREY MODELS FOR THE EVALUATION OF THE SEISMIC PERFORMANCE OF MULTI-STOREY ASYMMETRIC BUILDINGS

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SUMMARY

Of all the models used in the past for the development and evaluation of seismic design criteria for asymmetric structures, the most common has been the single-storey model with the same number of resisting planes as the structure it represents, same uncoupled dynamic properties and three degrees of freedom. When considering the coupled dynamic properties of these models, they are different to those of the structure they aim to represent. The only model that allows a correspondence of results with the structures represented is the parametric model proposed by Kan and Chopra. Based on this model, and with the purpose of evaluating and transforming to real buildings the massive amount of available results generated with single-storey models and investigating new cases of interest, this paper presents an approximate procedure that allows the definition of a 3-degree of freedom simplified "structure" representing the real structure and satisfying the requisites of the parametric model of Kan and Chopra. The non-linear force-displacement relationships for this model are obtained from the capacity curves of the real structure. Once the performance point of the approximate model is obtained, the seismic performance of the real building is obtained using the correspondence relationships to transform this performance point to the response of the real structure. Finally the forces and displacements corresponding to this performance point are recovered from the results of a pushover analysis.

INTRODUCTION

Most studies related to the torsional behaviour of buildings have been based on simplified single-storey models with three degrees of freedom (3DOF), Figs.1 and 2. The assumption on which these studies are based has been that these models represent, in the linear an non-linear range of behaviour, the response of multi-storey torsionally coupled buildings subjected to seismic forces, e.g., Bustamante and Rosenblueth [1] for linear models and Ayala and García [2] for non-linear models The information derived from these models is vast and has been extrapolated directly to multi-storey buildings. In fact, the validity of single-storey models to approximately represent multi-storey buildings response is questionable. Several authors have concluded that the single storey analogy to multi-storey buildings can be used only with shear buildings with constant stiffness ratios between lateral load resistant elements on each storey (in these

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structures the stiffness centres of all floors are located along a vertical axis), Yoon and Stafford-Smith [3].

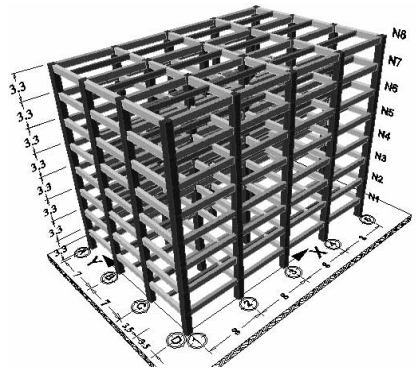


Figure 1. Multi-storey asymmetric building.

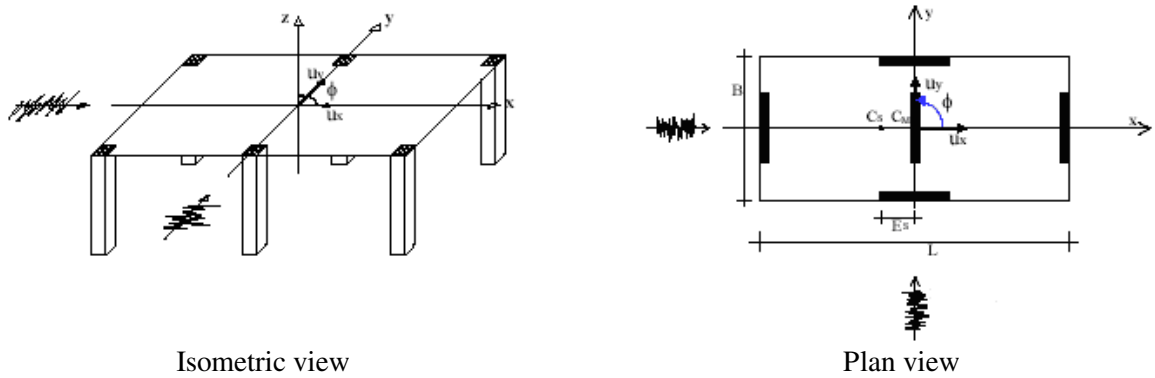


Figure 2. Equivalent single storey asymmetric model.

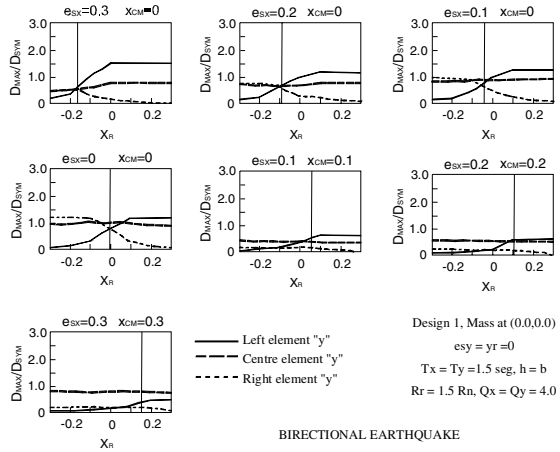


Figure 3. Illustration of the seismic performance of a single-storey 3DOF model.

With the aim to adequately interpret data from research of single-storey models, this paper presents a correspondence law between the results of an equivalent model; *e.g.* Fig.3, and the real building. Equations that allow the definition of single-storey models with 3DOF and natural frequencies approximating the frequencies of the corresponding coupled modes of the real building are presented. The same equations may be used to find the seismic response of real

structures from the responses of their equivalent linear elastic models. The assumptions used to formulate this equivalent model restrict its application to a class of buildings with certain geometric and asymmetry characteristics. Considering that the response of asymmetric buildings subjected to design demands beyond service conditions incursion in the non-linear range of behaviour, this paper establishes the concepts and steps required to formulate a non-linear single-storey 3DOF model similar to the elastic model defined analytically. This model is simpler than those based on conventional pushover analyses which associate the inelastic seismic response of a real structure to that of a simplified single degree of freedom model.

DEFINITION OF THE EQUIVALENT ELASTIC SINGLE-STOREY MODEL

In 1971 Kan and Chopra [4] developed a formulation to define a single-storey 3DOF system with eigenvalues that approximate the first three coupled frequencies of a real building. They presented a method to construct an approximation of the modal shapes of a torsionally coupled multi-storey building from a linear combination of the uncoupled modal shapes, obtaining an equation that allows the estimation of the maximum response from a modal spectral analysis.

For a building subjected to earthquake-like excitations, the dynamic equilibrium may be written as:

$$\begin{bmatrix} [m] & 0 & 0 \\ 0 & [m] & 0 \\ 0 & 0 & [m_\theta] \end{bmatrix} \begin{Bmatrix} \{\ddot{d}_x\} \\ \{\ddot{d}_y\} \\ \{\ddot{d}_\theta\} \end{Bmatrix} + \begin{bmatrix} [K_{xx}] & 0 & [K_{x\theta}] \\ 0 & [K_{yy}] & [K_{y\theta}] \\ [K_{x\theta}] & [K_{y\theta}] & [K_{\theta\theta}] \end{bmatrix} \begin{Bmatrix} \{d_x\} \\ \{d_y\} \\ \{d_\theta\} \end{Bmatrix} = -\begin{bmatrix} \ddot{u}_{g\,x}[m]\{1\} \\ \ddot{u}_{g\,y}[m]\{1\} \\ 0 \end{bmatrix} \tag{1}$$

where:
[m]: Translational mass matrix (diagonal).
$[m_\theta]=[m][r^2]$: Rotational mass matrix (diagonal).
$[r^2]$: Matrix of the squared gyration radii (Diagonal).
$[K_{xx}]$: Stiffness matrix partition corresponding to the translational stiffness in X direction.
$[K_{yy}]$: Stiffness matrix partition corresponding to the translational stiffness in Y direction.
$[K_{\theta\theta}]$ Stiffness matrix partition corresponding to the torsional stiffness.
$[K_{x\theta}]$: Stiffness matrix partition corresponding to the coupled Xθ stiffness.
$[K_{y\theta}]$: Stiffness matrix partition corresponding to the coupled Yθ stiffness.
$\ddot{u}_{gx}$ and $\ddot{u}_{gy}$: Ground acceleration in X and Y directions, respectively.
{1}: Unit vector.
$\{d_x\}$: Displacement vector in X direction.
$\{d_y\}$: Displacement vector in Y direction.
$\{d_\theta\}$: Rotation vector.

In general, Eq. 1 can be rearranged and modified accordingly to the transformation proposed in this work:

$$\begin{bmatrix} [m] & 0 & 0 \\ 0 & [m] & 0 \\ 0 & 0 & [m] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}_x\} \\ \{\ddot{u}_\theta\} \\ \{\ddot{u}_y\} \end{Bmatrix} + \begin{bmatrix} [K_{xx}] & [K_{xt}] & 0 \\ [K_{tx}] & [K_{tt}] & [K_{ty}] \\ 0 & [K_{yt}] & [K_{yy}] \end{bmatrix} \begin{Bmatrix} \{u_x\} \\ \{u_\theta\} \\ \{u_y\} \end{Bmatrix} = -\begin{bmatrix} \ddot{u}_{g\,x}[m]\{1\} \\ 0 \\ \ddot{u}_{g\,y}[m]\{1\} \end{bmatrix} \tag{2}$$

$$[K_{tt}] = [r]^{-1}[K_{\theta\theta}][r]^{-1} \tag{3}$$

$$[K_{xt}] = [K_{x\theta}][r]^{-1} \tag{4}$$

$$[K_{yt}] = [K_{y\theta}][r]^{-1} \tag{5}$$

$$\{u\} = \begin{Bmatrix} \{u_x\} \\ \{u_\theta\} \\ \{u_y\} \end{Bmatrix} = [R]^{-1} \begin{Bmatrix} \{d_x\} \\ \{d_\theta\} \\ \{d_y\} \end{Bmatrix} \tag{6}$$

$$[R] = \begin{bmatrix} [I] & 0 & 0 \\ 0 & [r]^{-1} & 0 \\ 0 & 0 & [I] \end{bmatrix} \quad , \quad [r] = \left[r^2\right]^{1/2} \tag{7}$$

where:
[R]: Transformation matrix.
[r]: Radius of gyration matrix (diagonal).
[I]: Identity matrix.

In the particular case of the single-storey 3DOF structure used to represent a multi-storey building, Eq. 2 becomes:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_\theta \\ \ddot{u}_y \end{Bmatrix} + \begin{bmatrix} k_x & \dfrac{-e_y k_x}{r} & 0 \\ \dfrac{-e_y k_x}{r} & \dfrac{k_\theta}{r^2} & \dfrac{e_x k_y}{r} \\ 0 & \dfrac{e_x k_y}{r} & k_y \end{bmatrix} \begin{Bmatrix} u_x \\ u_\theta \\ u_y \end{Bmatrix} = -m \begin{Bmatrix} \ddot{u}_{gx} \\ 0 \\ \ddot{u}_{gy} \end{Bmatrix} \tag{8}$$

where:
m: Translational mass for the 3DOF model.
r: Radius of gyration of the 3DOF model.
k_x: Translational stiffness in X direction.
k_y: Translational stiffness in Y direction.
k_θ: Rotational stiffness.
e_x: Eccentricity in X direction.
e_y: Eccentricity in Y direction.
u_i: Displacement of the i= x, y and θ degree of freedom.

With this equation it is possible to determine for a real building any required response variable, however in this work only the time history of displacements will be considered as variable of study, because, having determined the history of displacements of the equivalent model, the methods to obtain other variables are standard and can be consulted in the specialized literature, Chopra [5].

Equivalent model obtained from the Perturbation Theory

The eigenvalue problem of the single-storey equivalent model can be approximated using a second order perturbation of the multi-storey eigenvalue problem of the original model, Wilkinson [6] and Kan and Chopra [7]. The application of a perturbation to an eigenvalue problem expressed in its canonical form consists in approximating the matrix of coefficients as the sum of two matrices and its eigenvalues and eigenvectors by power series, Wilkinson [6]. In this way, the eigenvalue problem related with the original model can be expressed as:

$$([K_0]+[E]-\omega^2[M])\{\Phi\}=\{0\} \tag{9}$$

$$[K_0]:\begin{bmatrix}[K_{xx}] & 0 & 0\\ 0 & [K_{tt}] & 0\\ 0 & 0 & [K_{yy}]\end{bmatrix},\quad [E]:\begin{bmatrix}0 & [K_{xt}] & 0\\ [K_{xt}] & 0 & [K_{yt}]\\ 0 & [K_{yt}] & 0\end{bmatrix},\quad [M]:\begin{bmatrix}[m] & 0 & 0\\ 0 & [m] & 0\\ 0 & 0 & [m]\end{bmatrix} \tag{10}$$

where:

ω: Frequency.

$[K_0]$: Uncoupled stiffness matrix (unperturbed system).

[E]: Perturbation matrix (perturbed system).

[M]: Translational mass matrix.

$\{\Phi\}$: Modal shape.

$[K]\doteq[K_0]+[E]$

The parameters of the equivalent model derived from an application of the Perturbation Theory correspond to a second order approximation in the power series and are expressed in terms of the partitions, eigenvalues and eigenvectors of the uncoupled matrix. Using the Perturbation Theory the modal shapes of the original model can be expressed as a linear combination of those corresponding to the uncoupled system, Wilkinson [6] and Kan and Chopra [7]:

$$\{\Phi\}\doteq[\Psi]\{\alpha\} \tag{11}$$

where:

$$[\Psi]=\begin{bmatrix}\{\Psi_x\} & \{0\} & \{0\}\\ \{0\} & \{\Psi_\theta\} & \{0\}\\ \{0\} & \{0\} & \{\Psi_y\}\end{bmatrix} \tag{12}$$

$$\{\alpha\} = \begin{Bmatrix} \alpha_x \\ \alpha_\theta \\ \alpha_y \end{Bmatrix} \tag{13}$$

$$\{\alpha\}^T \{\alpha\} = 1 \tag{14}$$

where:

[Ψ]: Modal shapes of the unperturbed system ($[K_0]$).
$\{\Psi_x\}$: First modal shape corresponding to the [Kxx] partition.
$\{\Psi_y\}$: First modal shape corresponding to the [Kyy] partition.
$\{\Psi_\theta\}$: First modal shape corresponding to the [Ktt] partition.
{α}: Modal shape of the equivalent model.

In Eq. (11) the {α} vector may be interpreted as the vector of coupling coefficients (linear combination factors) needed to obtain the coupled modal shapes of the irregular building from its uncoupled modal shapes. Eq. (11) states the correspondence relationship between the response parameters of the single storey model and those of the real building.

Using the above approach, the eigenvalue problem associated to the original model may be expressed as:

$$(\left[\Psi^t\right][K]\left[\Psi\right] - \omega^2 \left[\Psi^t\right][M]\left[\Psi\right])\{\alpha\} = \{0\} \tag{15}$$

Developing

$$\begin{bmatrix} \omega_x^{\,2} - \omega^2 & \left\{\Psi_x^T\right\}\left[K_{xt}\right]\left\{\Psi_\theta\right\} & 0 \\ \left\{\Psi_x^T\right\}\left[K_{xt}\right]\left\{\Psi_\theta\right\} & \omega_t^2 - \omega^2 & \left\{\Psi_y^T\right\}\left[K_{yt}\right]\left\{\Psi_\theta\right\} \\ 0 & \left\{\Psi_y^T\right\}\left[K_{yt}\right]\left\{\Psi_\theta\right\} & \omega_y^{\,2} - \omega^2 \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_\theta \\ \alpha_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{16}$$

$$\left\{\Psi_x^T\right\}[m]\left\{\Psi_x\right\} = \left\{\Psi_\theta^T\right\}[m]\left\{\Psi_\theta\right\} = \left\{\Psi_y^T\right\}[m]\left\{\Psi_y\right\} = 1 \tag{17}$$

where:

ω_x, ω_y, and ω_t: Frequencies of the first modes of the uncoupled system ($[K_{xx}]$, $[K_{yy}]$ and $[K_{tt}]$ partitions respectively).

Establishing the eigenvalue problem of the single-storey 3DOF model:

$$\begin{bmatrix} k_x - \omega^2 m & -\frac{e_y}{r}k_x & 0 \\ -\frac{e_y}{r}k_x & k_t - \omega^2 m & \frac{e_x}{r}k_y \\ 0 & \frac{e_x}{r}k_y & k_y - \omega^2 m \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_\theta \\ \alpha_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{18}$$

Comparing term by term Eqs. 16 and 18:

$$k_x = \omega_x{}^2 m, \quad k_t = \omega_\theta{}^2 m, \quad k_y = \omega_y{}^2 m, \quad k_t = \frac{K_\theta}{r^2} \tag{19}$$

$$\frac{e_x}{r} = \frac{\left\{\Psi_y^T\right\}\left[K_{yt}\right]\left\{\Psi_\theta\right\}}{\omega_y{}^2 m} \tag{20}$$

$$\frac{e_y}{r} = -\frac{\left\{\Psi_x^T\right\}\left[K_{xt}\right]\left\{\Psi_\theta\right\}}{\omega_x{}^2 m} \tag{21}$$

Solving the eigenvalue problem of the equivalent structure, the frequencies obtained are an approximation to the first three natural frequencies of the original model, and the corresponding modal shapes are the coefficients which allow the modal shapes of the original model to be expressed as a linear combination of the modal shapes of the uncoupled system.

One feature of the model proposed by Kan and Chopra [7] is that it is parametric in nature. *i.e.,* the mass and stiffness matrices are modified in such way that they are factorized by the radius of gyration, not allowing the explicit definition of geometry, stiffness and mass.

Proposed transformation

In this work a linear transformation relating the mass and stiffness matrices to the Kan and Chopra [7] model is proposed. For the particular case of a building with equal mass at all levels and mass centres located along a vertical axis by assigning the mass and radius of gyration of any level of the original model to the equivalent model, it is possible to define the mass and transformation matrices as:

$$\left[m_0\right] = \begin{bmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{bmatrix} \tag{22}$$

$$[R]=\begin{bmatrix}[I] & 0 & 0\\ 0 & [r]^{-1} & 0\\ 0 & 0 & [I]\end{bmatrix} \quad , \quad [r]=\left[r^2\right]^{1/2} \tag{23}$$

where:

$[m_0]$: Translational masses matrix for the 3DOF model (containing the translational mass of any levels in original structure).
$m_{0:}$ Translational mass of the equivalent model.
[r]: Radius of gyration matrix of any floor of the original structure (diagonal).
[R]: Transformation matrix.

The eigenvalue problem for the equivalent system is:

$$\left([W_o]-\omega^2[m_0]\right)\{\alpha\}=\left(\begin{bmatrix} k_x & -\frac{e_y}{r}k_x & 0\\ -\frac{e_y}{r}k_x & k_t & \frac{e_x}{r}k_y\\ 0 & \frac{e_x}{r}k_y & k_y\end{bmatrix}-\omega^2\begin{bmatrix} m_0 & 0 & 0\\ 0 & m_0 & 0\\ 0 & 0 & m_0\end{bmatrix}\right)\begin{Bmatrix}\alpha_x\\ \alpha_\theta\\ \alpha_y\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{24}$$

where:

$[W_o]$: Equivalent stiffness matrix.

The canonical form of the eigenvalue problem associated to the equivalent model, illustrated in Fig 4, is:

$$\left([\Pi]-\omega^2[I]\right)\{\alpha\}=\left(\begin{bmatrix} \frac{k_x}{m_0} & -\frac{e_y}{r}\frac{k_x}{m_0} & 0\\ -\frac{e_y}{r}\frac{k_x}{m_0} & \frac{k_t}{m_0} & \frac{e_x}{r}\frac{k_y}{m_0}\\ 0 & \frac{e_x}{r}\frac{k_y}{m_0} & \frac{k_y}{m_0}\end{bmatrix}-\omega^2\begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)\begin{Bmatrix}\alpha_x\\ \alpha_\theta\\ \alpha_y\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{25}$$

where:

[Π] Coefficient matrix of the canonical form of the eigenvalue problem.
Multiplying Eq. 25 by matrix $[m_0]$, Eq. 22, leads to:

$$\left([W_o]-\omega^2[m_0]\right)[R]^{-1}[R]\{\alpha\}=0 \tag{26}$$

Pre-multiplying this equation by $[R]^{-1}$:

$$[R]^{-1}\left([W_o]-\omega^2[m_0]\right)[R]^{-1}[R]\{\alpha\}=0 \tag{27}$$

expanding:

$$\left([R]^{-1}[W_o][R]^{-1}-\omega^2[R]^{-1}[m_0][R]^{-1}\right)[R]\{\alpha\}=0 \tag{28}$$

the following matrix equation is obtained:

$$\left([K_C]-\omega^2\left[m_p\right]\right)\{\beta\}=0 \tag{29}$$

where:

$$[K_C]=[R]^{-1}[m_0][\Pi][R]^{-1} \tag{30}$$

$$\left[m_p\right]=[R]^{-1}[m_0][R]^{-1} \tag{31}$$

$$\{\beta\}=[R]\{\alpha\} \tag{32}$$

$[m_p]$:Acceleration coefficients matrix (Diagonal).
$[K_C]$:Displacement coefficients matrix (Diagonal).

As a result of the previous formulation, the dynamic equilibrium equation for a single storey 3DOF structure in its conventional form is:

$$\left[m_p\right]\left\{\ddot{d}\right\}+[K_C]\{d\}=-\left[m_p\right]\{A\} \tag{33}$$

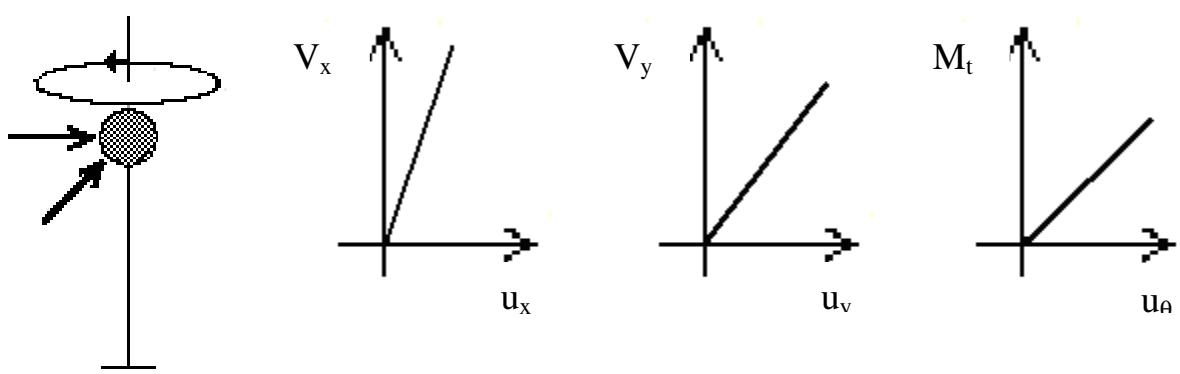


Figure 4. Equivalent 3DOF model.

EXAMPLE

Building description

To illustrate the application of the above procedure, a reinforced concrete mass asymmetric building is used. The building is regular in plan and in elevation with a rectangular plan, three bays in the transverse direction and four in the longitudinal direction (7 and 8 m long respectively) with storey heights of 3.3 m, beams sections of 0.4x0.8 m and column sections of 0.8x0.8 m. The considered elastic modulus is 2213600.0 kg/cm^2 and the masses in every floor located along a vertical axis, are 86.2141 ton*sec^2/m, Figs. 5 and 6. The building was designed by Chípol [8] in accordance with the current Mexico City code, DDF [9].

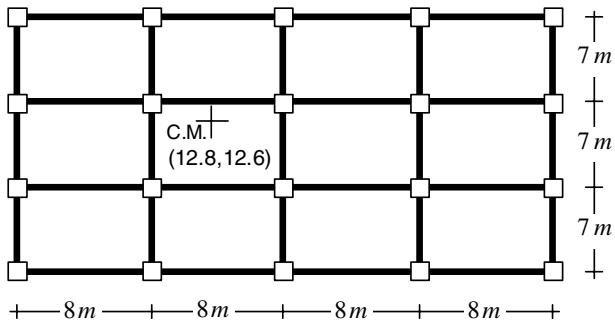


Figure 5. Plan view of the building.

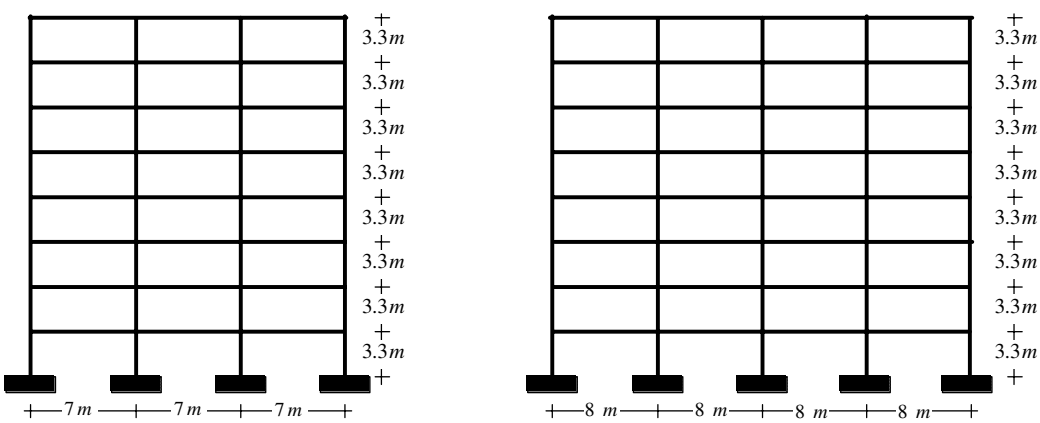


Figure 6. Lateral views of the building.

Equivalent model

In this work, the program to calculate the required parameters for the particular case of equal floor masses centred along a vertical axis was developed in Fortran language. With this program matrix [Π] defined by Eq. 20 is evaluated as:

$$[\Pi] = \begin{bmatrix} 45.350 & -8.399 & 0.000 \\ -8.399 & 74.110 & -13.960 \\ 0.000 & -13.960 & 50.420 \end{bmatrix} \tag{34}$$

The vibration periods of the equivalent model and the corresponding periods obtained from the original model are presented in Table 1:

The modal shapes associated to the equivalent model are:

$$[\alpha_o] = \begin{bmatrix} -0.2044 & -0.6809 & 0.7033 \\ 0.8961 & 0.1591 & 0.4145 \\ -0.3941 & 0.7149 & 0.5776 \end{bmatrix} \tag{35}$$

Table 1. Comparison of periods of the equivalent model *vs.* the periods from the original model.

	T_3	T_2	T_1
Original Model	0.69322	0.91352	0.98861
Equivalent Model	0.69317	0.91351	0.98858
Error (%)	0.00786	0.00085	0.00262

Modal shapes from the eigenvalue problem of the original model (complete stiffness and mass matrices), and the modal shapes derived from Perturbation Theory and the equivalent model are shown in Table 2. Relative error is calculated between modal shapes from the original model and the built modal shapes. Furthermore, the maximum error between the modal shapes is also presented.

Table 2. Modal shapes obtained from the Perturbation Theory and from the original model.

	Φ_3	Φ_2	Φ_1	Φ_{R3}	Φ_{R2}	Φ_{R1}	e_3 (%)	e_2 (%)	e_1 (%)
1	0.0011406	-0.0037888	0.0039152	0.0011378	-0.0037903	0.0039149	0.2455	0.0396	0.0077
2	0.0030286	-0.0100284	0.0103552	0.003011	-0.01003	0.0103596	0.5811	0.0160	0.0425
3	0.005015	-0.016585	0.0171175	0.0049787	-0.0165847	0.0171296	0.7238	0.0018	0.0707
4	0.0068655	-0.0227192	0.0234468	0.0068196	-0.022717	0.0234635	0.6686	0.0097	0.0712
5	0.0084526	-0.028035	0.0289421	0.0084157	-0.028034	0.0289551	0.4366	0.0036	0.0449
6	0.0097112	-0.0323136	0.0333746	0.0097008	-0.0323147	0.0333764	0.1071	0.0034	0.0054
7	0.010593	-0.0353672	0.0365482	0.0106187	-0.0353723	0.0365345	0.2426	0.0144	0.0375
8	0.0111419	-0.03728	0.0385383	0.011194	-0.0372886	0.0385138	0.4676	0.0231	0.0636
9	0.0022556	0.0040584	0.0032722	0.0022375	0.0040588	0.0032794	0.8024	0.0099	0.2200
10	0.0058997	0.0106408	0.0085838	0.0058657	0.0106403	0.008597	0.5763	0.0047	0.1538
11	0.0096813	0.0174864	0.0141103	0.0096394	0.0174858	0.0141279	0.4328	0.0034	0.1247
12	0.0131887	0.0238557	0.0192566	0.0131507	0.0238553	0.0192742	0.2881	0.0017	0.0914
13	0.0162231	0.0294004	0.0237422	0.0162059	0.0293974	0.023752	0.1060	0.0102	0.0413
14	0.0186576	0.0338581	0.0273525	0.0186632	0.0338551	0.0273536	0.0300	0.0089	0.0040
15	0.0204392	0.0371112	0.0299862	0.0204573	0.0371095	0.0299831	0.0886	0.0046	0.0103
16	0.0215844	0.0391943	0.0316702	0.0216062	0.0391935	0.0316669	0.1010	0.0020	0.0104
17	-0.0004281	0.0000767	0.0001984	-0.0004287	0.0000761	0.0001983	0.1402	0.7823	0.0504
18	-0.001127	0.0002017	0.0005221	-0.0011284	0.0002003	0.0005219	0.1242	0.6941	0.0383
19	-0.0018589	0.000332	0.0008606	-0.0018607	0.0003304	0.0008607	0.0968	0.4819	0.0116
20	-0.0025445	0.0004534	0.0011775	-0.0025462	0.0004521	0.0011777	0.0668	0.2867	0.0170
21	-0.003145	0.0005598	0.0014552	-0.0031463	0.0005586	0.0014553	0.0413	0.2144	0.0069
22	-0.003632	0.0006451	0.0016804	-0.0036325	0.0006449	0.0016802	0.0138	0.0310	0.0119
23	-0.0039862	0.000707	0.0018441	-0.003986	0.0007077	0.0018437	0.0050	0.0990	0.0217
24	-0.0042114	0.0007466	0.0019481	-0.0042106	0.0007476	0.0019476	0.0190	1.0013	0.9997

MAXIMUM ERROR (%)

0.8024	1.0013	0.9997

where:

Φ_1, Φ_2 and Φ_3: Modal shapes of the first, second and third modes, respectively, from the eigenvalue problem associated to the original problem.
Φ_{R1}, Φ_{R2} and Φ_{R3}: Modal shapes built from the equivalent model and the modal shapes of the unperturbed system.
$e_i = \left| \frac{(\Phi_i - \Phi_{Ri})}{\Phi_i} \right| \times 100\%$: Error between Φ_i and Φ_{Ri}, (i=1,2,3).

Considering the floor mass of the original building:

$$[m_0] = \begin{bmatrix} 86.210 & 0.000 & 0.000 \\ 0.000 & 86.210 & 0.000 \\ 0.000 & 0.000 & 86.210 \end{bmatrix} \tag{36}$$

For the transformation matrix:

$$[R_P]^{-1} = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 11.694 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} \tag{37}$$

From Eqs. 25 and 26 it is possible to obtain the matrices of displacement coefficients and acceleration coefficients, respectively:

$$[K_C] = \begin{bmatrix} 3909.624 & -8467.658 & 0.000 \\ -8467.658 & 873756.900 & -14074.117 \\ 0.000 & -14074.117 & 4346.708 \end{bmatrix} \tag{38}$$

$$[m_p] = \begin{bmatrix} 86.210 & 0.000 & 0.000 \\ 0.000 & 11790.000 & 0.000 \\ 0.000 & 0.000 & 86.210 \end{bmatrix} \tag{39}$$

DEFINITION OF THE EQUIVALENT INELASTIC SINGLE-STOREY MODEL

To obtain the inelastic response of a multi-storey irregular building with less effort than that involved in determining its inelastic response from a conventional non-linear dynamic analysis, the capacity spectrum concept is suitable. However, there are some concepts that must be considered. One is the fact that in a non-linear structural model it is only possible to obtain the lateral stiffness (uncoupled stiffness) in both orthogonal directions and the rotational stiffness from pushover analyses with the distributions of equivalent static lateral forces and floor moments applied at all levels of the structure, Fig. 7, in order to obtain three capacity curves expressed as roof displacement (rotation) *vs.* base shear (moment), Fig 8. From these capacity curves it is possible to derive the lateral and torsional stiffnesses needed in Eq. 8. The seismic response of this non-linear single-storey model may be obtained by any of the already available

step by step integration procedures. It is important to point out that the approximation of the seismic performance of real asymmetric buildings calculated with this method is highly dependent the distribution of the applied lateral loads and floor moments and must be performed carefully in order to obtain rational results, Ayala and Tavera [10].

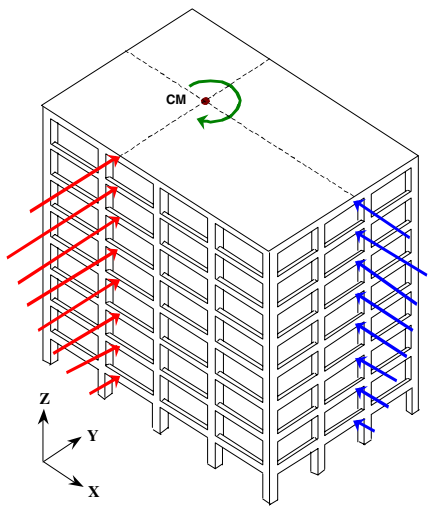
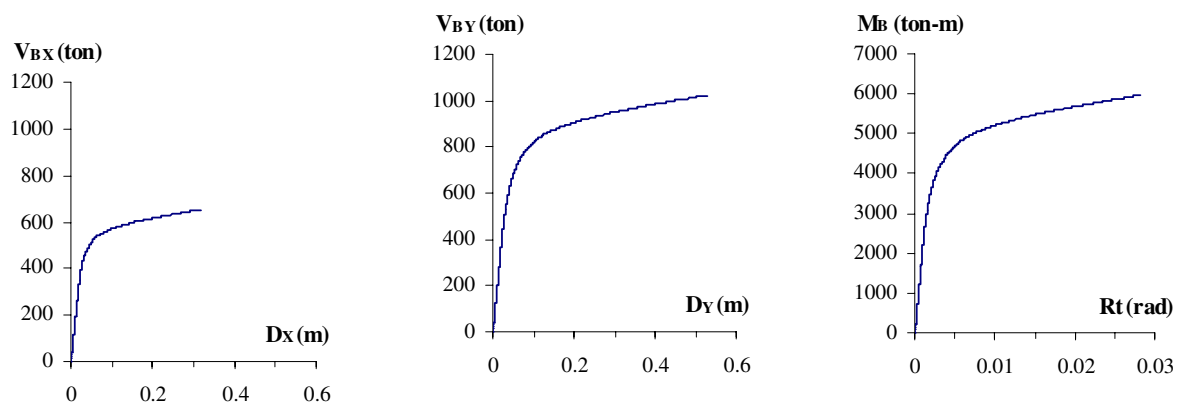


Figure 7. Direction of forces for the three pushover analyses of an asymmetric building.



a) Direction X. b) Direction Y. c) Direction θ

Figure 8. Illustration of base shear *vs.* roof displacement and base moment *vs.* roof rotation curves.

The procedure proposed is valid only for a family of multi-storey buildings with the same restrictions as s for the elastic cases. The fact that the matrices obtained do not allow the identification of its resistant planes of the prototype or its eccentricities makes it impossible to model the strength distribution and its characteristics in order to relate the inelastic response of the simplified model with the real model. These topics require further research to modify the procedure presented in this paper to formulate a single storey inelastic model and from its seismic response obtain the inelastic response of a real building.

CONCLUSIONS

The formulation of simplified single-storey 3DOF models equivalent to multi-storey asymmetric building models presented in this paper leads to dynamic systems with equal uncoupled and approximate torsionally coupled frequencies guaranteeing that the seismic responses of the simplified models also represent those of the prototypes. This characteristic opens up also the possibility of transforming the seismic performance results from these models to real asymmetric buildings. This result is of paramount important in the interpretation of results derived simplified single-storey models

The model formulated in this work is derived from an inverse transformation originally proposed by Kan and Chopra [4], with dynamic equilibrium equations with stiffness and mass matrices corresponding to those of an asymmetric multi-storey building subjected to bi-directional seismic demands. The matrices obtained represent the global values of mass and lateral and torsional stiffness of the asymmetric multi-storey building ignoring the in-plan distribution of resisting elements. This characteristic makes the results of numerous studies on single-storey models, with emphasis on the behaviour of their different resisting planes, partly useful as there is not possible to extrapolate results on the performance of these resisting planes to the corresponding planes in the actual asymmetric structure.

Regarding the proposed model, it is recommended to continue with the research to develop a procedure to define simplified models from which the performance of real multi-storey asymmetric buildings could be obtained. These models, if single-storey 3DOF, could be used to validate and reassess the results obtained since the 1980's, to investigate the seismic behaviour of mass and stiffness asymmetric buildings. These models should be capable of correctly representing the torsional coupling with consistent equations to allow the transformation of the structural properties of a real building to those of a corresponding simplified model and the seismic performance of this simplified model to the performance of the real structure.

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