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EFFECT OF CANTILEVER COLUMN FLEXURAL CHARACTERISTIC ON BRACED FRAME DISPLACEMENTS

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SUMMARY

The presence of stiff columns, which are continuous over several stories, can limit the possibility of a large story drifts in concentrically braced frame (CBF) buildings. Previous studies of CBF buildings with pinned-base columns show that concentration of drift in a story may be quantified in terms of the structural ductility and the column flexural stiffness. In real braced frames, columns are not pinned at their bases, but they have some moment resistance. By fixing the bases of the columns to the foundations the period of the structure changes and the both the columns themselves, as well as the CBF, resist load making a type of "dual system" with increased redundancy. This paper investigates the behavior of CBF structures with fixed base continuous columns. Expressions for drift concentration as a function of the column flexural stiffness and strength are obtained.

INTRODUCTION

Concentrically braced frames are generally designed considering member tension and compression forces. Bending forces, if they are considered in the elastic analysis for design, do not usually control the member sizes. However, if none of the members have any flexural resistance, then when braces in any story yield or buckle, large story drift concentrations may occur. In order for braced frames to behave well during dynamic inelastic displacements, continuity between the stories is beneficial. In real frames, this continuity is provided by seismic and gravity columns which are continuous over the frame height. Previous work by the authors has evaluated the relationship between column stiffness and frame drifts for various height frames subject to specified roof displacements when these continuous columns are pinned at the base (MacRae, Kimura and Roeder, 2004). It was shown that concentration of drift in a structure may be controlled by selecting appropriate column stiffnesses in design. However, in many real structures, continuous columns are not pinned, but they are likely to have moment fixity at their base. Their presence decreases the period of the structure and provides an extra load path for lateral force which may prevent collapse. This system, with braced frames without continuous columns together with fixed-based cantilever columns, is a type of "dual system" in which the braced frame could be represented by any shear-type structure, and the cantilever columns could be structural walls. This paper describes the

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behavior of these dual systems using lateral pushover analyses and dynamic inelastic time history analyses. Equations for estimating the drift concentration and moment demand are developed for a range of parameters. Minimum column stiffnesses and strengths to limit the story drift concentration are proposed.

EFFECT OF CANTILEVER COLUMN FLEXULRAL STIFFNESS AND STRENGTH ON TWO STORY BRACED FRAME

Estimation of Drift Concentration for Two Story Frame

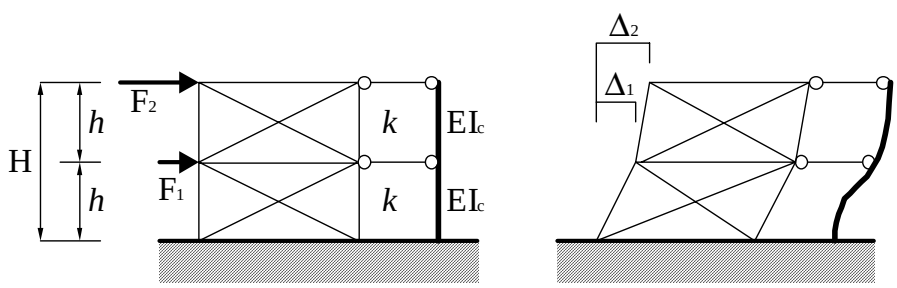
When a two-story braced frame with pinned column bases, with elastic-perfectly plastic unbonded braces (Clark, Peter W. et al. 2000) and constant member sizes in both stories is subject to a pushover analysis with an inverse triangular lateral force distribution, then the drift concentration factor, *DCF*, is given as the minimum of Equation 1 and Equation 2 (MacRae, Kimura and Roeder 2004). The *DCF* is the ratio of the maximum story drift to the maximum roof drift. Here, μ_t is the ratio of the displacement at the top of the structure, Δ_2, to the top displacement when yield occurs at any place within the CBF, Δ_{2y}, EI_c is the column flexural stiffness, *h* is the height of one story, and *k* is the stiffness of each story in the CBF.

$$DCF = \frac{10\mu_t + 55\alpha\mu_t - 4 - 10\alpha + 75\alpha^2\mu_t}{5\mu(1+5\alpha)(1+3\alpha)} \tag{1}$$

$$DCF = 1 + \frac{1}{15\alpha\mu_t} \tag{2}$$

$$\text{where } \alpha = \frac{EI_c}{kh^3} \tag{3}$$

The *DCF* for a braced frame with fixed-base columns may also be estimated using the column flexural stiffness, roof ductility, and the column flexural strength. Figure 1 shows (a) the model of the two story braced frame and (b) its deformation shape. This model has same stiffness and strength at each level.



(a) System with Gravity Column (b) Deformed Shape

Figure 1 Idealization of Two Story Frame with Continuous Column Deformation and Forces

Figure 2 describes the *DCF* of a CBF with fixed column bases. Pushover analysis was carried out using the computer program DRAIN-2DX (Prakash, Powell and Campbell, 1993). The moment strength, M_c is approximated as follows where *D* is the column depth and ε_y is the yield strain.

$$M_c = \frac{2\varepsilon_y EI_c}{D} \tag{4}$$

Three kinds of column were selected corresponding to steel column section depths of 200mm for Case 1, 3000mm for Case 2 and Case 3 is a RC shear wall with a depth of 4000mm. The column for Case 1)

almost keeps elastic when the CBF ductility is 4, and that for Case 3 yields as soon as α increases. The *DCF* is influenced by a non-dimensional column strength ratio, χ_c, given below. The strength ratio decreases from Cases 1 to 3 as *D* increases.

$$\chi_c = \frac{M_c h}{EI_c} = \frac{2\varepsilon_y h}{D} \tag{5}$$

The *DCF* for the braced frame with fixed base columns for very low α is the same as that expected if there is no column with or without a pinned base. This is associated with the bottom story of the frame only yielding. As α increases, the *DCF* slowly decreases. At higher column stiffness, indicated by the black triangle, there is a kink in the line and the *DCF* suddenly decreases. This corresponds to both stories of the frame yielding at the ductility at 4. This point, designated as α_c, is different for the 3 kinds of column. At high values of α, the *DCF* converges to about 1.35 for the 200m depth column (Case 1) which has a high moment strength ratio, χ_c. This value is similar to that expected from a cantilever column with no frame subjected to the inverted triangular force distribution. The frame moves over linearly so that *DCF* is close to unity for the concrete shear wall (Case 3) with low χ_c.

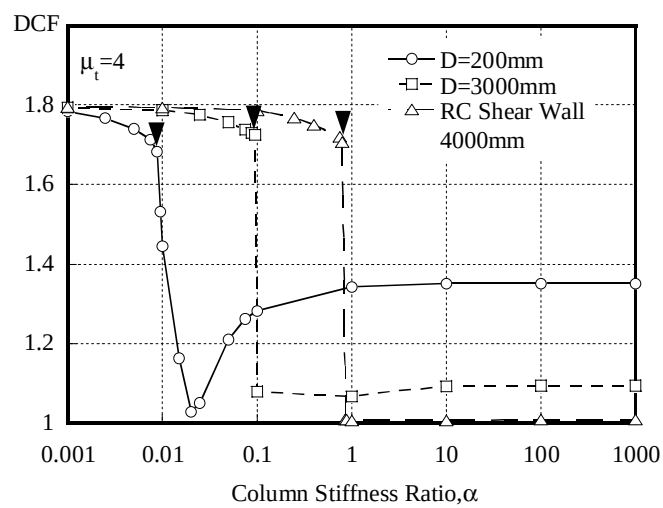


Figure 2 *DCF* of Two Story Shear Structure with Various Column Flexural Stiffness, α, and Roof Ductility, μ_t, using Static Pushover Analysis

Figure 3 shows the relationship between α_c and χ_c. The plots are the analytical results, and the curve is an empirical approximation given by:

$$\alpha_c = \frac{4.0*10^{-4}}{\chi_c} \tag{6}$$

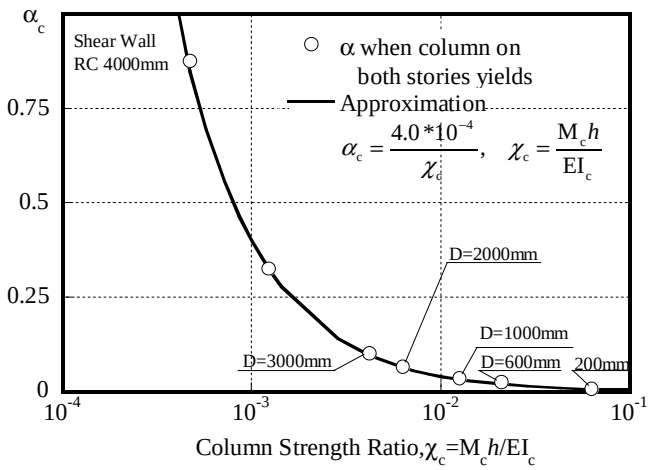


Figure 3 Relationship between α_0 and χ_c

Estimation for Drift Concentration Factor for Large α

The *DCF* for the braced frame with a fixed base column is approximated by two sets of equation. When α is larger than α_c, the *DCF* is assumed to be constant to approximate the behavior shown in Figure 2(b). Equation 7 is an approximation of the *DCF* when α is larger than α_c. Equation 7 is consists of 3 parts and it is dependent on the column base strength ratio, χ_c. The first part of Equation 7 is for the case of small column strength, and the third part is for the case of large column strength. The functions for *DCF* change at χ_{c1} and χ_{c2} where these values are given by Equation 8. The relationship between χ_{c2} and roof ductility, μ_t, is shown in Figure 4.

$$DCF = \begin{cases} 1 & \chi_c < \chi_{c1} \\ 1 + 0.35\dfrac{\chi_c - \chi_{c1}}{\chi_{c2} - \chi_{c1}} & \chi_{c1} < \chi_c < \chi_{c2} \\ 1.35 & \chi_{c2} < \chi_c \end{cases} \tag{7}$$

$$\chi_{c1} = 4.75*10^{-4}, \quad \chi_{c2} = 4.0*10^{-3}\mu_t \tag{8}$$

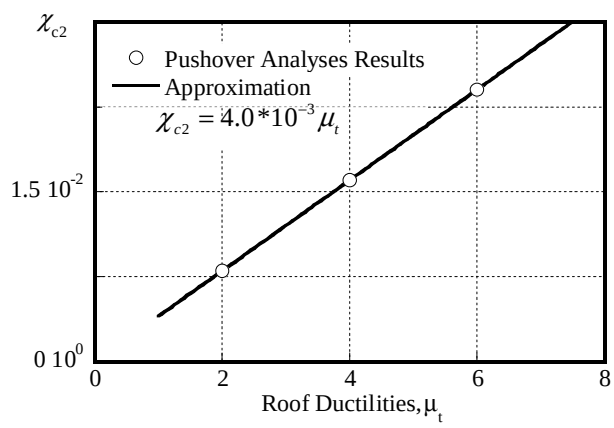
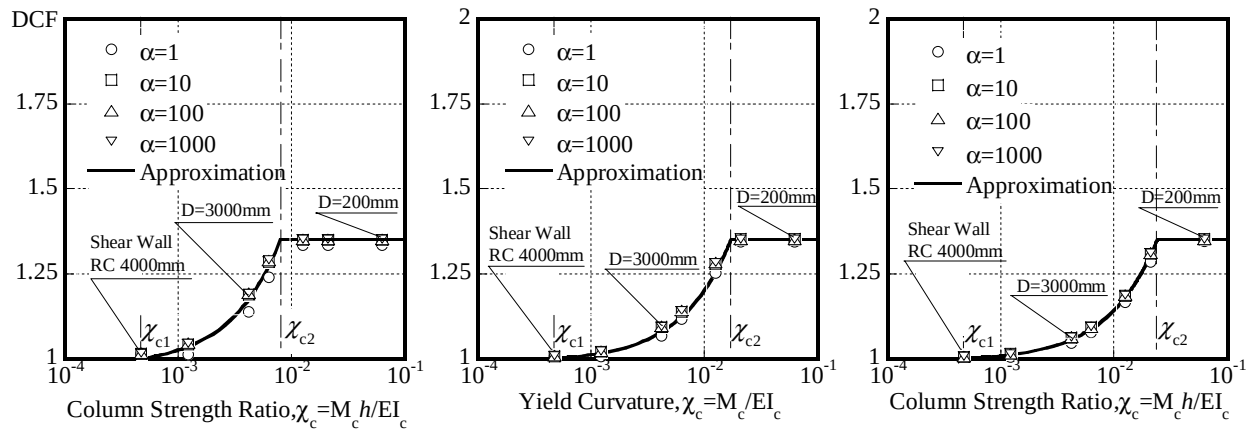


Figure 4 Relationship between χ_{c2} and μ_t

The DCF is shown for various column strength ratios, χ_c, in Figures 5(a)~(c) for ductilities of 2, 4 and 6 respectively. It may be seen that the empirical equations agree well with the pushover analysis results.



(a) μ_t=2 (b) μ_t =4 (c) μ_t =6

Figure 5 Relationship between *DCF* and χ_c

Estimation for Drift Concentration Factor for Small α

When α is smaller than α_c, *DCF* may be calculated based on Equation 1 where α is modified to α'. Here, α' is a function of α, α_c, and α_0 as shown in Equation 10 and α_0 is the value of α when Equations 1 and 2 are equal.

$$DCF = \frac{10\mu_t + 55\alpha'\mu_t - 4 - 10\alpha' + 75\alpha'^2\mu_t}{5\mu_t(1+5\alpha')(1+3\alpha')} \tag{9}$$

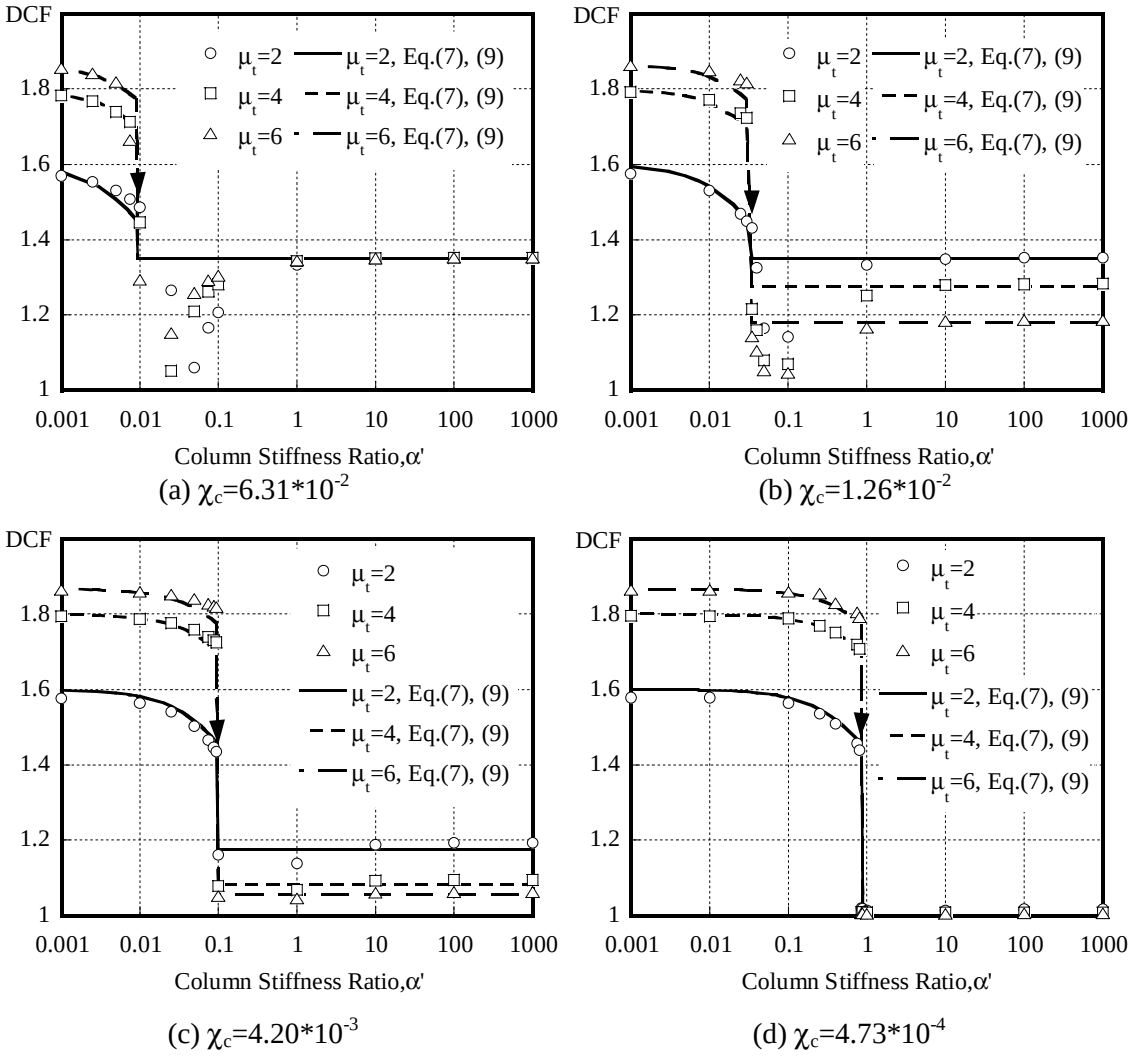
$$\alpha' = \alpha\left(\frac{\alpha_0}{\alpha_c}\right) \tag{10}$$

Figures 6(a)~(d) show the relationship between *DCF* and α' for χ_c= 0.0631, 0.0126, 0.00420 and 4.73x10^{-4} respectively using Equations 7 and 9 and pushover analyses results for μ_t = 2, 4 and 6. In the diagrams, the black triangle is the boundary point between Equations 7 and 9 which is when α' = α_c. Equation 7 is used to calculate the *DCF* for low α_c values and Equation 9 is used for high α_c values. It may be seen that the *DCF* is estimated well by these equations. Figure 6(d) shows that when χ_c is very low, the *DCF* for high α' is about 1. Figure 6(a) shows that when χ_c is high, the *DCF* at α' greater than about 0.01 it converges about 1.35. The parameters χ_c and α are important for estimation of the *DCF* for CBFs with fixed base columns.

EFFECT OF CANTILEVER COLUMN FLEXULRAL STIFFNESS AND STRENGTH ON MULTI STORY BRACED FRAME

Braced Frame Model

Figure 7 shows a multi-story frame model to be used in pushover analyses. It is also assumed that the frame has one continuous column representing the structural properties of all gravity and seismic columns in the building. The connections of the left braced frame are totally pinned and the continuous column is fixed at the base. The continuous column stiffness, EI_i, and the brace stiffness at each story are K_i. The frame is subjected to a lateral inverse triangular load distribution. Drift concentration describes the ratio maximum story drift, δ_i/h to maximum roof drift Δ_N/H in Figure 8. The continuous column flexural stiffness ratio and strength ratio for multi story frames are α and χ_c respectively.



(a) χ_c=6.31*10^{-2} (b) χ_c=1.26*10^{-2}

(c) χ_c=4.20*10^{-3} (d) χ_c=4.73*10^{-4}

Figure 6 Relationship between *DCF* and modified α'

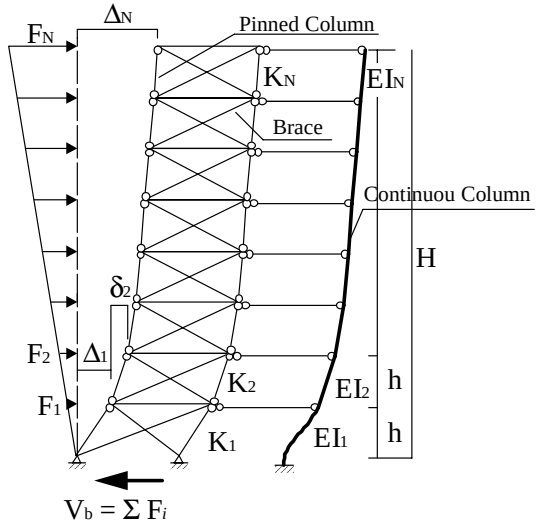
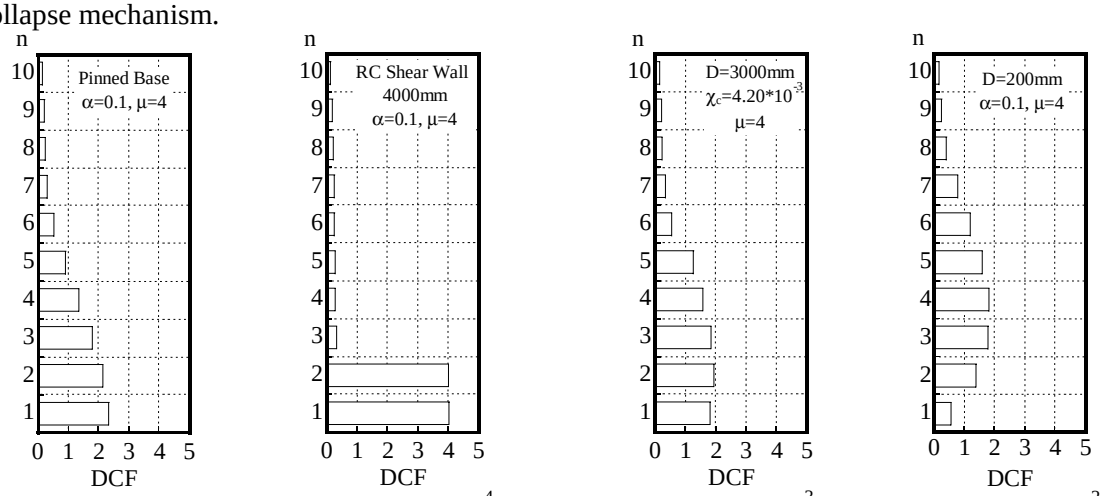


Figure 7 Idealization of Multi Story Frame with Continuous Column Deformation and Force

Behavior of Drift Concentration for 10 Story Frame

In the previous section for two story braced frames, the *DCF* was estimated by the column flexural stiffness, α and strength parameters, χ_c. In this section the relationship between *DCF* and column characteristic is investigated for multi story braced frames using push over analysis.

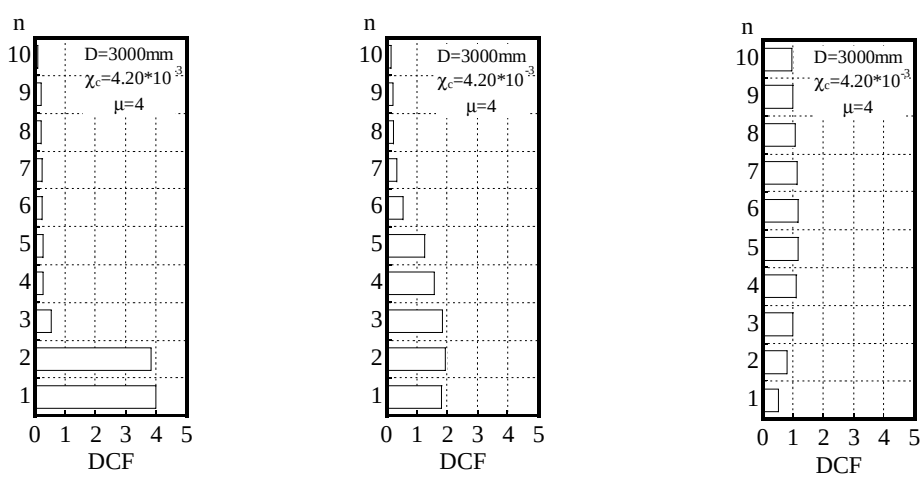
Figure 8 shows the *DCF* distribution over the height of the 10 story braced frame for column stiffness ratios, $\alpha = 0.1$ and a roof ductility $\mu_t = 4$. The four column strength ratios used χ_c are 0, $4.73*10^{-4}$, 0.00420 and 0.0631 respectively in Figures 9(a)~(d). Pinned base columns are represented by $\chi_c = 0$. The first story has a maximum drift in the case of pinned column (Figure 9(a)). The peak drift concentration occurs on 1st and 2nd stories in the case of low column strength ratio (Figure 9(b)), and the *DCF* is greater than 4. In the case of a higher column strength ratio (Figure 9(d)), the maximum *DCF* occurs on the fourth story, even though the maximum *DCF* occurs on 1st or 2nd story in the other cases. The location of peak drift depends on the collapse mechanism.



(a) χ_c=0 (Pinned Base) (b) $\chi_c=4.73*10^{-4}$ (c) $\chi_c=4.20*10^{-3}$ (d) $\chi_c=6.31*10^{-2}$

Figure 8 Distribution of Drift Concentration for 10 Story Frame with α=0.1 and μ=4

When the column strength is low, high *DCF* occurs on only a small number of floors. Then the column yields first at the base and the braces yield second. When the column strength is high, the column never yields before the braces yield. As the column can carry a large portion of the total shear force at large ductilities and the shear force which the column carries is largest on the first story and the maximum drift is restraint on 1st story.



(a) α=0.01 (b) α=0.1 (c) α=1

Figure 9 Distribution of Drift Concentration for 10 Story Frame with $\chi_c=4.20*10^{-3}$ and μ=4

Figure 9 shows the *DCF* distribution over the height for the 10 story braced frame with χ_c= 4.20*10^{-3}, μ_t=4 and α=0.01, 0.1 and 1. The *DCF* distribution for α=0.01 is the same as that for lower χ_c (=4.73*10^{-4}) and higher α(=0.1) such as Figure 9(b), as large drift concentration occurs on just 1st and 2nd stories. When α becomes high, the *DCF* decreases and the maximum *DCF* moves to the higher stories in the frame. It may be clearly seen that the *DCF* depends on the column flexural stiffness ratio, α and the column flexural strength ratio, χ_c.

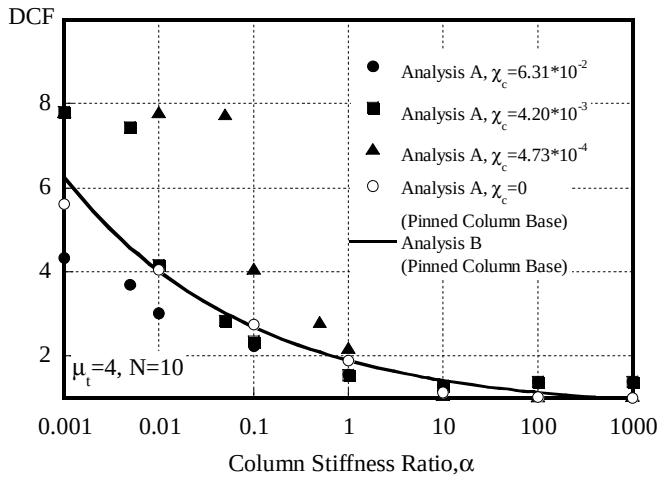


Figure 10 Comparison of *DCF* between Pinned and Fixed Column Base

Figure 10 shows the relationship between *DCF* and column stiffness ratio, α for braced frames with pinned and fixed base columns. Analysis A is the pushover analysis results, and Analysis B is drawn by the derived equations for the *DCF* with a pinned column base (MacRae, Kimura and Roeder, 2004). The black points are for fixed column at base, whose strength ratio, χ_c is respectively 4.73*10^{-4}, 4.20*10^{-3} and 0.0631, and the white points are for the pinned base column.

The *DCF* for the lowest column strength ratio, χ_c=4.73*10^{-4}, keeps high as α increases, and suddenly decreases at about α=0.1. Its behavior is similar to that for two story frames. As the value of χ_c is larger, *DCF* is smaller in α of less than 1. On the other hand *DCF* is constant in α of more than 1 even though χ_c value is different. The *DCF* for the braced frame with fixed column base can also consist of 2 kinds of closed form solutions.

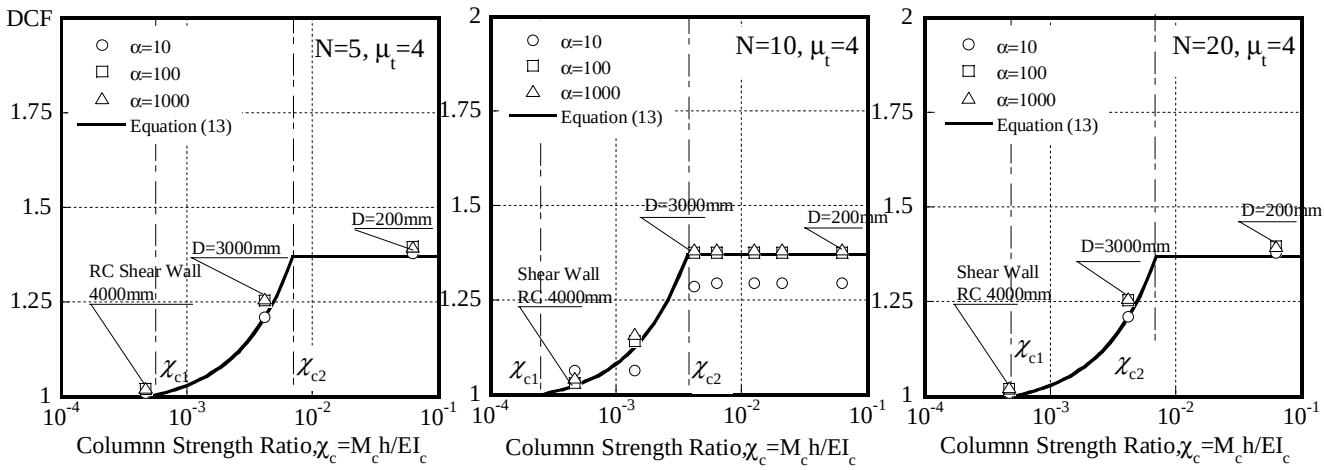
Estimation of Drift Concentration Factor for Multi Story Braced Frames for Large α

To estimate the *DCF* for the multi story braced frame with constant strength over their height with large α, the two parameters, χ_{c1} and χ_{c2} are similar to that for the two story frame. Equation 13 is for large α. The first part of Equation 13 is for the case of small column strength, and the third part is for the case of large column strength. Here, χ_{c1} is a function of the number of stories, *N*, and χ_{c2} is both a function of *N* and the roof ductility, μ_t as shown in Equation 14.

$$DCF_\chi = \begin{cases} 1 & \chi < \chi_{c1} \\ 1 + 0.37\dfrac{\chi - \chi_{c1}}{\chi_{c2} - \chi_{c1}} & \chi_{c1} < \chi < \chi_{c2} \\ 1.37 & \chi_{c2} < \chi \end{cases} \tag{13}$$

$$\chi_{c1} = \frac{2.5}{N} * 10^{-3}, \quad \chi_{c2} = \frac{(\mu - 0.5)}{N} * 10^{-2} \tag{14}$$

Plots based on the above equations are shown in Figures 11(a)~(c) respectively for 5, 10 and 20 story frames. The braced frames used in these diagrams are for models in which the strength on each story is the same over the height. The derived equations are for pushover analysis results. Results of approximation and pushover analyses are almost same as would be expected.



(a) N=5 (b) N=10 (c) N=20

Figure 11 Relationship between drift concentration factor, *DCF* and column strength ratio, χ_c

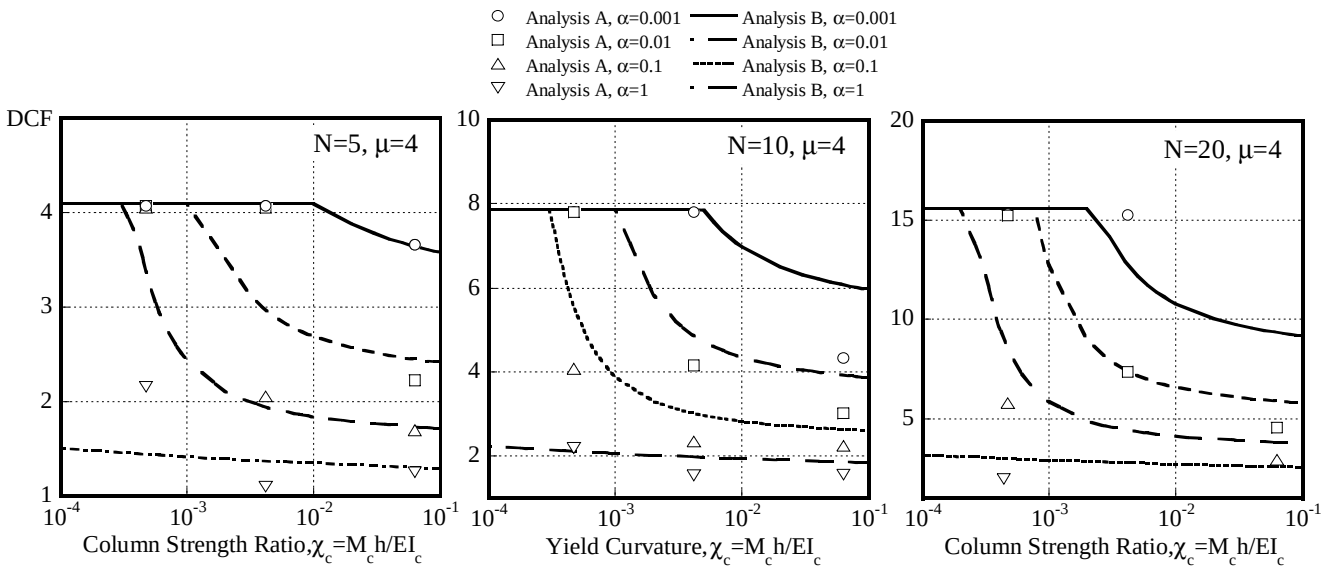
Estimation of Drift Concentration Factor for Multi Story Braced Frames for Small α

To estimate the *DCF* of the CBF with constant strength over their height with a fixed column base in the range of small α, a similar method to that for the two story frame can be applied. Equation 15(a) is based on the approximation for multi-story braced frame with pinned column base (MacRae, Kimura and Roeder 2004), and the coefficient of χ_c is added into this equation. Equation 15(b) is the value of the *DCF* at α=0, and it provides an upper bound of DCF.

$$DCF = \min \begin{cases} \dfrac{1+0.45(1+0.4/\chi_c^{0.1})(\mu_t-1)^{0.6}(1-\tau/3)\left(\dfrac{\tau^{0.32}}{\alpha^{0.22+0.00015/\chi_c}}\right)(N^{0.45}-1)}{\mu_t^{0.5}(1-\tau/3)} & (a) \\ DCF_{\alpha=0} & (b) \end{cases} \tag{15}$$

Figures 12(a)~(c) show the relationship between *DCF* and column strength ratio, χ_c respectively for frames with 5, 10 and 20 stories in the range of small α for pushover analysis. The f approximation represents the tendency of *DCF*-χ_c relationship.

Figure 13 shows the relationship between *DCF* and α for 10 story frame with same strength over heights at μ_t=4. The curves are drawn by Equation 13 and Equation 15, and the dots are the results of analyses. They are almost fitting as would be expected. When column strength and stiffness are very small, it should be careful to design the braced frame as the value of *DCF* becomes high such as that for α=0.



(a)N=5 (b) N=10 (c) N=20

Figure 12 Relationship between drift concentration factor, *DCF* and column strength ratio, χ_c

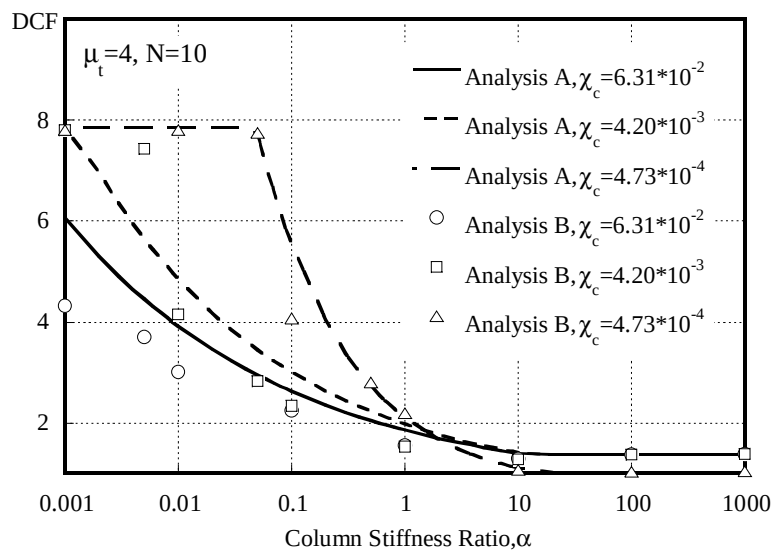


Figure 13 Relationship between drift concentration factor, *DCF* and column stiffness ratio, α

CONCLUSIONS

Pushover inelastic analyses were performed on the concentrically braced frame with fixed and pinned column bases to quantify the effect of continuous column stiffness and strength on drift concentration. It was shown that:

1) The peak drift concentration of a structure is influenced by the base-fixity of the continuous columns. When the column was pinned at the base, the *DCF* became unity as the column flexural stiffness ratio, α, increases. When the column is fixed at base, the *DCF* converges to a constant value between 1.0 and 1.37 for high α. This value of *DCF* depends on the column flexural strength ratio, χ_c.

2) Equations to estimate the *DCF* for 2-story and multistory CBFs with fixed column bases were developed. These equations may be used to form the basis for assessment and design of CBF structures to seismic loading.

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