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DAMAGE DETECTION IN BUILDING STRUCTURES

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SUMMARY

A method for detection and evaluation of damage based on the changes in the known modal shapes and vibration frequencies of building structures is presented. To locate and estimate the magnitude of the damage, measured in terms of the changes of the stiffness of the structural elements, the method uses the analytical model of the structure to represent its undamaged initial state. The modal shapes and vibration frequencies for the damaged state of the structure are used to build its lateral stiffness matrix from which the analytical model is adjusted by means of an iterative process by which the damaged structural elements are detected. The application of the method is illustrated by the evaluation of different simulated damage states of models of buildings.

INTRODUCTION

Due to different actions such as earthquakes, overloads, thermal effects or corrosion, the structural systems accumulate damage during their time life. From here the importance of having a reliable procedure that allows their structural evaluation, since if damage is not detected correctly, it can derive in the deterioration of the structural elements and in consequence to risk the stability of the structure. When the damage is visible, its physical detection can be carried out with relative easiness. However, in many cases this is not possible because, generally, the structural elements are not exposed in a direct way. In structures of buildings, for example, it can be necessary to remove panels and their cover.

In the last years several methods have been developed that use the changes in the modal shapes and vibration frequencies of a structure as data to evaluate the damage in their structural elements. These methods have the advantage of not requiring the direct exhibition of these elements and of being able to visualize the complete structure if it is properly instrumented. In consequence, it is possible to obtain a reduction in the time and cost of the evaluation, as well as a decrease in the impact of operation of the structure. For the localization and the calculation of the magnitude of the structural damage, these methods require the analytical model of the structure. These methods are denominated methods of detection of damage based on models. The method developed in this work belongs to this group and its application consists of three basic steps: a) construction of the analytic model of the real structure to

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establish its initial state without damage; b) estimation of the modal shapes and vibration frequencies for a latter state of the structure; and c) localization and estimation of the magnitude of the damage in the structural elements, using the initially built analytical model and the condensed stiffness matrix of the structure, obtained from the measured modal parameters.

DETECTION OF DAMAGE IN STRUCTURES USING THE METHOD OF THE TRANSFORMATION MATRIX

Formulation of the method

The global stiffness matrix of the analytical model of a structure can be represented as the assemble of the stiffness matrices of the structural elements that compose it. For a structure with *Ne* structural elements, the global stiffness matrix $[K_d]$ for the damaged state can be expressed as:

$$[K_d] = \sum_{i=1}^{Ne} x_i \, [Ke]_i \qquad (1)$$

where x_i is a non-dimensional parameter that represents the contribution of the stiffness of the structural element *i* to the global stiffness matrix of the structure ($0 \leq x_i \leq 1$); $[Ke]_i$ is the stiffness matrix without damage of that element. A structural element is considered without damage if the value corresponding of *x* is equal to 1.0.

In real situations there is a great difference among the number of degrees of freedom of the analytic pattern and those whose are possible to measure in the structure to determine its dynamic characteristics. For these cases, the damage detection method uses the matrix of geometric transformation to condense the degrees of freedom of the analytic pattern with respect to those measured in the structure. In the context of this work, the rigid body displacements of the storey slabs. The condensed stiffness matrix related to these degrees of freedom for the damaged state of the structure $[\bar{K}_d]$, is:

$$[\bar{K}_d] = [T_d(x)]^T [K_d][T_d(x)] \qquad (2)$$

where $[T_d(x)]$ is the transformation matrix expressed in function of the parameter *x*, because it is obtained from the partition of the global stiffness matrix of the damaged structure Guyan [1]. The matrix of stiffness can also be written as:

$$[\bar{K}_d] = \sum_{i=1}^{Ne} x_i [\bar{K}e(x)]_i \qquad (3)$$

where $[\bar{K}e(x)]_i = [T_d(x)]^T [Ke]_i [T_d(x)]$. The matrices $[\bar{K}_d]$ and $[\bar{K}e(x)]_i$ of the previous equation are of size *Nm*x*Nm* (*Nm* is the number of degrees of freedom measured in the structure); and due to their symmetry these matrices have $nti = Nm(Nm+1)/2$ independent terms. Developing equation 3 for each one of the independent terms of these matrices it is obtained:

$$\{\bar{k}_d\} = [S_k(x)]\{x\} \qquad (4)$$

where $\{\bar{k}_d\}$ is the vector of size *nti*x*1* containing the independent terms of the condensed stiffness matrix of the structure; $[S_k(x)]$ is the matrix of order *nti*x*Ne* containing the independent terms of the matrices

$[\overline{K}e(x)]_i$; $\{x\}$ is the vector of size *Ne*x*1* whose elements represent the contribution of the stiffness from each structural element, to the condensed stiffness matrix of the structure.

In order to adjust $[\overline{K}_d]$ with the stiffness matrix obtained from the modal shapes and vibration frequencies measured in the structure, as a first approach, it can be assumed that the transformation matrix $[T_d(x)]$, does not differ of the corresponding to the undamaged state of the structure. In this way, an iterative procedure establishes in which the damaged structural elements are detected by successive approaches.

On the other hand, experimentally measured modal shapes and vibration frequencies are affected by noise. Due to this, the stiffness matrix of the structure $[\overline{K}_m]$, obtained from these parameters, Baruch [2], and the matrix $[\overline{K}_d]$ would not be necessarily the same. The magnitude of the error *E* that measures the difference between both matrices, can be calculated as:

$$E = \left\| [\overline{K}_d] - [\overline{K}_m] \right\| \tag{5}$$

where $|\bullet|$ represents any matrix norm. This norm can also be measured with its corresponding vector norm when developing the previous equation for the independent terms of the matrices $[\overline{K}_d]$ and $[\overline{K}_m]$, in the following way:

$$E = \left\| \{\overline{k}_d\} - \{\overline{k}_m\} \right\| \tag{6}$$

where $\{\overline{k}_m\}$ is the vector of size *nti*x*1* that contains the independent terms of the matrix $[\overline{K}_m]$. Knowing that $\{\overline{k}_d\} = [S_k(x)]\{x\}$, the previous equation can also be written as

$$E = \left\| [S_k(x)]\{x\} - \{\overline{k}_m\} \right\| \tag{7}$$

When the matrix $[\overline{K}_m]$ is affected by noise in the measurements, in general, it cannot be expressed as a linear combination of the matrices $[\overline{K}e(x)]_i$ (equation 3). In this case, the magnitude of *E* will be different from zero. In order to obtain the minimum value of this error, the vector represented by $\{\overline{k}_d\} - \{\overline{k}_m\}$ should be perpendicular to the space column $[S_k(x)]$, Strang [3]. By applying this reasoning, the solution of the equation 4 can be obtained from an optimization problem, Fierro [4], when solving

$$[S_k(x)]\{x\} \leq \{\overline{k}_m\} \tag{8}$$

subject to:

$$[S_k(x)]^T [S_k(x)]\{x\} = [S_k(x)]^T \{\overline{k}_m\}$$

$$\{0\} \leq \{x\} \leq \{1\}$$

The previous equations represent the necessary condition to find the solution that minimizes the value *E* and, consequently, the difference between the condensed stiffness matrix obtained with the modal parameters measured in the structure and the adjusted one for its analytical model. By solving this equation, the matrix of damaged global stiffness of the structure is obtained (equation 1). When the correspondent transformation matrix is determined, a new approach for the equation 2 is obtained. The algorithm that allows to carry out this iterative procedure in which the damaged structural elements are detected is presented later on.

Considerations on the method of the transformation matrix

As it can be observed, with the proposed method, the structural damage detection problem is reduced to solve a system of *nti* linear equations for *Ne* unknowns. Because the value of *nti* is a function of the number of degrees of freedom that is possible to know in building structures; it is possible that the resulting system of equations is underdetermined or ill-conditioned. Thus, in order to solve equation 4, it could be necessary to use additional mathematical considerations. An approach for it is to use the singular value decomposition, SVD, of a matrix, Golub et al [5]. This decomposition for the matrix $[S_k(x)]$ is given as:

$$[S_k] = [U][\Sigma][V]^T \tag{9}$$

where $[U]$ is an orthogonal matrix of size *nti*x*nti*; $[V]$ is an orthogonal matrix of size *Ne*x*Ne* and $[\Sigma]$ is a diagonal matrix of size *nti*x*Ne* whose terms greater than or equal to zero, are called singular values of the matrix $[S_k(x)]$.

When a matrix $[S_k(x)]$ is ill-conditioned, it is useful to substitute the smallest singular values in $[\Sigma]$ for zeros. The number of singular values that are eliminated is obtained defining the minimum value that can take a singular value previously. Thus, the singular values smaller than this value are rejected. The SVD, is used in the proposed algorithm of detection of damage to modify the direct solution of equation 4 for those cases in which the matrix $[S_k(x)]$ is ill-conditioned.

On the other hand, when the system of equations represented by equation 4 is underdetermined, its solution can be obtained reducing the number of unknowns if the structural elements that have the same magnitude of damage are factored such that a unique factor *x* is associated to them. This can be based on a priori knowledge of the structure, Hjelmstad et al [6]. With this consideration, a linear system of equations similar to the one represented by equation 4 is obtained; however, for this case, the matrix $[S_k(x)]$ is of size *nti*x*Ng* (where *Ng* the number of elements with the same damage magnitude) and the vector $\{x\}$ of size *Ng*x*1*. The vector $\{\bar{k}_d\}$ conserves the same size (*nti*x*1*).

Algorithm

In order to determine the damaged structural elements of a structure with the proposed method, the following iterative process is used:

1. The matrices $[Ke]_i$ and $[T_d(x)]$ are obtained for the undamaged state of the structure.
2. The number of necessary iterations for the convergence of the algorithm is established (in the examples of application of the method that are presented later on, it is observed that this number can be five).
3. The matrices $[\overline{Ke}(x)]_i = [T_d(x)]^T [Ke]_i [T_d(x)]$ are calculated
4. The matrix $[S_k(x)]$ is formed.
5. The decomposition $[S_k(x)] = [U][\Sigma][V]^T$ is carried out.
6. It is solved $[S_k(x)]\{x\} \approx \{\bar{k}_m\}$
7. For the vector $\{x\} = \{x\}_n$, obtained of solving the system of equations of the previous step, it is calculated:

$\{x\}_{n+1} = \beta\{x\}_n + (1-\beta)\{x\}_{n-1}$

where β is the convergence factor and $\{x\}_{n+1}$ is the damage obtained as a fraction of the one calculated in the iterations *n* and *n-1*.

8. The matrices $[K_d]$ and $[T_d(x)]$ associated to the vector $\{x\}_{n+1}$ are calculated.
9. $[\overline{K}_d]$ is obtained and the norm $E = \| [\overline{K}_m] - [\overline{K}_d] \|$ is calculated.
10. Return to the step 3 in order to complete the defined number of iterations in the step 2.

By using the initial hypothesis that damage, step 1, does not exist, the iterative procedure converges to the state of damage of a structure defined by the vector $\{\overline{k}_m\}$. This is achieved if the transformation matrix that is used in step 3, for the iteration *n+1*, is calculated like a fraction of the sum of the damage obtained in iterations *n* and *n-1*, step 7. In this way, the transformation matrix presents a gradual change that allows, by successive approaches, the detection of the damaged structural elements.

In practical cases, the modal parameters of the structure, obtained from experimental measurements, are affected by noise. For this, the condensed stiffness matrix $[\overline{K}_m]$ obtained with these parameters and the stiffness matrix of the analytical model $[\overline{K}_d]$, adjusted for this state, will differ in a certain norm *E* (equation 5). Because the magnitude of this norm depends mainly on the errors made in the measurement of the modal parameters, it is difficult to fix a permissible value of this to stop or to continue the process in step 10 of the above algorithm. With this in mind and considering that the iterative process described is convergent to the state of damage of a structure, it is more convenient to halt this process establishing an initial number of iterations, step 2. Thus, the best solution $\{x\}$ corresponds to the matrix $[\overline{K}_d]$ for which, during the iterative process, the minimum value of the error *E* is obtained.

APPLICATION OF THE METHOD

STC Building

Several cases of simulated damage of a building located in the lake area of Mexico City are presented (STC Building). This building is regular in plan and it is formed by frames in the longitudinal direction and coupled shear walls in the transverse direction (figure 1). Because most of the lateral forces in the transverse direction are taken by the shear walls, the biaxial effect that can be presented in the structural elements of the longitudinal frames is minimum, Villaverde [7]. By this reason, the behaviour of this building in its longitudinal direction can be properly analysed by means of the analytical model of an interior plane frame.

Figure 2 shows the cases of simulated damage and the obtained results using the analytical model of a plane frame in the longitudinal direction of the STC building. Cases of simulated damage in elements located in specific parts of the frame are presented. From cases I and II, the symmetrical structural elements were simulated with same magnitude of damage. On the contrary, for case III, the damage in the elements is asymmetric. In all the studied cases, by using the proposed method, the damaged structural elements were located correctly and the magnitude of damage was calculated without relative error.

Figure 2 it is also used to present the trend of the error *E* during the iterative process applied to each one of the studied cases of damage. This figure shows the variation of this error for different values of the convergence factor β (0.25, 0.50 and 0.75). In all the cases it is observed that the smallest values of *E*, is reached in the fourth iteration, and correspond to β=0.25. However, in the fifth, its minimum value, of the order of 1×10^{-7}, is the same for the three β values proposed. Thus, it is possible to observe that the iterative procedure proposed converges to any state of damage of the structure if one knows the condensed stiffness matrix that defines it. This convergence is independent of the value of the β factor used, as well as of the quantity of structural elements and of the magnitude of the damage in them.

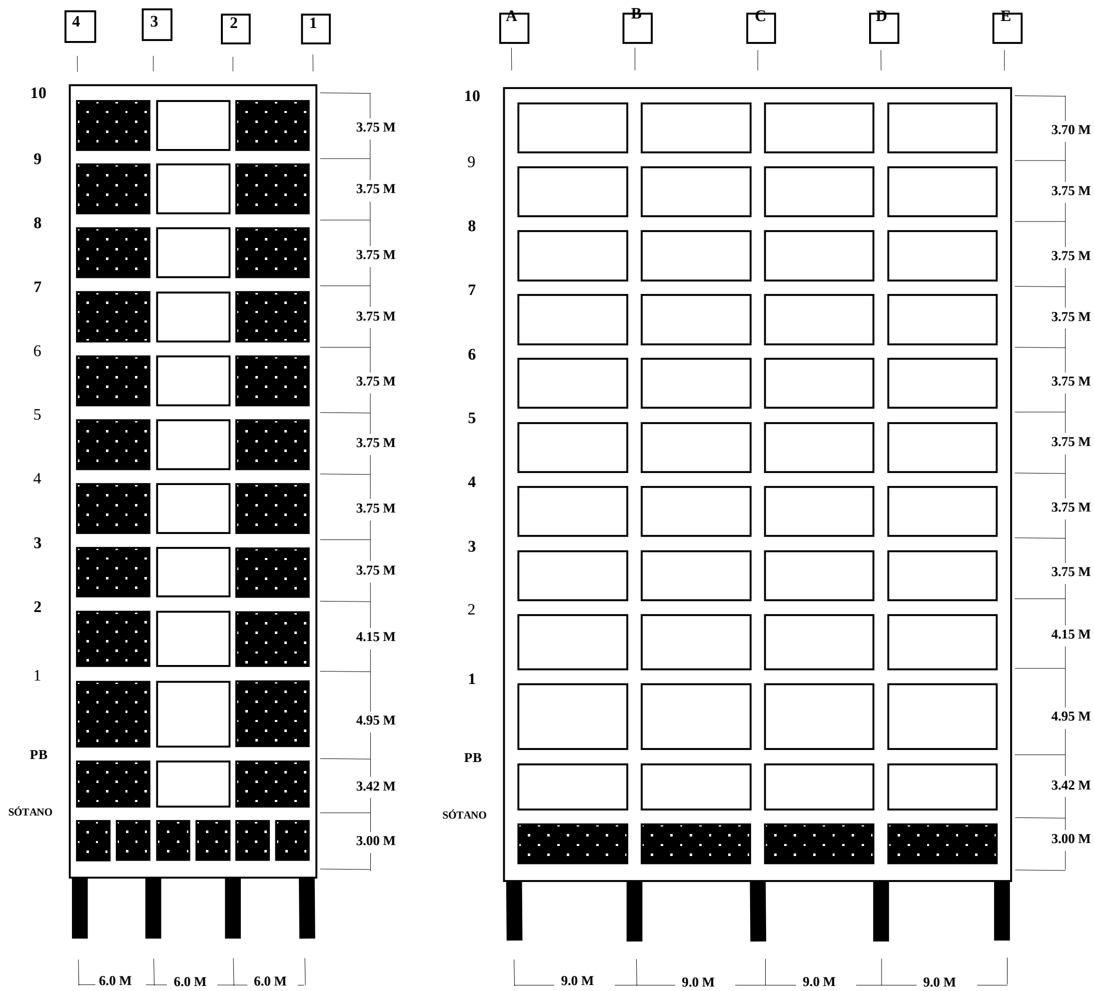

Figure 1. STC Building.

EFFECT OF UNCERTAINTIES IN THE MEASUREMENTS ON THE EVALUATION OF DAMAGE

In order to consider the uncertainties due to the presence of noise in the measurement of the dynamic characteristics in the evaluation of a state of damage of a real building, the modal shapes obtained for a state of simulated damage are perturbed with different levels of noise, Sohn and Law [8]. Thus for the modal shape *j*, the perturbed modal form $\{\phi_j^p\}_d$ is built as:

$$\{\phi_j^p\}_d = \{\phi_j\}_d\left(1+\frac{N}{100}R\right)$$

where $\{\phi_j\}_d$ is the *j-th* modal shape of the analytical model for the simulated damage state; N is the level of noise in percentage; R is a random number with normal probability distribution function, with zero mean, and variance one.

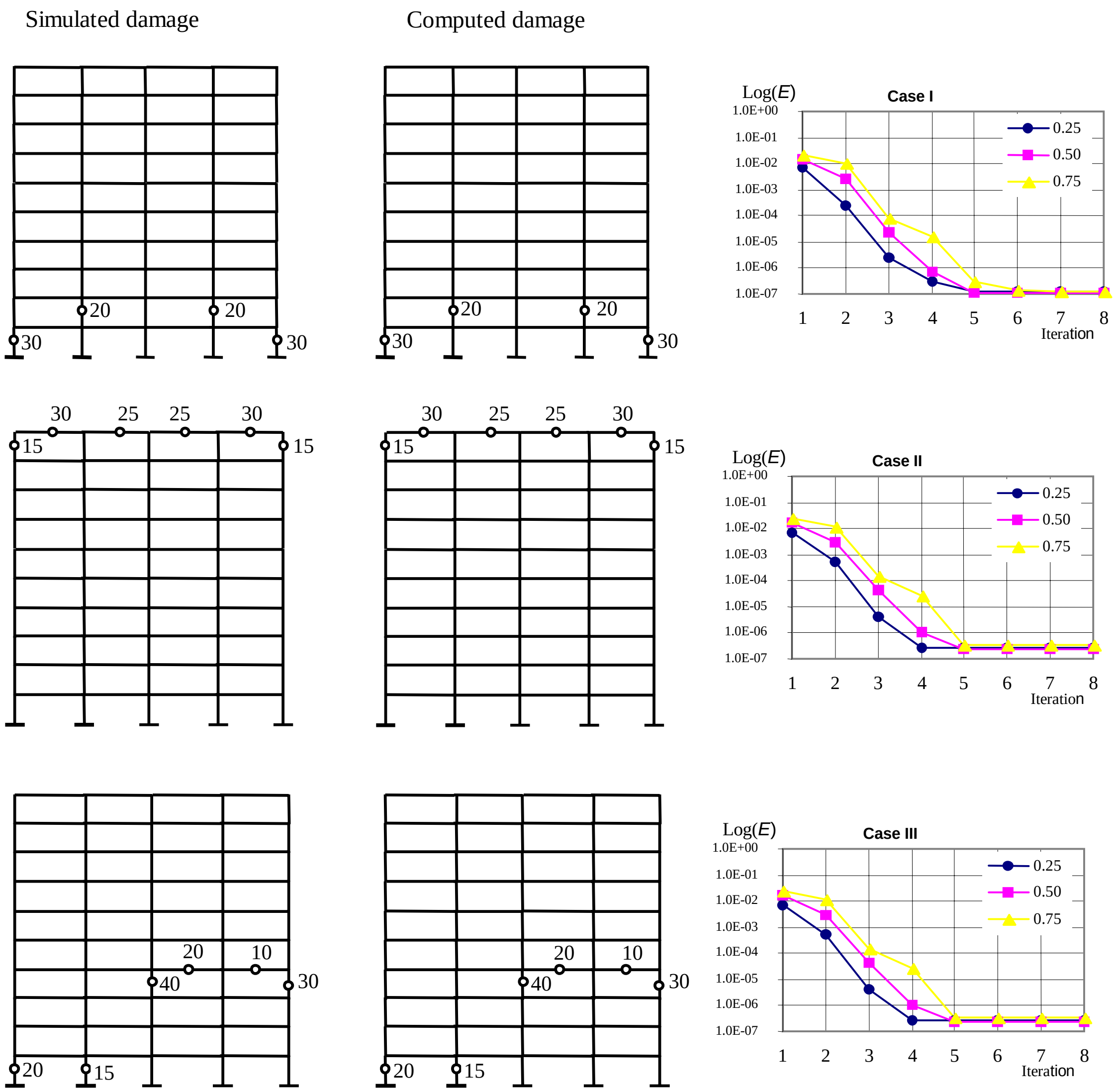

Figure 2. Studied cases of damage of the STC Building.

The 10-storey frame and the case of simulated damage used to study the effect of the uncertainties in the measurements on the structural damage computed with the proposed method are shown in figure 3.

The modal shape of the analytical model for the state of simulated damage were perturbed with five levels of noise: 0.5, 1, 3, 5, and 10 percent that were defined as cases A, B, C, D and E; respectively. With these modal shapes a stiffness matrix $[\overline{K}_m]$ was built for each case. These matrices are the input data for the evaluation of structural damage with the proposed method.

The results of the detected damage for the five cases studied are presented in figure 3. It is observed that when the stiffness matrices are used as data for cases A, B and C, the damaged structural elements are located correctly. The maximum relative error made in the calculation of the magnitude of the damage, for these cases, was of -5.6 percent and corresponds to case C, where for a value of the simulated damage of 45 percent in the beam of the fifth level, a calculated damage of 45.5 percent was obtained. However, case D shows as damaged the beam of the third level, and for case E, those beams of the third and of the sixth, which do not correspond to the simulated damage state. Also, in this last case the damaged columns of the storeys two, five and eight, were not detected.

It is important to point out that in cases D and E, the accuracy in the calculation of the magnitude of the damage diminishes. This is because the stiffness matrix $[\overline{K}_m]$ cannot be expressed, in general, as the contribution of the stiffness of each structural element for the state of simulated damage. However, it is observed that for cases A, B and C, the damage calculated with the proposed method is quite acceptable.

Determination of the structural damage magnitude by using SVD

The approach to improve the direct solution of equation 4 using the SVD was applied to all the cases studied in the previous section. Figure 4 shows the trend of the 30 singular values of the matrix $[S_k(x)]$ during the iterative process applied for the solution. It is observed that in this process a slight change exists among the first 20 and one more noticeable among the remaining 10 singular values of $[S_k(x)]$. Because, for practical reasons, these last are very close to zero, about $1x10^{-8}$, it can be considered that matrix $[S_k(x)]$, for these problems, is ill conditioned. In the same figure it is shown the typical trend of the error E for the studied cases. It can be seen that when all the singular values of the matrix $[S_k(x)]$ are considered, the magnitude of E does not have a defined trend; however, when only the 20 first singular values are used, E falls quickly until reaching a minimum value of the order of $1x10^{-7}$; being achieved with a better solution.

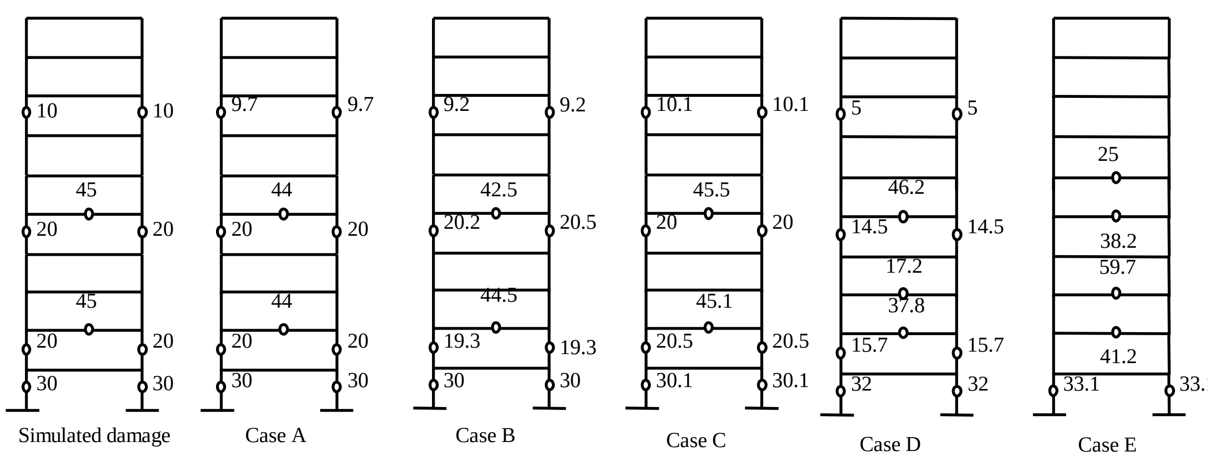

Figure 3. Effect of uncertainties in the measurements on the estimated structural damage.

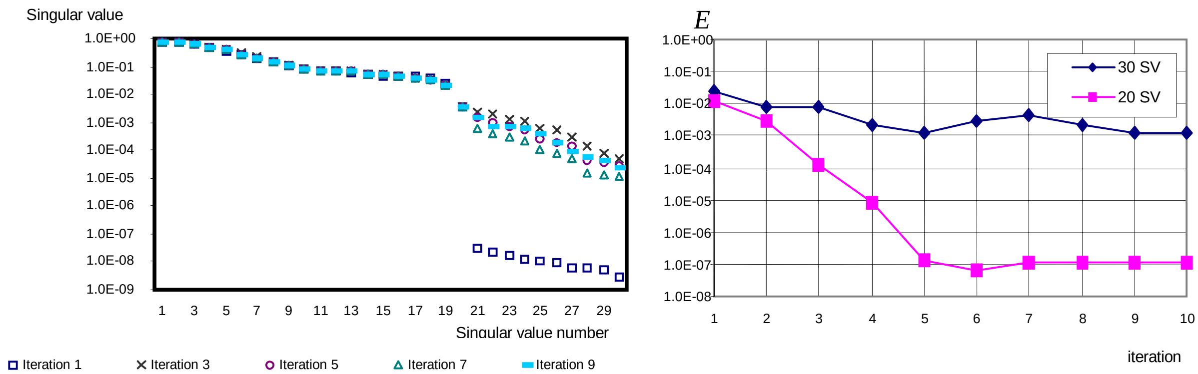

Figure 4. Trend of the 30 singular values of the matrix during the iterative process.

CONCLUSIONS AND RECOMMENDATIONS

A method of detection of damage in structures applied to models of plane frames of buildings was presented. The localization and the calculation of the damage, defined as the loss of stiffness at structural element level, are determined by means of an iterative process convergent to the state of damage of the structure. From the results obtained for the cases of damage studied in the present work, the following conclusions can be established:

The transformation matrix method proposed produces an excellent estimation of the magnitude of the damage for the studied cases where the noise in the modal parameters of the damaged structure was not considered; and a trend to diminish with the increment of this, that which does not depend on the method but on the adjustment of the stiffness matrix.

The initial hypothesis of considering that the geometric transformation matrix that condenses the global stiffness matrix of the structure, is similar for the damaged and for the undamaged state, allowed the development and the application of an iterative convergent procedure to any state of damage of the structure; that, as it was observed in the presented cases, does not depend on the used convergence factor.

On the contrary to other methods of detection of damage where the computational effort required is increased considerably with the number of damaged structural elements with the method developed this increment is minimum, because the number of iterations needed to converge to the damaged state of a structure is independent of the quantity of structural elements and of the magnitude of the damage in them.

The techniques of matrix decomposition can be useful tools in the improvement of the direct solution of not well conditioned singular systems of equations. In the studied cases, the SVD was applied to the matrix of coefficients obtained with the proposed method to determine the magnitude and localization of the structural damage.

It was shown that the accuracy in the localization and in the calculation of the magnitude of the structural damage is limited by the precision with which the modal parameters of the damaged structure are measured. Also, in a method of detection of damage based on models, as the one presented here, the correspondence between the response of the theoretical structural model and the behaviour of the real structure is an indispensable factor for the evaluation of the structural damage. In a more general form, it is probable that simplifications in the model, due to not very realistic consideration of non-structural elements, floor systems, soil-structure interaction, etc, reduce the accuracy of a mathematical model. For these cases, for the evaluation of damage, it is necessary to build an analytic model that represents the possible the behaviour of the structure.

Finally, methods of detection of damage have the potential of becoming useful tools in repair decisions, reinforcement and design of structural systems. In reinforced concrete structures the damage is presented as cracking, plastic articulations, buckling of the steel, loss of adherence, lack of the cover, etc, for which it is necessary to relate the physical state of the structure with the damage calculated in mathematical models.

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