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COMBINATION RULES FOR THE EFFECTS OF THE HORIZONTAL COMPONENTS OF EARTHQUAKES: A CRITICAL EVALUATION

A. REYES-SALAZAR[1], J.A. JUÁREZ-DUARTE[2], A. LÓPEZ-BARRAZA[1] and A. HALDAR[3]

SUMMARY

The Square Root of the Sum of the Squares (*SRSS*) and the 30-percent (*30%*) combination rules, commonly used to estimate the effect of both horizontal components of earthquakes, are re-evaluated. The maximum seismic responses of several moment resisting steel frames are estimated as realistically as possible by simultaneously applying both horizontal components. Then, the accuracy of the above-mentioned rules and others is evaluated. More than two thousands nonlinear seismic analyses of frames, with several hundred and in some cases a few thousands degrees of freedom, are performed. The numerical study indicates that both rules may underestimate the combined effect. It is observed that the underestimation is more for the *SRSS* than for the *30%* rule. In addition, the underestimation is more for inelastic than for elastic analyses. The underestimation cannot be correlated with the height of the frames or the predominant period of the earthquakes. A basic probabilistic study is performed in order to estimate the accuracy of several rules in the evaluation of the combined response. It is observed that some rules estimate the combined response for elastic analysis but they fail for inelastic analysis. Based on the results obtained in this study, it is concluded that the design requirements for the combined effect of the horizontal components, as outlined in some code-specified seismic design procedure, need to be modified. New combination rules are proposed.

INTRODUCTION

In most seismic design codes, approximate yet conservative simplified equivalent lateral load procedures are allowed to independently estimate the responses due to the two horizontal components of earthquakes. The overall response is then estimated using some combination rules. Building designed according to codes for gravity loads provides a high factor of safety in the vertical direction. In addition, the vertical component is significantly out of phase with the horizontal components. Consequently, the effect of the vertical component is usually neglected and the problem is apparently simplified. However, how to combine the effect of the horizontal components is an issue of considerable interest to the profession. The combination rules are

1→Professor, Facultad de Ingeniería, Universidad Autónoma de Sinaloa, reyes@uas.uasnet.mx
2→Graduate students, Facultad de Ingeniería, Universidad Autónoma de Sinaloa
3→Professor, Department of Civil Engineering and Engineering Mechanics, University of Arizona

different in many international codes [1, 2, 3]. The most commonly used combination rules are the Square Root of the Sum of the Squares (*SRSS*) and the 30-percent (*30%*) combination method. Since the basic equivalent lateral load procedure is conservative, the assumption is that these combination rules are also conservative. The accuracy of these rules needs a re-evaluation and is the subject of this paper.

There have been a significant number of studies regarding the evaluation of the responses of structures considering both horizontal components. Wilson and Button [4] presented a simple method to determine the critical angle of simplified structures without considering any correlation between the horizontal ground motion components. Correnza and Hutchinson [5] analyzed one-story models with and without transverse elements subjected to one and to bi-directional earthquakes. López and Torres [6] studied the critical angle of incidence for a one story one bay elastic concrete building. De Stefano and Faella [7] studied the biaxial inelastic response of simplified single mass two-degree-of-freedom concrete structures. Fernández-Dávila et al [8] studied the seismic elastic response of concrete buildings considering three degrees of freedom per floor. Nonetheless, the general limitations on these studies are that elastic analysis and/or concrete frame were used and that the models were too simplified (a few stories and plane frames connected by rigid diaphragms, etc.). Consequently, the inelastic behavior of all the structural elements (including beams and columns), energy dissipation, and contribution of higher modes were not properly considered. In addition, steel frames modeled as multi degree of freedom (MDOF) systems have not been studied. In this paper, moment resisting steel frames (MRSF) modeled as complex MDOF systems are studied.

A MRSF is usually designed as a strong-column weak-beam (SCWB) system. In order to simplify the seismic analysis, the beams of these structures are usually considered to be infinitely rigid. If a SCWB steel structure is modeled as a frame with rigid diaphragms, one of the most important sources of energy dissipation, i.e. dissipation of energy at plastic hinges, won't be considered and the structural behavior will be modified. It has been shown [9] that the force reduction factors depend on the amount of dissipated energy which in turn significantly depends on the number and plastic hinges formed at the ends of the beams. Moreover, numerical studies [10] showed that, for response evaluation of SCWB steel buildings, assuming conventional rigid diaphragms might significantly underestimate the structural response. Consequently, MRSF should be analyzed as multi degree of freedom (MDOF) systems.

In this study, the *SRSS* and the *30%* rules, commonly used in code-specified seismic design procedures to evaluate the maximum effect of both horizontal components of earthquakes, are re-evaluated. Using a time domain nonlinear finite element program developed by the authors and their associates [11, 12], the maximum inelastic seismic responses of several SCWB steel structures are estimated as realistically as possible by simultaneously applying both horizontal components. Then, the accuracy of the above-mentioned combinations rules and others, in the estimation of the combined response, is evaluated. The response is expressed in terms of maximum average interstory displacements, the maximum total base shear, and the maximum axial loads at interior, exterior, and corner columns at the base of the frames. Several moment resisting steel frame structures, representing different dynamic characteristics, are used in the study. The frames are modeled as complex multi degree of freedom (MDOF) systems. Consequently, energy dissipation and higher modes responses are explicitly considered. Six degrees of freedom per node are considered. The frames are excited by several recorded time histories, which were selected to represent the different characteristics of strong motions. The effect of the vertical component of the earthquakes is neglected and the horizontal components are assumed to be uncorrelated [13].

METHODS AND MATHEMATICAL MODELS

Most of the currently available finite-element based nonlinear analysis techniques for frames are based on an assumed displacement field. In order to capture the effect of change in axial length of an element due to

large deformation, several elements are needed to model each member. The necessity for a large number of elements, coupled with the use of a numerical integration scheme to obtain the tangent stiffness matrix for each element several times during the analysis, makes this approach uneconomical.

Considering its efficiency, particularly for steel frame structures, the assumed stress-based finite element method [11, 14, 15] is used in this study. Using this approach, an explicit form of the tangent stiffness matrix is derived without any numerical integration. Fewer elements can be used in describing a large deformation configuration without sacrificing any accuracy. Furthermore, information on material nonlinearity can be incorporated in the algorithm without losing its basic simplicity. It gives very accurate results and is very efficient compared to the displacement-based approach. The procedure has been studied and extensively verified with existing theoretical and experimental results. Details of the algorithm are not given here due lack of space.

Based on an extensive literature review, it is observed that viscous Rayleigh-type damping is commonly used in the profession and is used in this study too. The consideration of both the tangent stiffness and the mass matrices is a rational approach to estimate the energy dissipated by viscous damping in a nonlinear seismic analysis. The mass matrix is assumed to be concentrated-type and the step-by-step numerical integration procedure with the Newmark β method is used to solve the nonlinear governing equations of the problem.

A computer program has been developed to implement the procedure. The program was extensively verified using information available in the literature. The structural response behavior in terms of members' forces (axial load, shear forces and bending moments), total base shear and interstory displacement can be estimated using this computer program.

STRUCTURAL MODELS AND EARTHQUAKES

Four three-dimensional steel moment-resisting frame structures are used in the study. The geometry of the frames is shown in Fig. 1 and the corresponding member sizes in Table 1. These frames will be denoted hereafter as Models 1, 2, 3 and 4. For each model, four plane frames are considered: two interior (M_{xi} and M_{yi}) and two exterior (M_{xe} and M_{ye}). The story height for all the models is a constant of 3.66 m and their bay width is 7.32 m in both directions. The plane frames in both directions were designed according to the UBC standards and then modified following the strong column-weak beam (SCWB) concept. With the exception of exterior joints and the joints located on the top floor, the ratio of the sum of the plastic moments of the beams framing into a given beam-column joint to the sum of the plastic moments of the columns framing into the same joint, ranges from 0.68 to 0.91. The fundamental periods of the models in the major direction (*X* direction) are 0.21, 0.67, 1.15 and 1.32 sec, respectively. The corresponding values for the minor direction are 0.34, 1.04, 1.61 and 1.93 sec., respectively. In all these frames, the columns are assumed to be made of Grade-50 steel and the girders of A36 steel. In the seismic analysis of these frames, equivalent nodal forces were calculated, as required for the assumed stress-based finite element formulation used in this study. One node was placed at the mid-span of each of the girders. Each node is considered to have six degrees of freedom. The number of degrees of freedoms considered is 240, 720, 1920 and 3660, for Models 1, 2, 3 and 4, respectively. These four models with different dynamic characteristics are subjected to twenty strong motion earthquakes which are scaled down or up in such a way that all the frames approximately develop a maximum interstory displacement of 1.5%. The earthquakes are given in Table 2. They are presented in an increasing order for their predominant period and are denoted hereafter as Earthquakes 1 through 20.

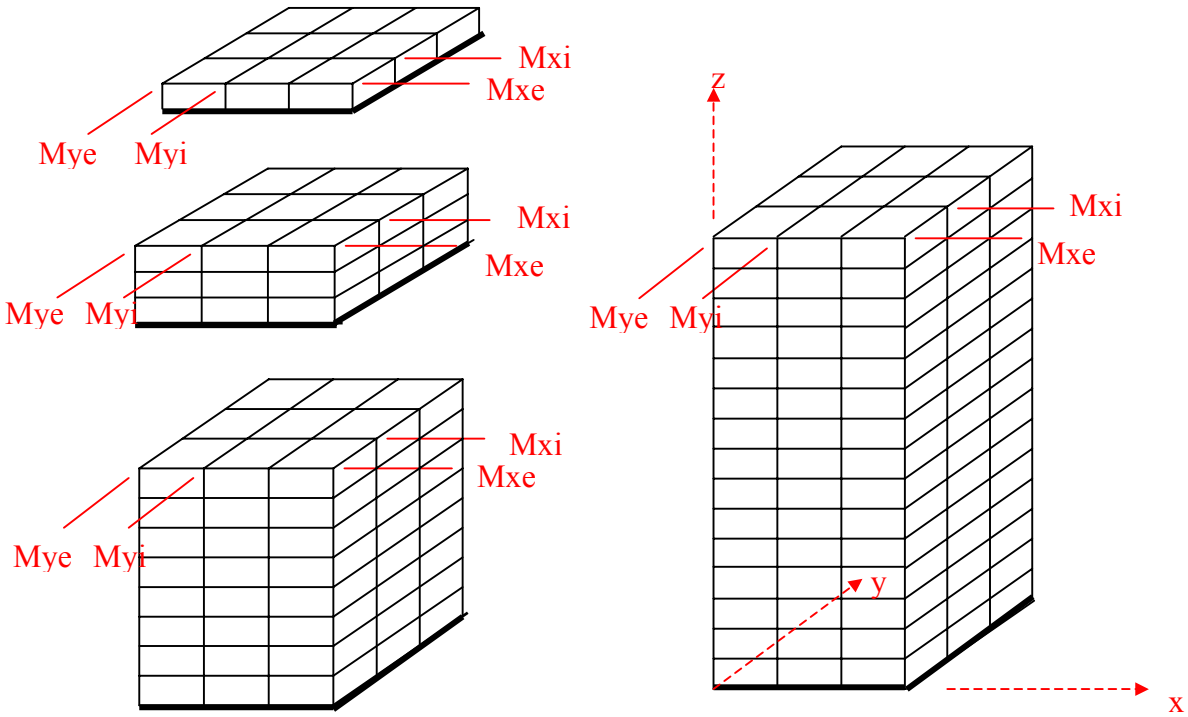


Fig. 1. Structural models

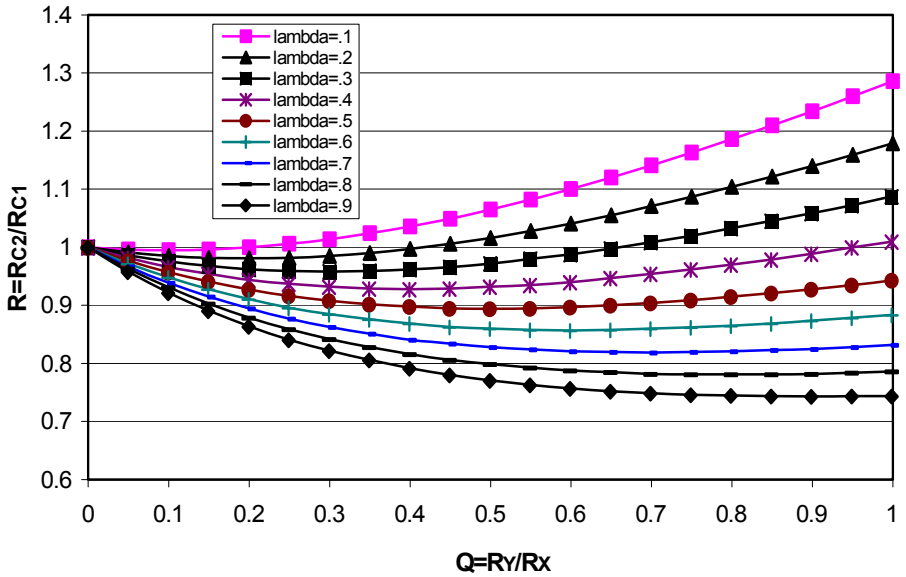


Fig. 2. Variation of the ratio of the *SRSS* and *LAMBDA* combinations

Table 1. Member sizes

MODEL	FRAME	STORY	EXT COL	INT COL	GIRDERS
1	Mxe	1	W14x74	W14x109	W21x44
	Mxi	1	W14x99	W14x145	W21x57
	Mye	1	W14x74	W14x99	W14x26
	Myi	1	W14x109	W14x145	W14x38
2	Mxe	1-2	W14x82	W14x99	W21x73
		3	W14x82	W14x99	W18x40
	Mxi	1-2	W14x99	W14x159	W18x71
		3	W14x99	W14x159	W18x71
	Mye	1-2	W14x82	W14x99	W18x40
		3	W14x82	W14x99	W16x26
	Myi	1-2	W14x99	W14x159	W18x71
		3	W14x99	W14x159	W16x40
3	Mxe	1-2	W14x120	W14x159	W24x94
		3	W14x109	W14x159	W24x94
		4-5	W14x109	W14x145	W24x84
		6-7	W14x82	W14x132	W24x84
		8	W14x82	W14x132	W21x50
	Mxi	1-2	W14x159	W14x211	W24x131
		3	W14x145	W14x211	W24x131
		4-5	W14x145	W14x193	W24x117
		6	W14x132	W14x176	W24x104
		7	W14x132	W14x176	W24x104
		8	W14x132	W14x176	W21x68
	Mye	1-2	W14x120	W24x117	W24x55
		3	W14x109	W14x159	W24x55
		4-5	W14x109	W14x145	W21x57
		6-7	W14x82	W14x132	W18x46
		8	W14x82	W14x132	W16x31
	Myi	1-2-3	W14x159	W14x211	W24x68
		4-5	W14x145	W14x193	W21x73
		6-7	W14x132	W14x176	W18x71
		8	W14x132	W14x176	W16x40
4	Mxe	1-2-3	W14x283	W14x370	W27x217
		4-5	W14x257	W14x342	W27x194
		6-7	W14x233	W14x311	W27x178
		8-9	W14x211	W14x283	W27x161
		10-11	W14x193	W14x257	W27x146
		12-13	W14x176	W14x233	W27x114
		14-15	W14x145	W14x176	W21x68
	Mxi	1-2-3	W14x398	W14x500	W27x258
		4-5	W14x370	W14x455	W27x258
		6-7	W14x342	W14x398	W27x235
		8-9	W14x311	W14x370	W27x217
		10-11	W14x283	W14x342	W27x194
		12-13	W14x257	W14x311	W27x161
		14-15	W14x211	W14x233	W21x93
	Mye	1-2-3	W14x283	W14x398	W27x129
		4-5	W14x257	W14x370	W27x114
		6-7	W14x233	W14x342	W27x102
		8-9	W14x211	W14x311	W27x94
		10-11	W14x193	W14x283	W27x84
		12-13	W14x176	W14x257	W27x84
		14	W14x145	W14x211	W24x76
		15	W14x145	W14x211	W21x44
	Myi	1-2-3	W14x370	W14x500	W27x146
		4	W14x342	W14x455	W27x146
		5	W14x342	W14x455	W27x129
		6	W14x311	W14x398	W27x129
		7	W14x311	W14x398	W27x114
		8-9	W14x283	W14x370	W27x114
		10	W14x257	W14x342	W27x114
		11	W14x257	W14x342	W27x102
		12	W14x233	W14x311	W27x102
		13	W14x233	W14x311	W24x84
		14	W14x176	W14x233	W24x84

		15	W14x176	W14x233	W21x50

COMBINATIONS RULES

Let us define R_X is the maximum effect (moment, shear, etc.) at a particular point in a particular member of a given structure arising from the horizontal component in the X direction of a given earthquake. Let us also define R_Y as the peak value of the same effect arising from the horizontal component of the earthquake in Y direction. Then, the combined effect can be calculated as the most unfavorable of:

$$R_{C1} = R_X + \lambda\, R_Y \quad or, \quad R_{C1} = \lambda\, R_X + R_Y \tag{1}$$

If λ =0.3 in Eq. 1, it represents the *30%* combination rule. According to the *SRSS* rule, the combined response is given by

$$R_{C2} = \sqrt{R_X^{\ 2} + R_Y^{\ 2}} \tag{2}$$

The accuracy of the above procedures is evaluated by comparing the R_{C1} (for λ=0.3) and R_{C2} values with the theoretical response. The accuracy of using other combination rules is also evaluated. The theoretical solution is obtained from the simultaneous application of both horizontal components.

At this stage, it is important to establish a relationship between the R_{C1} and R_{C2} parameters. Let us assume that R_X is the larger of the two peak effects and consequently R_Y will be the smaller one. If Q denotes the ratio of the smaller to the larger effect, it is $Q = R_Y / R_X$, then R_{C2} can be expressed as:

$$R_{C2} = \sqrt{R_X^{\ 2} + (QR_X)^2} = R_X \sqrt{1+Q^2} \tag{3}$$

$$\frac{R_{C2}}{R_X} = \sqrt{1+Q^2} \tag{4}$$

In the same manner R_{C1} can be expressed as:

$$R_{C1} = R_X + \lambda\, QR_X \tag{5}$$

$$\frac{R_{C1}}{R_X} = 1 + Q\lambda \tag{6}$$

Therefore, the ratio of the two combined responses, defined as R, is given by

$$R = \frac{R_{C2}}{R_{C1}} = \frac{\sqrt{1+Q^2}}{1+Q\lambda} \tag{7}$$

The values of R are plotted in Fig. 2 for several values of Q and λ. It is observed from this figure that for λ= 0.3, R_{C2} and R_{C1} are close each other. If the *SRSS* combination rule were the exact solution, the minimum error introduced by using λ = 0.3 would be zero for Q = 0.67. The maximum underestimation error would be about 4% for Q = 0.3 and the maximum overestimation error would be about 8% for Q = 1.0. In the authors' understanding, the 30% combination rule is based on the assumption that the *SRSS* gives the exact solution. However, as shown below, the combined response according to the *SRSS* rule can be quite different from that of the theoretical solution. Thus, if λ = 0.3 is used in Eq. 1, significant errors can be introduced according to this rule.

EVALUATION OF THE 30% COMBINATION RULE.

The four models are excited for the twenty earthquakes given in Table 2 and the accuracy of the *30%* combination rule is evaluated. The horizontal component with the largest peak ground acceleration is

selected to be applied to the strong direction of the models and the other one is applied to the weak

Table 2. Earthquake models

EARTHQUAKE NUMBER	EARTHQUAKE NAME	STATION	PREDOMINANT PERIOD (Sec.)
1	EL SALVADOR 2001	RELACIONES EXTERIORES	0.11
2	EL CENTRO	ELC7	0.19
3	NORTHRIDGE	LOS ANGELES, WADSWORTH V.A. HOSPITAL	0.25
4	MÉXICO 1985	CAYACO, MICHOACÁN, MÉXICO	0.29
5	NORTHRIDGE	TOPANGA FIRE STATION	0.31
6	NORTHRIDGE	IRVINE, 2603 MAIN	0.38
7	EL CENTRO	ELC8	0.39
8	NORTHRIDGE	LOS ANGELES, BRENTWOOD V.A. HOSPITAL	0.49
9	NORTHRIDGE	LOS ANGELES, GRIFFITH OBSERVATORY	0.51
10	NORTHRIDGE	LOS ANGELES, WADSWORTH V.A. HOSPITAL	0.55
11	MÉXICO 1985	VILLITA, MICHOACÁN, MÉXICO	0.55
12	MÉXICO 1985	ATOYAC, MICHOACÁN, MÉXICO	0.58
13	NORTHRIDGE	HAWTHORNE FAA BLDG.	0.60
14	EL CENTRO	ELC0	0.68
15	MÉXICO 1985	APATZINGAN, MICHOACÁN, MÉXICO	0.91
16	EL SALVADOR 2001	AHUACHAPAN	1.03
17	MÉXICO 1985	CHILPANCINGO, GUERRERO, MÉXICO	1.05
18	EL CENTRO	ELC1	1.29
19	EL CENTRO	ELC5	2.10
20	EL CENTRO	ELC2	2.20

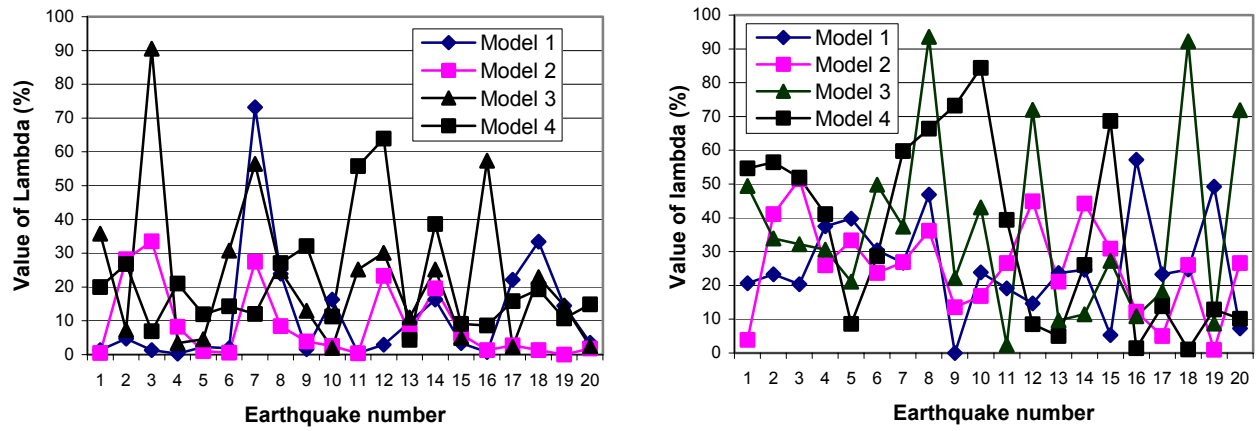
direction. Elastic and inelastic analyses are considered. The λ parameter given in Eq. 1 is estimated for the maximum values of axial loads at ground level columns, the average interstory displacements, and the total base shear. Results according to the elastic analysis for axial load at interior columns are presented in Fig. 3a, for all the models. It is observed that, for a given model, the values of λ significantly vary from one earthquake to another without shown any correlation. In addition, for a given earthquake, no correlation is observed between λ and the fundamental period of the models. The most important observation that can be made is that the 30% value is exceeded in many cases. If the values of λ are significantly larger than 30%, this rule could underestimate the combined structural response.

Values of λ for interior columns and inelastic analysis, for all the models, are presented in Fig. 3b. The major observations made for elastic analysis also apply to inelastic analysis. The only additional observation that can be made is that, for the case of axial loads, the number of cases in which the *30%* value is exceeded is much larger for the inelastic than for the elastic analysis. For an example, for Model 4, this number is *4* for the elastic analysis but the corresponding number is *10* for the inelastic case. Results for corner and exterior columns are similarly estimated, but they are not shown because of lack of space. However, results indicate that the number of cases of underestimation is larger for corner than for exterior columns, which in turn is larger than for interior columns.

Results for the total base shear and the interstory displacements are similarly estimated but they are not shown here. It is also observed that λ values significantly vary from one earthquake to another and from one frame to another. No correlation is observed between the λ values and the predominant periods of the earthquakes or between the λ values and the fundamental periods of the models.

It is important to note that the accuracy of evaluating the combined response by using $\lambda = 0.3$, will significantly depend on the values of the Q parameter. As observed from Eq. 5 (with $\lambda = 0.3$), if Q tends to zero, for any value of λ the combined response R_{C1} will tend to R_X. Thus, if the "real" λ is equal, larger or smaller than 0.3, the combined response will be correctly estimated. On the other hand, if Q tends to one, the combined response will tend to 1.3 R_X. Consequently, if the "real" lambda is larger than 0.3, the

combined response will be underestimated.



a) Elastic analysis

b) Inelastic analysis

Fig. 3. Values of λ for interior columns

From the above results, it is concluded that the λ values depend on the fundamental period of the frames, the predominant period of the earthquakes, the selected response parameter, and the type of analysis. The most important observation so far is that the values of λ can be significantly larger than 30%.

SRSS AND *30%* COMBINATION RULES VS THEORETICAL SOLUTION

The combined effect, according to the *SRSS* and the *30%* rules, is estimated and compared to that given by the theoretical solution. In addition, the combined effect is calculated by using λ = 40%, 50%, and 60%, instead of 30%, and by assuming the combined response to be 1.2 times R_{max}, where R_{max} is the maximum of R_X and R_Y. These four additional combination rules will be denoted hereafter as the *40%, 50%, 60%* and the *1.2* R_{max} rules, respectively. For comparison purposes, the following error terms are defined:

$$E_{SRSS} = \frac{SRSS\ value - theoretical\ value}{theoretical\ value} \tag{8}$$

$$E_{30\%} = \frac{30\%\ value - theoretical\ value}{theoretical\ value} \tag{9}$$

$$E_{40\%} = \frac{40\%\ value - theoretical\ value}{theoretical\ value} \tag{10}$$

$$E_{50\%} = \frac{50\%\ value - theoretical\ value}{theoretical\ value} \tag{11}$$

$$E_{60\%} = \frac{60\%\ value - theoretical\ value}{theoretical\ value} \tag{12}$$

$$E_{1.2R\max} = \frac{1.2R_{\max}\ value - theoretical\ value}{theoretical\ value} \quad \textbf{(13)}$$

where the terms *theoretical value, SRSS value, 30% value, 40% value, 50% value, 60% value, and 1.2Rmax*, represent the combined effect according to the theoretical solution, *SRSS* rule, λ = 30%, λ = 40%, λ = 50%, λ = 60%, and the maximum peak value increased by 20%, respectively. The rules given for Eqs. 9 through 13, will be also referred as the λ´s rules. A negative error in any of the equations implies that the combination rule under consideration underestimates the combined effect of both components; in other words, the result is unconservative. On the other hand, a positive value of the error indicates that the combination rule overestimates the combined effect, and thus is conservative. The errors are calculated for axial loads at ground level columns, interstory displacements in both directions and total base shear in both directions, for all the models. Elastic and inelastic analyses are considered.

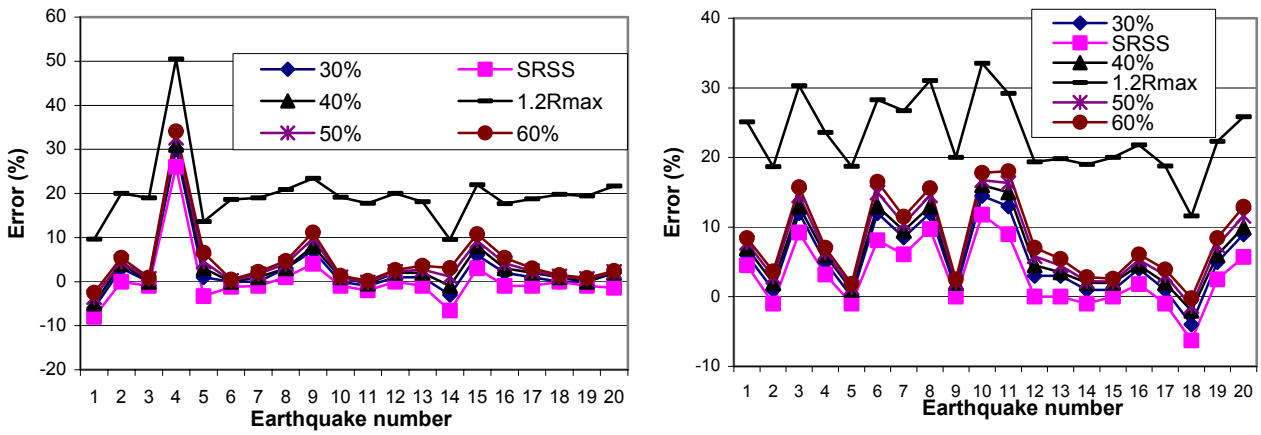
The errors for axial loads at exterior columns of Model 3 and elastic analysis are shown in Fig 4. It is observed that both the 30% and the *SRSS* rules, commonly used in seismic design procedures, may underestimate the combined response. For this frame, the magnitude of the negative errors are small. However, as discussed below, the errors can be much larger for other cases. In general, the curve for the *30%* rule is over the corresponding curve for the *SRSS* rule. In other words, the *30%* rule is more conservative than the *SRSS* rule. The implication of this is that the values of the *Q* ratio, presented in Fig. 2**,** are smaller than 0.67 in all the cases. No correlation is observed between the magnitude of the errors and the predominant period of the earthquakes.

Results shown in these figures indicate that increasing λ from 30 to 40% does not necessarily reduce the magnitude of the negative error with respect to that of 30%. This is supported by the results shown in Fig. 2 and Eq. 5 where it is observed that, as stated in earlier sections, the combined response depends on the value of the *Q* parameter. If *Q* is small, increasing λ from 30% to 40% and even to 60% may result in negligible increments of R_{C1} and consequently the error remains practically the same. Fig. 4 also indicates that the *1.2Rmax* combination rule, in general, reasonably estimate the combined response. As for the other rules, no correlation is observed between the magnitude of the errors and the predominant period of the earthquakes.

The errors for the axial load at corner and interior columns of Model 3 and columns of the other models are similarly estimated but are not shown here. The major observations made for Model 3 also apply to these models. From an analysis of the results for all columns and models, it is observed that the underestimation error may be up to 40% for the *30%* and *40%* rules, in many cases. This value may be up to 20% for the 50% and 60% rules. It is also observed that the $1.2R_{max}$ rule reasonably estimate the combined response for exterior columns. The error can be up to 20% for interior and corner columns, but only for a few cases.

The errors for interstory displacements in the *X* direction are presented in Fig. 5 for Models 1, 2, and all the combination rules. Unlike the error for axial load at columns, both the *30%* and the *SRSS* combination rules accurately estimate the combined effect in terms of this parameter. It is observed that for these two rules, the error ranges between 5 and –5% in most of the cases. Only in one case, the magnitude of the negative error is 9%. This is expected. In the case of interstory displacements and the elastic analysis, one of the peak responses (R_X or R_Y) is very large compared to the other. Then, R_{C1} and R_{C2} given by Eqs. 1 and 2, respectively, will be very close each other and at the same time will be close to the theoretical solutions. Consequently, the introduced errors according to Eqs. 8 and 9 will be very small. As for the case of axial loads, the underestimation is more for the *SRSS* rule than for the *30%* rule. It is also observed that the $1.2R_{max}$ rule, reasonably estimate the combined response in all the cases. In

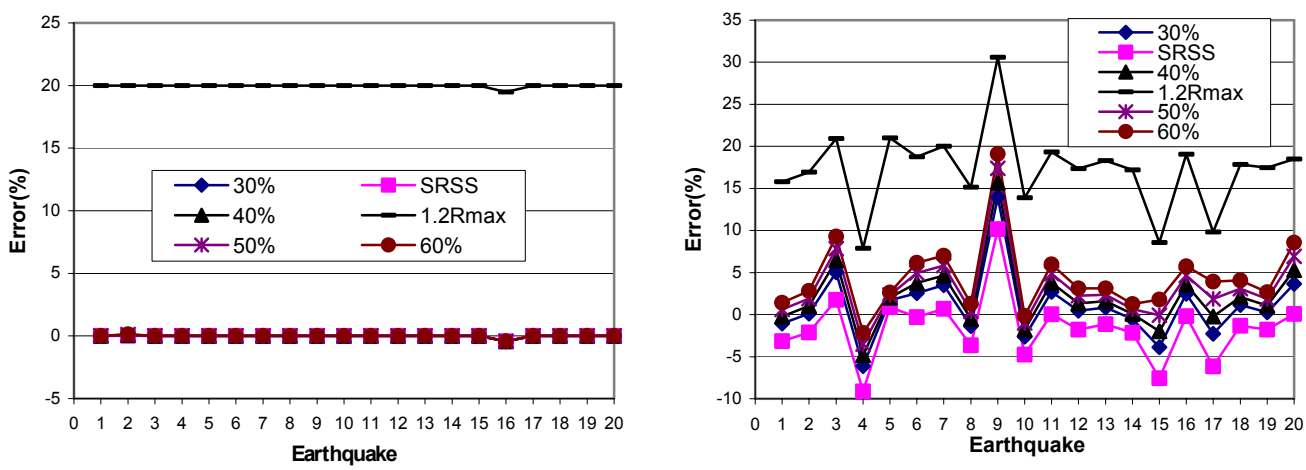
addition, since R_{C1}, R_{C2} and the theoretical solution are close each other and because of Eq. 13, the introduced error according to this rule is close to 20% for most of the cases. The corresponding errors for Models 3 and 4, for all the models in the *Y* direction, and for base shear in both directions are similarly estimated. The results are similar to those of interstory displacements in the *X* direction. The only additional observation that can be made is that the scatter in the error values tends to increase with the fundamental period of the frames.



a) Lateral *X*

b) Lateral *Y*

Fig. 4 Error, axial load, Model 3, lateral columns, elastic



a) Model 1

b) Model 2

Fig. 5 Error, interstory displacements Models 1 y 2, elastic

Inelastic analyses are also performed and the introduced error for interstory displacements, base shear, and axial loads at base columns are evaluated, but the results are not shown. Only the major observations are given here. Results indicate that for the axial load and for the elastic analysis, for a given rule, no correlation is observed between the magnitude of the errors and the predominant period of the

earthquakes. It is also observed that both the *30%* and *SRSS* rules may underestimate the combined response in some cases. However, the number of cases and their corresponding negative errors are larger for the inelastic case. In addition, for these two rules the underestimation is in general larger for corner and interior than for exterior columns. It is shown that for exterior columns the *1.2Rmax* rule reasonably estimates the combined response. For corner and interior columns, however, unlike for elastic analysis, this rule may significantly underestimate the combined response. The error is 60% in some cases. It indicates that the results for the elastic analysis may be quite different from those of inelastic analysis. The implication of this is that the results obtained from the elastic analysis of steel frames structures subjected to strong motions may be a very crude approximation.

For interstory displacements, as for the case of elastic results, the *30%* and *SRSS* rules in general estimate the combined effect very well. Whenever the combined effect is underestimated (negative errors), the corresponding error is relatively small. The underestimation is always more for the *SRSS* than for the *30%* rule. Results also indicate that the *1.2Rmax* rule, reasonably estimate the combined effect. The errors for interstory displacements in the *Y* direction and the base shear in both directions are similar to those of interstory displacements in the *X* direction.

In summary, weather elastic or inelastic analysis is used, the combined interstory displacements and the total base shear are reasonably estimated by all the rules. The *30%* and the *SRSS* rules, however, can underestimate the combined response in terms of axial loads. The $1.2R_{max}$ rule reasonably estimates the combined axial load for elastic analysis but it can significantly underestimated the response for inelastic analysis, particularly for corner and interior columns.

PROBABILISTIC ANALYSIS

Results of the study show that the combined response according to the different rules, for interstory displacements, axial loads, and total base shear, significantly vary from one earthquake to another and from one model to another, even though the maximum deformation produced on the models is approximately the same for all time histories. In addition, the error is different from each rule, which in turn may be quite different from the theoretical solution. This indicates that the seismic response of frames is highly sensitive to the characteristics of the time histories used (frequency content and strong motion duration). This is particularly true for inelastic analysis. This shows the necessity of treating the introduced error as a random variable.

In the design of an engineering system, it is usually stated that its *capacity* should be greater than d*emand* [16]. Different terminology is used to describe these concepts depending upon the specific problem under consideration. In structural engineering, for example, *capacity* is usually expressed in terms of resistance, and *demand* in terms of applied loads or their effect. However, the parameters related to *capacity* and *demand* are, in general, random variables. Therefore, the associated uncertainty needs to be quantified. The primary task in the design of a system is to ensure satisfactory performance. Satisfactory performance cannot be absolutely guaranteed. Instead, assurance can only be given in terms of the probability of success (p_s) according to some performance criterion. In engineering, this probabilistic assurance of performance is referred as reliability. Another way of look at the problem is to consider the probability of failure (p_f), which is also commonly defined as risk. Risk and reliability are complementary terms.

The problem of reevaluating the *SRSS* and the *30%* rules can be circumscribed in the abovementioned context. Even though strictly speaking, it is not exactly a *demand and capacity problem,* this formulation can give probabilistic bases to estimate the accuracy of the combination rules in evaluating the combined

response. Any mathematical model satisfying the properties of Probability Density Function (PDF) and Cumulative Distribution Function (CDF) can be used to evaluate the uncertainties in the problem. However, if a random variable cannot have negative values, the lognormal distribution will be appropriate, since it automatically eliminates the possibility of negative values. As shown below, this is the case in this study.

For the problem under consideration, one can say that the combined axial load at a given column according to the *SRSS* rule is the capacity (*C*), and the corresponding exact solution is the demand (*D*). Then, according to Equation 2,

$$C = R_{C2} = \sqrt{R_X{}^2 + R^2{}_Y} \tag{14}$$

If it is assumed that both, *D* and *C* are statistically independent lognormal variables [that is, $LN(\eta_D, \varsigma_D)$ and $LN(\eta_C, \varsigma_C)$], then another random variable *Y* can be defined as [16]:

$$Y = C / D \tag{15}$$

or

$$Z = \ln Y = \ln C - \ln D \tag{16}$$

Since *C* and *D* are lognormal, *ln C*, *ln D,* and consequently *Z,* are normal [$Z \sim N(\mu_Z, \sigma_Z)$]. It can be shown that [16] the probability of success is given by

$$p_s = \Phi\left(\frac{\eta_C - \eta_D}{\sqrt{\zeta_C{}^2 + \zeta_D^2}}\right) \tag{17}$$

where Φ is the CDF of the standard normal variable and η and ς are the parameters of a lognormal variable.

As stated earlier, the accuracy of the abovementioned combination rules is not exactly a "demand and capacity" problem. In a real problem, it is accepted that the p_s value of a properly designed engineering system should be relatively large. For example, the p_s value for a steel beam designed according to the Load and Resistance Factor Design (LRFD) is expected to be about 0.999. For the problem under consideration however, this value does not have to be so large to indicate success. If the value of the axial load for a given column were exactly the same for the theoretical solution and a given combination rule, for all the earthquakes, then both solutions would have the same mean an standard deviation and the corresponding value of p_s would be 0. Consequently, it would indicate that the rule under consideration is *acceptable* in the evaluation of the combined response. Thus, if p_s is 0.5 or larger, it will be said that rule is *acceptable*, otherwise it will be *no acceptable.*

The probability of success (p_s) in the estimation of the combined effect in terms of axial load, according to the different combination rules, is calculated for all the frames by using Eq. 17. The results for both, elastic and inelastic analysis are presented in Table 3. The p_s values are estimated for individual models (M1, M2, M3 and M4) and for all the models (GBL). It is observed that, for elastic analysis, all the rules give values of the p_s parameter larger than 0.5 for most of the cases. Only in a few cases p_s resulted in values smaller than 0.5. Whenever it is the case, it is quite close to 0.5. Thus, the accuracy of all the

rules in the evaluation of the combined response in terms of the axial loads is *acceptable* for elastic analysis. Results also indicate that, increasing λ from 30 to 40% does not significantly increase the

Table 3. Probabilistic results, axial load

TYPE OF ANALYSIS		LATERAL *X*					CORNER					INTERIOR					LATERAL *Y*				
		M1	M2	M3	M4	GBL	M1	M2	M3	M4	GBL	M1	M2	M3	M4	GBL	M1	M2	M3	M4	GBL
ELASTIC	SRSS	0.494	0.525	0.502	0.530	0.504	0.560	0.532	0.544	0.464	0.497	0.517	0.512	0.556	0.528	0.519	0.575	0.518	0.530	0.499	0.503
	30%	0.518	0.547	0.516	0.563	0.511	0.590	0.529	0.544	0.465	0.497	0.540	0.524	0.565	0.522	0.523	0.599	0.514	0.553	0.515	0.510
	40%	0.529	0.563	0.522	0.583	0.515	0.622	0.560	0.594	0.505	0.511	0.563	0.538	0.594	0.565	0.539	0.664	0.546	0.562	0.523	0.514
	50%	0.540	0.578	0.528	0.603	0.519	0.656	0.648	0.692	0.544	0.531	0.585	0.552	0.621	0.607	0.555	0.722	0.576	0.571	0.530	0.518
	60%	0.551	0.591	0.534	0.623	0.523	0.688	0.655	0.713	0.583	0.542	0.607	0.565	0.646	0.646	0.570	0.773	0.605	0.580	0.537	0.522
	1.2Rmax	0.676	0.680	0.635	0.007	0.556	0.679	0.534	0.556	0.483	0.501	0.632	0.566	0.592	0.529	0.544	0.655	0.517	0.701	0.609	0.546
INELASTIC	SRSS	0.338	0.527	0.567	0.532	0.507	0.394	0.499	0.470	0.429	0.491	0.370	0.492	0.470	0.404	0.470	0.488	0.494	0.506	0.503	0.501
	30%	0.359	0.548	0.617	0.577	0.515	0.389	0.515	0.470	0.438	0.492	0.328	0.511	0.494	0.424	0.474	0.504	0.510	0.531	0.521	0.508
	40%	0.367	0.556	0.637	0.600	0.518	0.452	0.531	0.535	0.487	0.506	0.351	0.539	0.515	0.451	0.485	0.511	0.530	0.544	0.528	0.512
	50%	0.375	0.564	0.655	0.623	0.522	0.524	0.548	0.599	0.535	0.514	0.498	0.566	0.536	0.478	0.394	0.518	0.540	0.557	0.535	0.516
	60%	0.386	0.572	0.673	0.645	0.526	0.583	0.563	0.659	0.581	0.526	0.504	0.590	0.557	0.504	0.504	0.524	0.563	0.570	0.542	0.520
	1.2Rmax	0.602	0.702	0.900	0.740	0.555	0.400	0.550	0.483	0.467	0.498	0.286	0.556	0.553	0.469	0.495	0.625	0.544	0.622	0.652	0.550

Table 4. Probabilistic results, interstory displacements and base shear

COMBINATION RULE	BASE SHEAR										INTERSTORY DISPLACEMENT									
	ELASTIC					INELASTIC					ELASTIC					INELASTIC				
	M1	M2	M3	M4	GBL	M1	M2	M3	M4	GBL	M1	M2	M3	M4	GBL	M1	M2	M3	M4	GBL
SRSS	0.500	0.468	0.458	0.497	0.469	0.436	0.458	0.530	0.508	0.478	0.535	0.492	0.510	0.394	0.448	0.447	0.49	0.497	0.514	0.498
30%	0.500	0.505	0.510	0.508	0.504	0.508	0.489	0.552	0.523	0.509	0.535	0.401	0.520	0.401	0.457	0.450	0.506	0.513	0.530	0.500
40%	0.500	0.508	0.514	0.513	0.506	0.512	0.499	0.561	0.529	0.512	0.535	0.489	0.526	0.405	0.494	0.451	0.513	0.519	0.537	0.503
50%	0.500	0.511	0.518	0.517	0.508	0.495	0.517	0.569	0.534	0.514	0.535	0.503	0.531	0.409	0.507	0.452	0.520	0.526	0.543	0.507
60%	0.500	0.513	0.522	0.521	0.509	0.945	0.522	0.577	0.540	0.516	0.535	0.521	0.536	0.413	0.513	0.453	0.527	0.532	0.550	0.513
1.2Rmax	0.665	0.587	0.640	0.605	0.567	0.593	0.592	0.739	0.646	0.578	0.684	0.576	0.635	0.500	0.567	0.572	0.576	0.644	0.624	0.614

values of p_s. In general, p_s is larger for the *30%* rule than for the *SRSS* rule. The $1.2R_{max}$ rule gives larger values of p_s than the other rules for lateral columns. For the *50%* and *60%* rules, larger values of p_s are obtained, in general, for corner and interior than for exterior columns.

Unlike the elastic analysis case, the p_s parameter for inelastic axial loads takes values much smaller than 0.5 for several cases. The *30%* and *40%* rules give small values of p_s for exterior columns of Model 1 in the *X* direction. The $1.2R_{max}$ also give small values (p_s can be as small as 0.286) for corner and interior columns and consequently is neither *acceptable*. The *50%* and *60%* rules are *acceptable* for axial loads for interior and corner columns. It is observed that smaller values of p_s are obtained in general for Model 1 than for the others and that, as for the elastic case, the *30%* rule gives values larger than those of the *SRSS* rule.

Values of p_s in terms of interstory displacements and total base shear, for both elastic and inelastic analysis are shown in Table 4. Results indicate that the $1.2R_{max}$ rule is *acceptable* in the calculation of the combined response for these two parameters, but the λ's rules are not *acceptable* in general.

CONCLUSIONS

The 30-percent (*30%*) and the Square Root of the Sum of the Squares (*SRSS*) rules, commonly used in seismic design procedures to evaluate the maximum effect of both horizontal components of earthquakes, are re-evaluated. Four 3D moment resisting steel frames, modeled as complex multi degree of freedom structures, representing different dynamic characteristics, are considered in the study. Using a time domain nonlinear finite element program developed by the authors, the maximum inelastic seismic responses of the models in terms of several parameters are evaluated by simultaneously applying both components. Then, the abovementioned combination rules and others are evaluated. The numerical study indicates that both, the *SRSS* and the *30%* combination rules, may underestimate the combined effect in terms of axial loads. It is observed that the underestimation is more for the *SRSS* than for the *30%* rule. In addition, for axial loads the underestimation is more for inelastic analysis than for elastic analysis. It indicates that the results for the elastic analysis may be quite different from those of the inelastic analysis. The implication of this is that the results obtained from elastic analysis of steel frames structures subjected to strong motions may be a very crude approximation. The underestimation cannot be correlated with the height of the frames or the predominant period of the earthquakes. The $1.2R_{max}$ rule estimates the combined axial load for elastic analysis reasonably well but it can significantly underestimated the response for inelastic analysis. It is also shown that the combined effect in terms of interstory displacements and total base shear is correctly estimated by all the rules. Based on the results obtained in this study, it is concluded that the design requirements for the combined effect of the horizontal components, as outlined in some code-specified seismic design procedure, need to be modified. It is suggested that the $1.2R_{max}$ rule can be used to obtain the combined response in terms of interstory displacements, base shear, and axial loads at exterior columns. The 50*%* rule can be used in the estimation of the axial loads at interior and corner columns.

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