**13th World Conference on Earthquake Engineering**
**Vancouver, B.C., Canada**
**August 1-6, 2004**
**Paper No. 1983**

# SEISMIC SHEAR DEMAND ON DUCTILE CANTILEVER WALL SYSTEMS IN MULTISTOREY STRUCTURES

**A. RUTENBERG [1], E. NSIERI[2] and R. LEVY[3]**

## SUMMARY

The seismic base shear demand on multistorey structures comprising ductile cantilever walls is reexamined. Two effects are studied: (1) shear force amplification due to higher vibration modes; (2) shear redistribution among walls through load transfer from yielded walls to those still elastic. The shear amplification factor is shown to be much larger than specified by seismic codes, resulting in a new design formula. It is also shown that base shear on each wall depends on the hinge formation sequence at their bases. Allocating shear forces per relative flexural rigidity or base moment capacity appreciably underestimates shear force demand on the walls, particularly the shorter ones. Influence of shear deformation and in-plane floor slab flexibility is noted.

## INTRODUCTION

The design seismic shear forces on the cantilever structural walls comprising the lateral load resisting system of multistorey building are reexamined. The two main factors affecting the shear demand are discussed: (1) higher natural vibration modes and (2) shear force redistribution from walls in which plastic hinges have already formed at their bases to those still remaining elastic.

As users of some modern seismic codes (e.g. EUROCODE 8 [1]) well know, the effect of higher vibration modes is considered by multiplying the base shear resulting from the lateral load distribution leading to the base moment by a shear amplification factor $\omega_v$, whose value depends on the number of storeys, and which in some codes is applicable only to structures designed to withstand high ductility demands. However, as will be shown subsequently, base shear amplification is an increasing function of ductility, i.e., of the strength reduction factor R (q in EUROCODE 8 [1]), and it is not negligible already at low ductility demands. In this work the effect on base shear amplification of higher vibration modes is studied

[1] Professor Emeritus, Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Technion City, Haifa 32000, Israel
[2] Graduate Student, Faculty of Civil and Environmental Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel
[3] Associate professor, Faculty of Civil and Environmental Engineering, Technion - Israel Institute of Technology, Haifa 32000, Israel

parametrically using a suite of 20 historic accelerograms, and is presented as function of the fundamental period T and R.

Structural engineers routinely distribute the lateral shear forces among the resisting flexural cantilever walls in proportion to their moments of inertia. This, of course, is true in the linear range, but redistribution – following formation of plastic hinges – results, somewhat unexpectedly, in the flexible walls resisting more than their elastic share. Since the shear strength capacity of walls in the plastic hinge region is appreciably lowered, this effect should be considered in design. This effect is to some extent considered by some codes (e.g. EUROCODE 8 [1] and NBCC [2]) through the requirement that the shear capacity be made proportional to the flexural capacity. This is due to the fact that with decreasing wall length the flexural strength falls more slowly than the flexural rigidity (power of 2 vs. power of 3 in rectangular members), so that the more flexible wall has to be designed for more than its moment share. However, this requirement is far from being the answer to the substantial increase in the shear demand in multistorey structures due to its redistribution. Herein the effect of redistribution on the base shear amplification and on the in-plane forces it induces in the floor slabs is first illustrated by means of a 2-wall 2-storey model. An analysis procedure, yielding the results of static cyclic incremental or pushover analysis, is then proposed to predict the peak seismic wall forces for given total base shear when the plastic hinges are confined to the wall bases. The approach is compared with the results of parametric studies on 10-storey structures having 4 walls of different lengths using the same suite of accelerograms. The effects of shear flexibility of the walls and in-plane flexibility of the floor slabs on shear redistribution are also considered in the numerical example.

## EFFECT OF HIGHER VIBRATION MODES ON ISOLATED WALLS

The vertical distribution of the lateral forces over the height of the structure specified by building codes for the equivalent lateral force procedure is usually an inverted triangle, and in some codes (e.g. IBC 2000 [3]) it becomes increasingly concave with increasing fundamental period. These loading shapes are quite adequate for predicting the moment distribution, but are far off the mark for the peak shear forces in flexural wall structures. This fact was not generally recognized until the mid 1970's. The work of Blakeley et al [4] on isolated walls demonstrated that the higher modes of vibration not only increase the *elastic* shear demand relative to that commensurate with the moment demand, but they more appreciably amplify the base shear after the formation of a *plastic* hinge at the wall base. Their work brought about a change in the static provisions of the New Zealand seismic code (New Zealand Standards Association [5]), and in turn in several other seismic codes. It is interesting to note that code-writing bodies in the USA have not been quick to follow, but the Commentary to the 1999 edition of the Recommended Lateral Force Requirements of the Structural Engineers Association of California (SEAOC [6]) already recommends adopting the amplification factor $\omega_v$ advocated by the New Zealand code. This factor is made dependent on the number of storeys n, as follows:

$$V_a = \omega_v V_d$$
$$\omega_v = \begin{cases} 0.9 + \dfrac{n}{10} & n \le 6 \\ 1.3 + \dfrac{n}{30} \le 1.8 & n > 6 \end{cases} \tag{1}$$

in which $V_a$ is the amplified value of the base shear $V_d$ as evaluated by means of the equivalent lateral force procedure. Note that the SEAOC 1999 commentary [6] proposes factoring $\omega_v$ by 0.85 because the

Blue Book applies a reduction factor of 0.85 to concrete strength. Note also that Eurocode 8 [1] has a different amplification formula based on the work of Keintzel [7].

Later studies (e.g. Seneviratna & Krawinkler [8]) show quite clearly that the expected amplification is much larger than that resulting from Eqn. 1, and that the extent of amplification is not only a function of the fundamental period T, as implied by the dependence of $\omega_v$ on the number of storeys, but also on the expected ductility demand, which in seismic codes is the main contributor to the force reduction factor R (or q). A parametric study was carried out recently at the Technion by Nsieri [9], using an ensemble consisting of 20 records. This ensemble has an exceedance probability in the Los Angeles area of 10% in 50 years, or a return period of 475 years (Somerville [10]), and was developed for the SAC project that was initiated in response to the damage to steel structures caused by the January 1994 Northridge earthquake. The 2-dimensional version of the computer code RUAUMOKO (Carr [11]) was used in the analysis assuming five percent tangent stiffness Rayleigh damping in the 1st and 5th modes.

Figure 1 shows the variation of the ***mean*** amplification factor $\omega_v^* = V_a / V_d$ with T for several values of R, assuming elastic-plastic moment-curvature relationship and plastic hinge formation only at the base. Note that $V_d$ is taken here as triangularly distributed at floor levels over the height of the building causing flexural yielding $M_y$ at the base, namely:

$$V_d = \left\{ \frac{3n}{(2n+1)H} \right\} M_y \tag{2}$$

where H = building height. It is evident that $V_a$ is increasing with T and R. A simple linear approximation for the mean value of $V_a$ when $n \geq 5$ is given below:

$$V_a = \left[ 0.75 + 0.22 \left( T + R + TR \right) \right] V_d \tag{3}$$

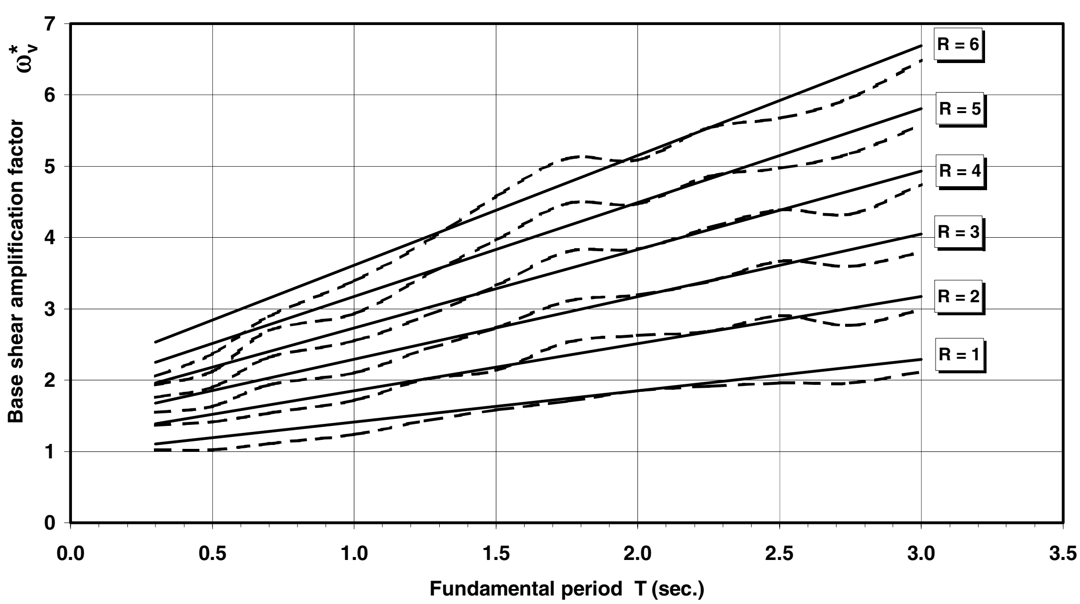


**Figure 1. Base shear amplification factor $\omega_v^*$ vs. T for several values of R**

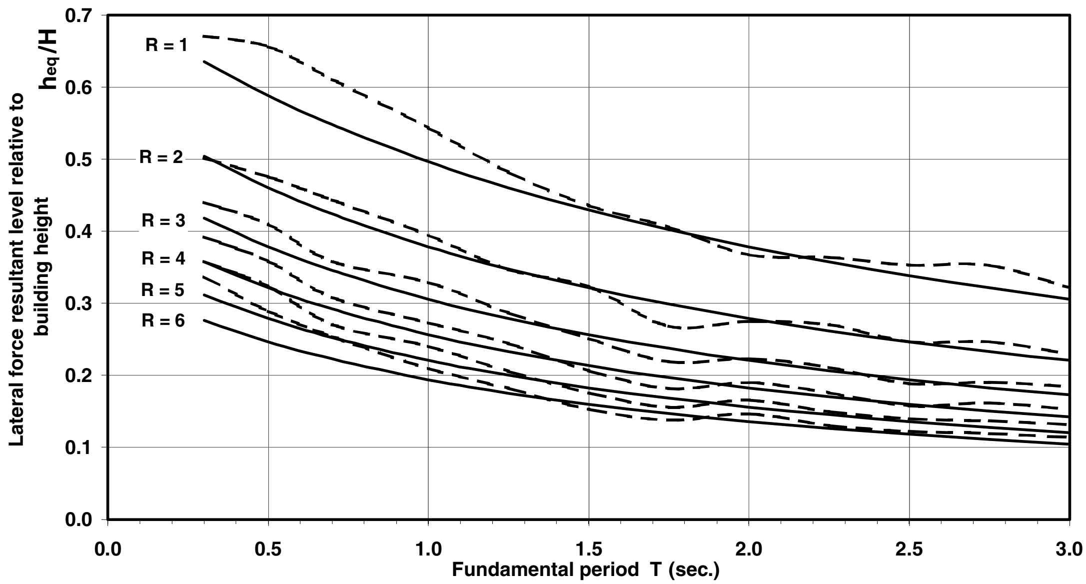


**Figure 2: Lateral force resultant level relative to building height vs. T for several values of R**

Note that the coefficients of variation from Eqn. 3 were found to be circa 40% for most of the range. Another way of looking at the data is presented in Fig. 2, in which the height of the resultant base shear relative to the building height, $h_{eq}/H = M_y/(V_{max}H)$, is depicted. Note also that this amplification formula is applicable only to the equivalent lateral force procedure, since modal analysis already includes the effect of higher modes in the linear range. Hence, for shear forces obtained from modal analysis $V_a$ can be obtained by dividing Eqn. 3 for the actual R value, by the amplification for R =1.

## SHEAR FORCE DISTRIBUTION IN SYSTEMS WITH SEVERAL WALLS

Consider first a one-storey structure consisting of several walls having different lengths that are connected by a roof diaphragm (rigid in its own plane but otherwise flexible). The system is loaded horizontally at roof level. Within the elastic range the shear force distribution among the walls is proportional to their stiffnesses. However, following yielding of the longer wall, the additional load will be carried by the shorter ones, as can be seen in Fig. 3. It is, therefore, evident that design for shear on the basis of relative stiffness underestimates the shear forces on the shorter wall, since the shear force capacity should be made proportional to the flexural strength. This point is known, and, as noted, has been addressed in the technical literature as well as in seismic codes.

Consider now the two-storey system supported by the two walls shown in Fig. 4a. Wall 1 is fully fixed at its base, whereas Wall 2, which is longer and hence stiffer, is hinged. The two horizontal pin-ended rigid members model the floor slabs connecting the two walls. For simplicity assume that the external horizontal force acts only at roof level. Note that the hinge at the base of Wall 2 is in fact a plastic one, and has formed at a horizontal force level H. An additional force $\Delta$H in the same direction will act only on Wall 1 provided that Wall 2 follows its deflected shape without resistance. However, this requires that another hinge forms in Wall 2 at first floor level, which usually is not the case. To fix ideas, let Wall 2 be very much stiffer than Wall 1. Therefore, under the force increment $\Delta$H, Wall 2 will enforce on Wall 1 its straight-line deflected shape at every floor level. The resulting forces on the system and the deflected shapes are shown in Fig. 4b, assuming simple flexural behaviour (i.e., ignoring shear deformation), rigid floors and no foundation rotation. It is seen that the additional shear force on Wall 1 $\Delta V_1(0) = 2.5\Delta H = 1.25\ \Delta M(0)/h$, ($\Delta M(0)$ = base moment increment due to $\Delta H$, h = storey height), and the shear on Wall 2 is reduced by $1.5\Delta H$. In fact, for a rigid Wall 2 and a large number of equal storeys, it can be shown (e.g. Karman & Biot [13]) that $\Delta V_1(0) \cong (3- \sqrt{3})\Delta M(0)/h$, which is quite close to the two storeys value, and in this case is independent of the vertical distribution of the incremental horizontal loading.

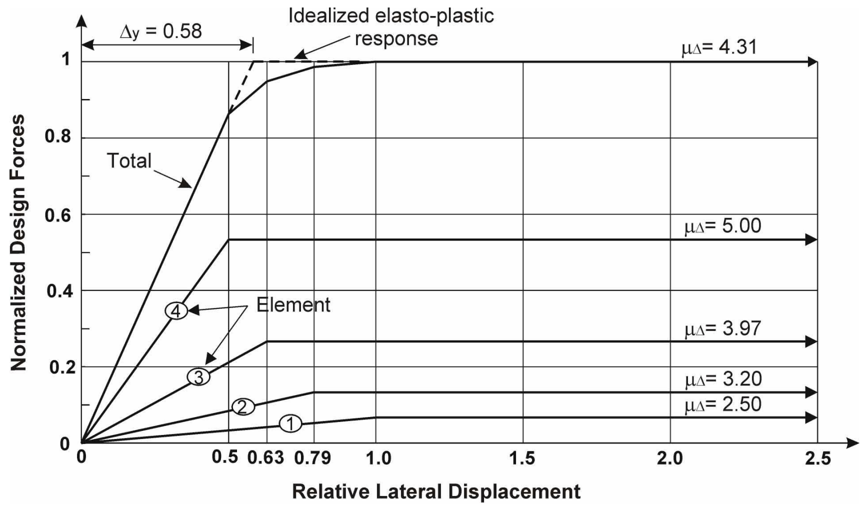


**Figure 3. Force-displacement relationships for 4 one-storey walls having equal roof displacement [12]**

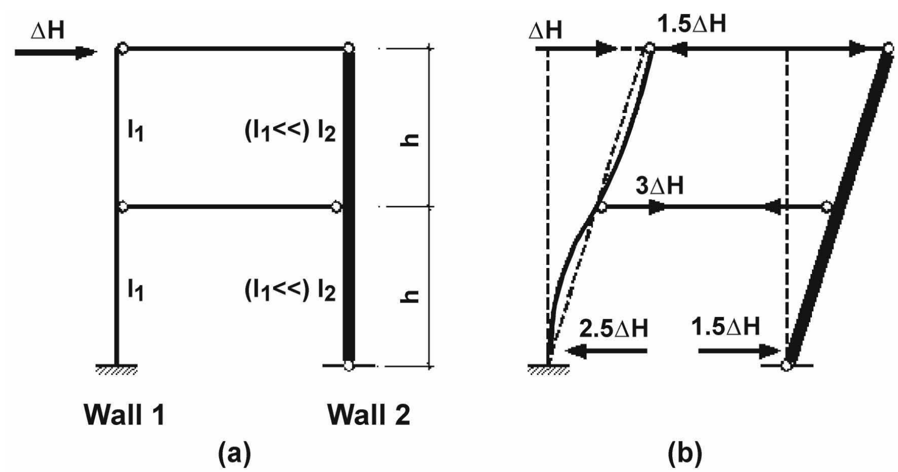


**Figure 4. Two-storey wall system: (a) Properties and loading (b) Floor forces and deflected shapes [14]**

In an elastic-perfectly-plastic system all this takes place without a change in the plastic hinge moment because the sense of curvature remains the same. Real systems are, of course, much more complicated than that: they are unlikely to be fully fixed at the foundation level (in fact they may even rock), shear deformations may become important with falling aspect ratio, floors are not infinitely rigid in their own plane, and increased wall rotation may take place due to tensile steel strains below the level of assumed fixity. Also, the assumption of concentrated plasticity is a very crude approximation for walls of some depth, as the hinge lengths will increase with increasing $\Delta H$, and the transition of the force-displacement curve to the plateau is not abrupt, as assumed in this model. Therefore, in many instances this effect can be much less pronounced than in this case, also because Wall 2 is not necessarily very much stiffer than Wall 1. Yet, this simple model does show that the shear forces acting on the shorter wall may be much larger than those evaluated on the basis of the usual assumptions. In other words, it may not be correct to assume, as is often done in practice, that shear force demand is proportional to bending moment demand, even in cases in which the effect of higher modes of vibration is considered.

An important aspect of the moment and shear transfer to the unyielded walls is the large force acting on the first floor diaphragm $\Delta N$, which, for a stiff wall with a plastic hinge at its base and large number of storeys, can reach the value of $6(2-\sqrt{3})\Delta M(0)/h$. Note that the force depends on the base moment, rather than on the base shear. This floor force can manifest itself either as an axial force when the lines of action of the walls coincide or as shear when the walls are parallel.

For increasing statically applied lateral load it is possible to follow the post first-yield shear force distribution among a number of walls – as plastic hinges form at the base of each one - either by means of successive applications of Eqns. 4, successive static linear analyses, or pushover analysis. These equations take the following form:

$$\Delta V_{if}(0) = \left[\Delta V(0) + \frac{\alpha \Delta M(0) \sum I_h}{h \sum I_f}\right] \frac{I_{if}}{\sum I} \tag{4a}$$

$$\Delta V_{ih}(0) = \left[\Delta V(0) + \frac{\alpha \Delta M(0)}{h}\right] \frac{I_{ih}}{\sum I} \tag{4b}$$

In which $\Delta V_{if}(0)$ and $\Delta V_{ih}(0)$ are the respective shear forces and $I_{if}$ and $I_{ih}$ are the respective moments of inertia of the i-th fixed and hinged walls, $\sum I$ and $\sum I_h$ are the moment of inertia sums of all walls and the hinged walls respectively. In multistorey buildings the second term in Eqn.4 is usually much larger than the first one, so that in fact $\Delta V_i(0)$, as $\Delta N$, the first floor diaphragm force, depends on the base moment rather than on the base shear.

It is more difficult to predict the peak shear forces in the walls during an earthquake by means of the above approaches. The main difficulty lies in the fact that at the lateral load level for which all the walls have yielded the shear forces acting on them, except on the most flexible one, are not at their peak values - as can be inferred from the simple 2-storey example presented above. On the other hand, pushover analysis with load reversal - cyclic pushover, as shown in Fig. 5, can estimate the sought peak values.

However, pushover analysis requires that the base shear on the system be known. One can use, of course, the $\omega_v$ values given in seismic codes, but, as already shown, these are not conservative; hence the use of Eqn. 3 is recommended. The applicability of Eqn. 3 is predicated on the assumption that the total base shear acting on a group of walls is equal to that on an isolated wall whose stiffness and strength are equal to the respective sums of the properties of the individual walls. This assumption was checked for the 20-record ensemble used for the analyses, and was found to be fully justified.

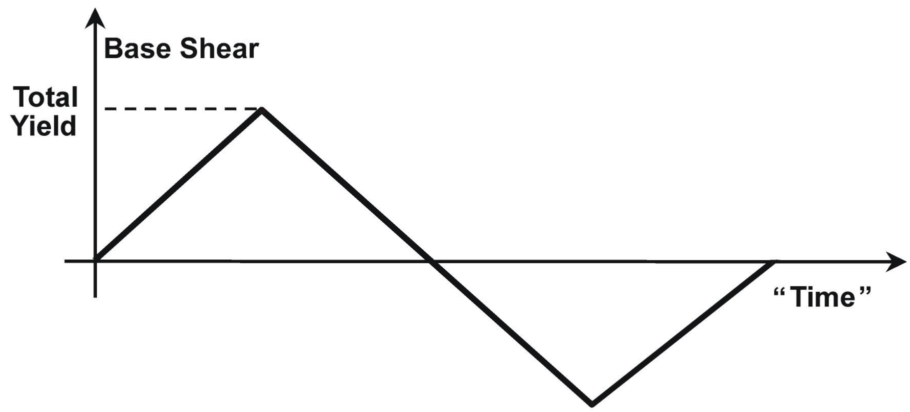


**Figure 5. Cyclic pushover load shape as function of "Time"**

The influence of shear deformation in the walls and of the in-plane flexibility of the floor diaphragms has also been considered. As expected both reduce the redistribution of shear in the walls. Some results are presented in the following section.

## NUMERICAL EXAMPLE

This example compares the results of the static cyclic pushover analysis with those of time history analyses for the symmetric 10-storey wall structure shown in Fig. 6. The 4 walls represent one half of the lateral load resisting system. A simplified lateral force distribution over the height of the building at peak base shear is shown on the left hand side of the figure. The relevant structural properties of the walls are given in Table 1. It is assumed that the floor slabs are rigid in their own plane and completely flexible out of plane. The walls are modelled as elastic-perfectly plastic and the mass of the structure was adjusted to give a fundamental vibration period T = 1.0 sec. Rayleigh damping of 5% in the 1st and 5th modes using the *tangent* stiffness matrix was assumed.

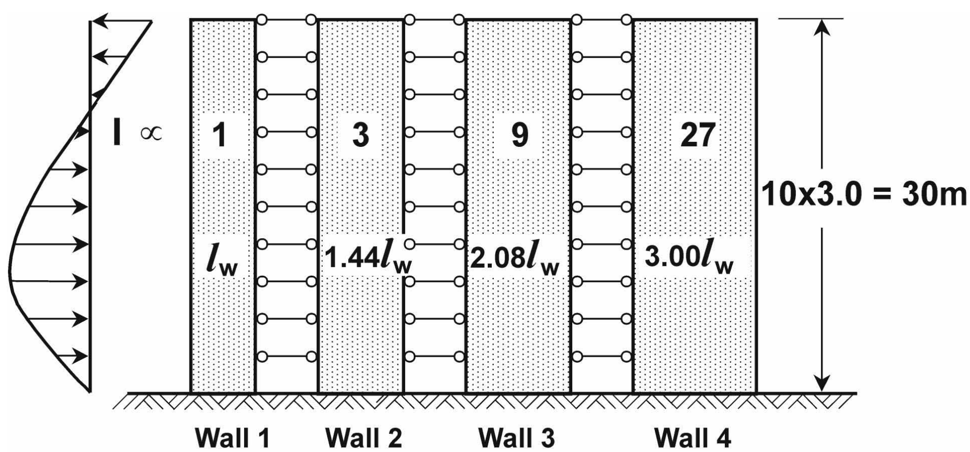


**Figure 6. Four walls lateral load resisting system for 10-storey building**

**Table 1: Properties of the 4 Walls**

| Wall | $l_w$ (m) | EI(kNm$^2$) | I / ΣI | $M_y$ (kNm) | $M_y/\Sigma M_y$ |
|---|---|---|---|---|---|
| 1 | 2.50 | 2.52x10$^6$ | 0.025 | 4018 | 0.0609 |
| 2 | 3.61 | 7.56x10$^6$ | 0.075 | 8356 | 0.1268 |
| 3 | 5.21 | 22.68x10$^6$ | 0.225 | 17385 | 0.2637 |
| 4 | 7.50 | 68.04x10$^6$ | 0.675 | 36165 | 0.5486 |
| Σ | | 100.8x10$^6$ | 1.000 | 65924 | 1.0000 |

The static approximation of the peak shear forces at the base of the walls was carried out by cyclic pushover analysis up to a peak base shear $V_{max} = V_a$ (Eqn. 3) for R = 4, which was applied at the level $h_{eq} = M_y/V_{max}$ above the base ($M_y$ = combined yield moment of the 4 walls). The dynamic analyses were carried out using RUAUMOKO (Carr [11]) with the same ensemble of the SAC 10/50 20 accelerograms, again assuming 5% percent tangent stiffness damping in the 1st and 5th modes.

In both analyses it was assumed that plastic hinges could form only at the wall bases. Figure 7 shows the variations of the total base shear and the shear forces on Walls 1 and 4 with "time", and Fig. 8 shows these variations with roof displacement. Note the small residual displacements and shear forces. A comparison of the pushover analysis with the mean time-history results is given in Table 2. This table also presents the shear forces that the structure would be designed for using routine procedures. It is quite evident that the cyclic pushover analysis yields very reasonable estimate of the seismic shear force demand on the walls.

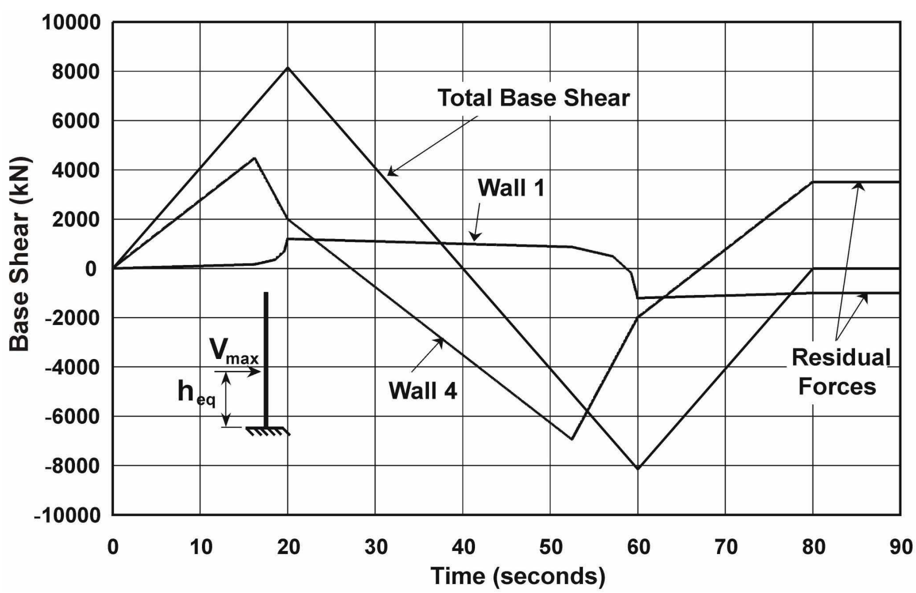


**Figure 7. Cyclic pushover results for Walls 1 and 4: Base shear vs. Time**

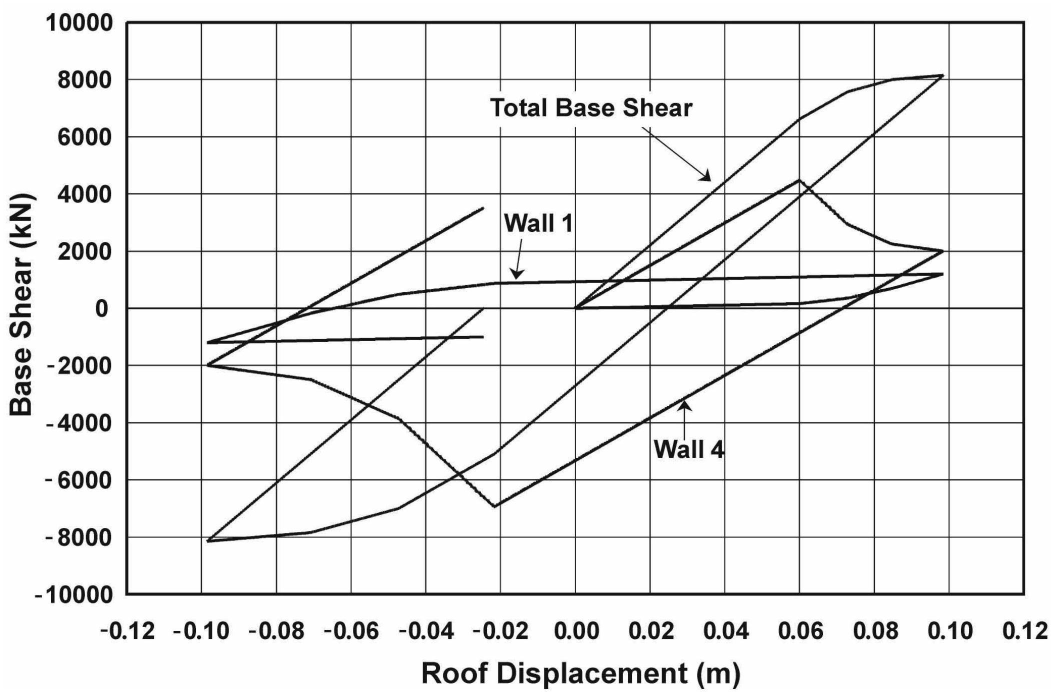


**Figure 8. Cyclic pushover results for Walls 1 and 4: Base shear vs. Roof displacement**

**Table 2: Comparison of base shear peak values (kN)**

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | Base Shear | $\Sigma V$ |
|---|---|---|---|---|---|---|
| Mean | 1187 | 2077 | 3274 | 7595 | 8234 | 14133 # |
| Pushover analysis | 1208 | 2118 | 3560 | 7009 | 8256 | 13895 |
| Stiffness proportional | 206 | 618 | 1853 | 5557 | 8234 | 8234 |
| Strength proportional | 501 | 1044 | 2171 | 4518 | 8234 | 8234 |

# non-simultaneous

The effect of shear deformation on the mean (again for the SAC LA10/50 20 records) base shear in the walls is shown in Table 3. It is seen that even when the shear rigidity is based on cracked sections the effect is relatively minor. As expected, shear flexibility affects the longer walls (smaller aspect ratio) more than the shorter ones. Note that cyclic pushover predicts quite well (with the possible exception of $V_4$) the peak shear also for these cases.

The influence of in-plane floor flexibility for two different assumptions on the distribution of mass among the walls is presented parametrically in Fig. 9 for the El Centro (1940) NS record factored by 1.85. It is seen that with falling in-plane floor stiffness the effect of mass distribution becomes more pronounced. Note that floor rigidities lower than $10^6$ kN/m are not realistic for reinforced concrete floors.

Table 3: Effect of shear deformation on base shear (kN)

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | Base shear | ΣV |
|---|---|---|---|---|---|---|
| **Shear deformation neglected** | **1187** | **2077** | **3274** | **7595** | **8234** | **14133** |
| **Pushover analysis** | **1208** | **2118** | **3560** | **7009** | **8256** | **13895** |
| **Shear deformation (uncracked member)** | **1120** | **1842** | **2950** | **6213** | **8315** | **12125** |
| **Pushover analysis** | **1112** | **1866** | **3040** | **5524** | **8256** | **11542** |
| **Shear deformation (cracked member)** | **1014** | **1663** | **2453** | **5110** | **8820** | **10240** |
| **Pushover analysis** | **1007** | **1660** | **2394** | **4279** | **8256** | **9340** |

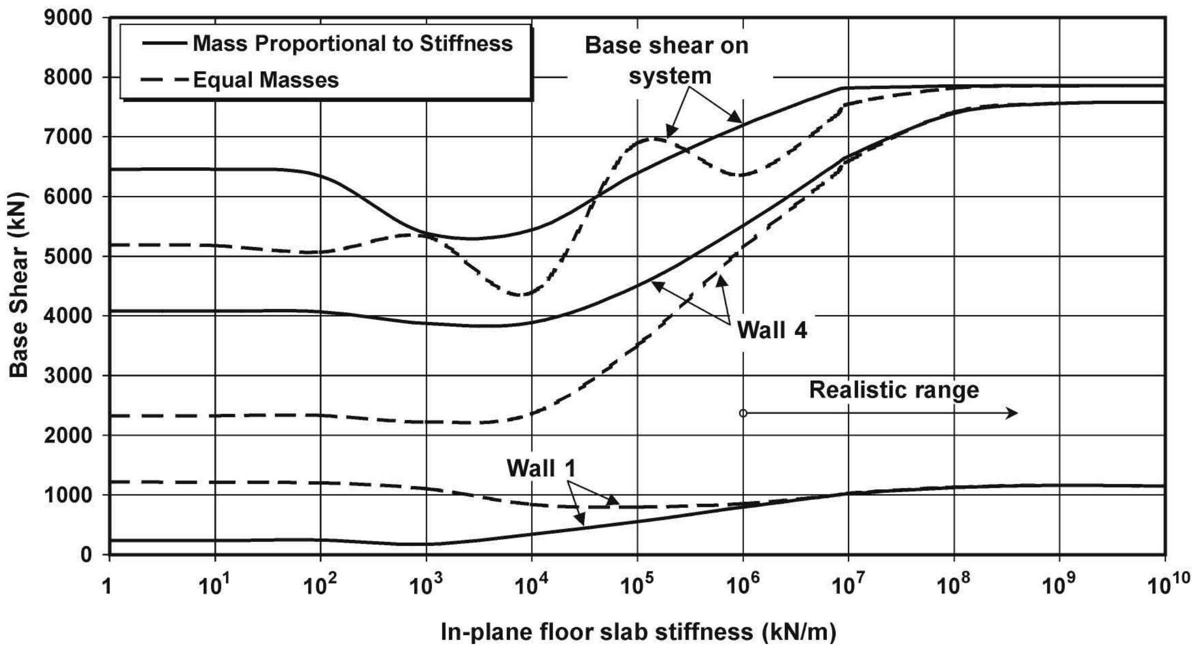


**Figure 9. Base shear vs in-plane floor stiffness for two mass distributions: factored El Centro record**

The combined effect of shear deformation and in-plane floor flexibility is summarized in Table 4, assuming the floor diaphragm stiffness = $7.5 \times 10^6$ kN/m between two adjacent walls and floor mass distribution proportional to wall flexural rigidities. It is seen that some reduction in the shear demand is to be expected.

**Table 4: Effect of shear deformation and in-plane floor flexibility on base shear (kN); factored El Centro record**

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | Base shear |
|---|---|---|---|---|---|
| Shear deformation & in-plane flexibility neglected | 1149 | 2048 | 2970 | 7584 | 7856 |
| Shear deformation - uncracked member & in-plane floor flexibility | 967 | 1531 | 2593 | 5968 | 6918 |
| Shear deformation - cracked member & in-plane floor flexibility | 907 | 1430 | 2351 | 5289 | 7782 |

## SUMMARY AND CONCLUSIONS

This paper is concerned with the seismic shear force demand on flexural walls serving as the lateral load resisting system in multistorey buildings. Two main factors affecting the shear demand were studied: (1) higher modes of vibration in the linear and post yield ranges and (2) shear force redistribution from walls with plastic hinges to those still remaining elastic. It has been shown by parametric analysis, using a suite of 20 accelerograms, and pushover analysis that the shear amplification factors specified in seismic codes are not conservative, particularly for walls designed for high ductility demands, i.e., for large strength reduction factors. Note that this observation is not only valid for structures designed by the equivalent lateral force procedure, but also for those designed using modal analysis. Note, however, that since modal analysis already provides the correct shear amplification for elastic response, i.e., for R=1, the amplification reflects only the nonlinear effects. A simple formula based on the data leading to Eqn. 3 can replace the one given in seismic codes.

The study on multistorey multi-wall systems shows that non-simultaneous yielding at the bases of walls has a significant effect on the base shear distribution among the walls. It is demonstrated that the shear demand on the flexible walls is likely to be much larger than is commensurate with their relative stiffness, or even with their relative flexural strength. Shear deformation in the walls and the in-plane flexibility of the floor diaphragms mitigate to some extent these effects. Yet, modification of seismic code provisions to account for shear force amplification with increasing natural period and ductility, as well as redistribution due to successive formation of plastic hinges, is called for.

## REFERENCES


1. Eurocode 8. "Earthquake resistant design of structures". Pt. 1.3 General rules – specific rules for various materials and elements (ENV 1998), CEN, Brussels, 1995.
2. 2. NBCC. "National Building Code of Canada". Associate Committee on the National Building Code. National Research Council of Canada, Ottawa, Ont, 1995.
3. IBC. "International Building Code". International Code Council, Falls Church, Virginia, 2000.
4. 4. Blakeley, R. W. G., Cooney, R. C. and Megget, L. M. "Seismic shear loading at flexural capacity in cantilever wall structures". *Bulletin, New Zealand National Society Earthquake Engineering*, 1975, 8, 278-290.
5. New Zealand Standards Association. "NZS 3101 Code of Practice for the design of Concrete Structures (Parts 1 & 2)". Standards Association of New Zealand, Wellington, 1982 & 1995.

6. SEAOC. “Recommended lateral force requirements and commentary”. Structural Engineers Association of California”. Sacramento, Calif, 1999.
7. Keintzel, E. “Ductility requirements for shear wall structures in seismic areas”. Proc. 8th World Conf. Earthq. Engng., San Francisco, 1984, 4, 671- 677.
8. Seneviratna, G. D. P. K. and Krawinkler, H. “Strength and displacement demands for seismic design of structural walls”. Proc. 5th US Natl. Conf. Earthq. Engng., Chicago, 1994, 2, 181-190.
9. Nsieri, E. “The seismic nonlinear behaviour of ductile reinforced concrete cantilever wall systems: shear forces”, M.Sc. thesis in preparation, Technion – Israel Institute of Technology, Haifa, Israel, 2004.
10. Somerville, P. et al. “Development of ground motion time histories for Phase 2 of the FEMA/SAC steel project”. Report SAC/BD-97/4, SAC Joint Venture, Sacramento, Calif., 1997.
11. Carr, A. J. “RUAUMOKO – Program for inelastic dynamic analysis”. Department of Civil Engineering, University of Canterbury, Christchurch, 2000.
12. Paulay, T. & Restrepo, J. “Displacement and ductility compatibility in buildings with mixed structural systems”. *Journal, New Zealand Structural Engineering Society,* 1998, 11(1), 7-12.
13. Karman, T. von & Biot, M. A. “Mathematical Methods in Engineering”. McGraw-Hill, New York, 1940.
14. Rutenberg, A. & Leibovich, E. “On the lateral force distribution among structural walls in multistorey buildings”. *Bulletin, New Zealand Society Earthquake Engineering*, 2002, 35(4), 231-242.