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ANALYTICAL STUDY ON COMPOSITE WALL-FRAME BUILDING SEISMIC PERFORMANCE

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SUMMARY

A new composite wall system, consisting of two RC wall panels, short flexural members and coupling girders of wide flanges, is introduced. In this system, two RC panels are comparatively stiff members, which behave in elastic manner without cracking during large earthquake. Short flexural members that link RC panels to the base and roof girders, and wide flanges coupling RC panels which act as shear links, deform under earthquake load. Taking advantage of the wall system to prevent collapse and soft story, a simple design method, which focuses on overturning moment resisted by the wall system is introduced. Beam-column frames, which are combined with wall systems, is uniform frames that has the same section along height. Analytical results are presented for four series of fiber model FEM analyses. Based on results of pushover analysis using composite wall models, the shear link girders yield at roof drift angle of 0.0025rad. The new wall system is effective in leveling story deformation along height. Hysteretic damping of the shear link girders reduces overall story drift of the building. Drift of uniform frames is larger than 0.02 rad. if base shear capacity or column overstrength is small. Combined with the new composite wall system, maximum story angle is reduced to 0.01 or less.

INTRODUCTION

Reinforced concrete wall frame is a frame system used broadly in the world. In many countries, this system is applied to low-rise and high-rise buildings. RC walls mainly resist to shear force in low-rise buildings. In taller buildings, overturning moment is large, which may be more critical than shear force for the design of RC walls. Three patterns of mechanism is considered in designing mid- and high-rise RC wall frames. Fig. 1 summarizes the three failure modes. In one case (Fig. 1a), shear failure occurs to RC walls, a soft story case, which is generally disfavored because of structural diseconomy and brittle behavior. In tall buildings, RC walls may fail at the wall base due to large overturning moment (case b, Fig. 1b). In another mechanism, grade beams may fail due to moment (case c), as shown in Fig. 1c. Of these three mechanism patterns, case b and c are preferred because the behavior is relatively ductile.

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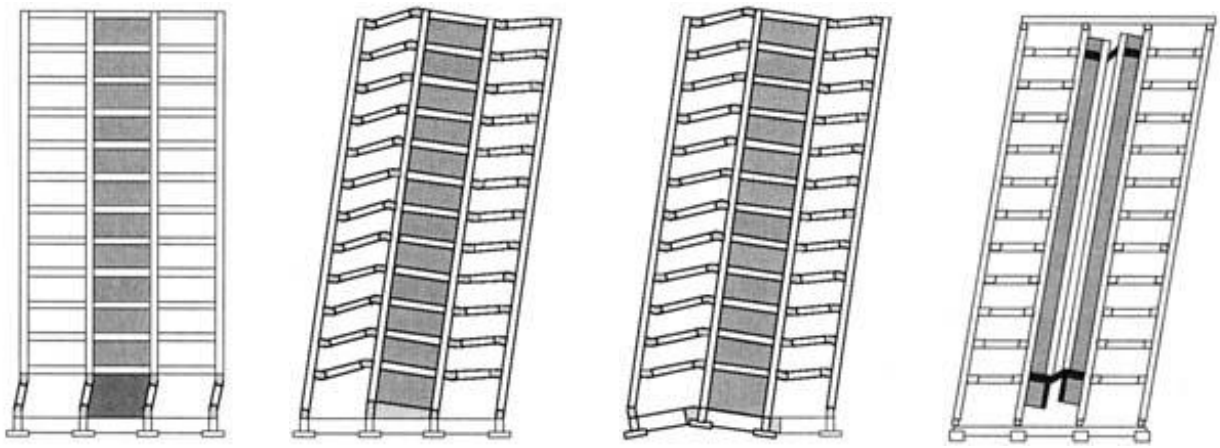
While structural benefit of moment-critical RC wall frames is widely recognized, several problems are also pointed out by researchers, e.g. rehabilitating damaged bases is difficult. Repairing damaged walls will be also cumbersome if hinges formed at the wall base stretch over multiple stories. Estimating stiffness of wall-frames is difficult, especially when the soil is not rigid. Another issue concerns deformation of RC walls under seismic load. As in Fig. 1b, tension side of a wall is lifted to a significant level by earthquake load, which does not conform to deformation of adjacent plane frames. Three-dimensional analysis may be required in this case. Despite these problems, mid- and high-rise RC wall-frames are designed such that a) flexural capacity of the RC wall, which is smaller than its shear capacity by substantially large margin, exceeds overturning moment induced by design earthquake load. Thus designed RC walls will resist horizontal force in a more ductile manner, b) maintaining comparatively large axial resisting capacity up to a fairly large deformation level, and prevent story collapse.



a) Shear Failure b) Flexural Failure c) Rocking d) Composite Wall

Figure 1 Mechanism of Wall-Frame

Composite Wall system

Avoiding story collapse is a critical approach to ensure reliability of a building's structural performance. Other approaches to improve seismic performance of building frames may be c) to avoid creating soft stories and d) to increase hysteretic damping in order to decrease roof drift. New composite wall system presented in this paper is a system that is proposed reflecting the feature and functions expected for RC walls, which are a) through d).

Fig. 1d schematically explains the configuration of the proposed composite wall system. The system is composed of two RC wall panels (RC Panels), coupling girders (Shear Link Girders), and elements located at the top and bottom of the wall (Wall Top/Base Hinge Element). Two RC Panels are connected at second and sixth floor level by Shear Link Girders. Shear force acting in the Shear Link Girders is transferred to the RC Panels, which is then transmitted to the Wall Base Hinge Elements, as the RC Panels are not anchored to the grade beam. As a result, large axial force is acting in the Wall Base Hinge Elements, which resists to the overturning moment.

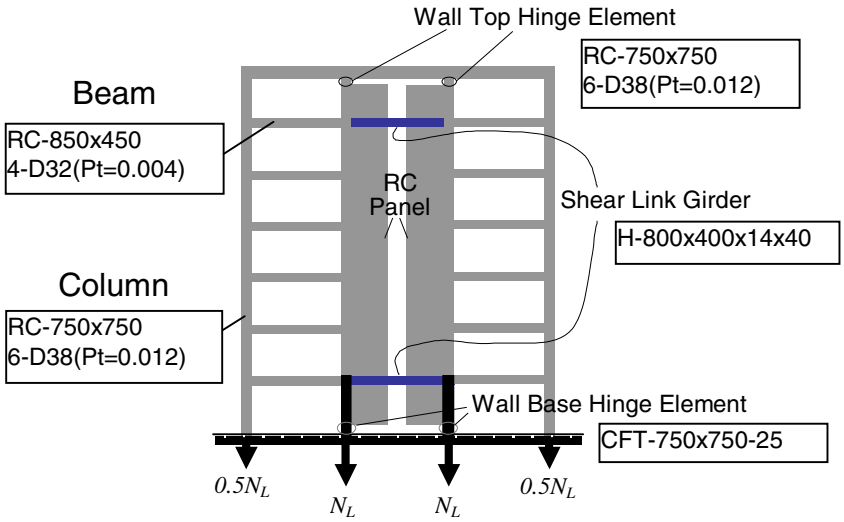
Features of each element consisting the Composite Wall are summarized in the following;

RC Panel: In this discussion, it is assumed that the RC Panels do not crack and thus behave in elastic manner during seismic excitation. Such RC Panels may be modeled as equivalent elastic braces using technique proposed by Tomii, et. al. By controlling the shear capacity of the Shear Link Girder, the level of stress induced in the RC Panels will remain below its crack-initiating stress level. Authors carried out a series of test on Composite Wall specimens to verify that it is actually possible to design such manner (Sakino, 2004). Both flexural and shear deformation of the RC Panels are small compared with the whole

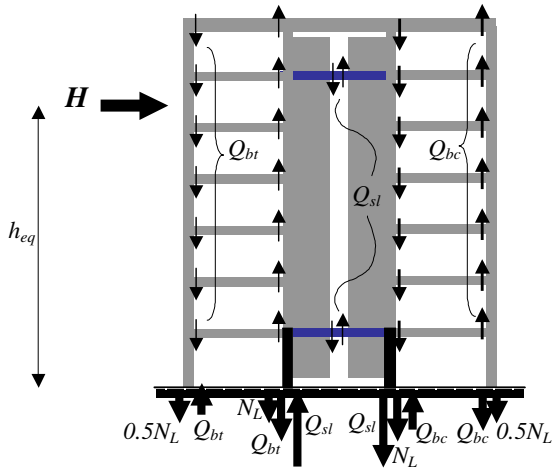
deformation of the Composite Wall.

Shear Link Girder: The Shear Link Girders are designed to shear yield by controlling their shear span ratio. The Shear Link Girders yield at relatively small roof drift level, and during severe seismic excitation, experience cumulative plastic deformation. Rolled wide flanges, known for its tolerance for low cycle fatigue, may be utilized. The Shear Link Girder is meaningful because of its large hysteretic energy absorption capacity and because it caps the level of stress induced in the RC Panel as well as the tension transmitted to the Wall Base Hinge Element.

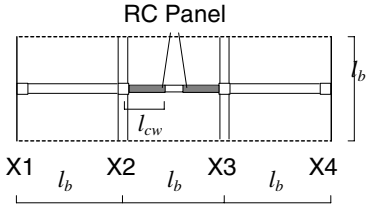
Wall Base/Top Hinge Element: Wall Base Hinge Elements should be capable of transmitting large axial and shear force to the base. Concrete filled steel tubular (CFT) members are known to possess large axial and shear capacity. Demand for Wall Top Hinge Elements is not as severe as the Base Element. RC may be used, provided that adequate amount of reinforcement is placed, which will ensure proper amount of flexural ductility of the element.



a) Components of Composite Wall and Sections of Models for Composite Wall and Uniform Frame



b) Composite Wall – Uniform Frame under Earthquake Load



c) Plan of Composite Wall – Uniform Frame Span

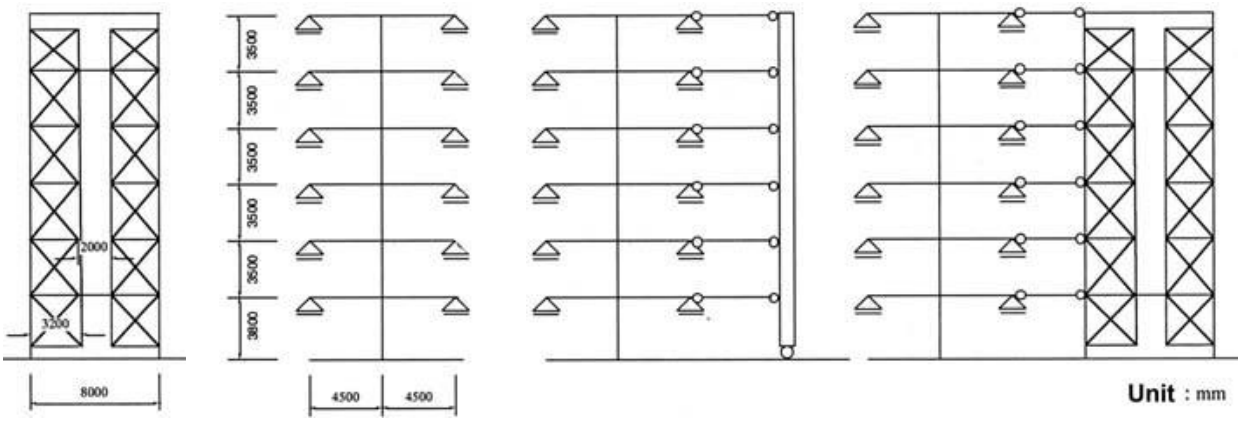
Figure 2 Composite Wall – Uniform Frame

The assembly composed of these members is referred as Composite Wall, hereafter.
Assuming that each element behaves in the manner as explained above, deformation of the proposed new wall system consists mainly of flexural deformation the Wall Base and Top Hinge Elements and shear deformation of the Shear Link Girders after the Shear Link Girders' shear yielding at a small deformation level. Its behavior in response to horizontal force will be insensitive to the force distribution pattern, and similar to what is schematically expressed in Fig. 1d. Story deformation will be distributed to each story equally along the height. Concept of this new system is a stiff bar with large hysteretic damping , which straightens story angle along the height.

Simple Design Method Considering Overturning Moment and Wall-Frame Composed of Uniform Beam-Columns and Composite Walls

A simple design method proposed by authors is applied to design Composite Wall-frames.
In designing a Composite Wall-frame, we apply a strategy where overturning moment, rather than shear and moment induced in members, is focused on. The assumption at the base of this design approach is that neither story collapse nor soft story occurs in buildings incorporating the Composite Walls during seismic excitation. With this assumption, checking columns' overstrength is unnecessary. Section of beams and columns may be the same along height (such moment frames are called Uniform moment frames hereafter).
Detailed method applying this design approach is explained in the following using a Composite Wall – Uniform frame as shown in **Fig. 2a**. The model frame has six stories and 3 bays located inside the building (not perimeter). Gravity load applied to floors is w kN/m^2. Beam spans are the same for each span in X and Y direction (l_b). First, compression induced by gravity load acting at the column base is calculated, which is $0.5N_L$ (N_L=$6wl_b^2$) for the columns at the perimeter (X1 and X4) and N_L for the Wall Base Hinge Elements. When earthquake load is applied to the building to the right in **Fig. 2b**, compression in the columns on the tension side of the perimeter (X1) is reduced to $0.5N_L$-Q_{bt}, where Q_{bt} is the uplift force transmitted from the beams. Compression in the Wall Base Hinge Element on the left (X2) is N_L+Q_{bt}-Q_{lg}, where Q_{lg} is the sum of shear acting in the Shear Link Girders. Behavior of RC columns under tensile force is not desirable. Assuming that when design earthquake load is applied, the building frame is in its mechanism as shown in Fig. 1d and axial force acting in the X1 and X2 columns is naught, the value for Q_{bt} is determined (Q_{bt} = $0.5N_L$). Value for Q_{lg} is also determined (Q_{lg} = $1.5N_L$), for which the Shear Link Girder is designed. Total of moment carried in the beam spans X1-X2 and X2-X3 is $2N_Ll_b$. The rest of overturning moment is carried by the beam span X3-X4. Beams in this span are designed such that span moment, $Q_{bc}l_b$ (Q_{bc} is total of shear acting in the beams of X3-X4 span), is enough to carry the rest of overturning moment. Finally, axial force induced in the columns governs design of the column section. Once sections are determined, pushover analysis is carried to check whether the frame's base shear capacity exceeds the target base shear. If the base shear capacity does not exceed the target base shear, strength of beam is increased by augmenting the amount of reinforcement. Base shear capacity of the modified frame is checked by another pushover analysis. This routine is iterated until base shear capacity exceeds the target value. If column flexural strength is smaller than beams, base shear capacity obtained by the first pushover analysis will not exceed the target base shear. However, flexural strength of RC columns in lower stories is larger than those in upper stories. Generally, iteration of pushover and modification of beam sections won't be more than a few times.
The objective of this paper is to study seismic performance of building frames composed of Composite Walls and Uniform Moment Frames based on analytical results. Numerical analysis was carried using analytical models shown in Fig.3. First, the paper describes the seismic behavior of Composite Walls. Second, performance of uniform fishbone frames is studied, which is compared with its performance connected to a Composite Wall. Finally, a RC wall-frame is redesigned using Composite Walls to verify performance of more realistic Composite Wall-frame. In the discussion, base shear capacity coefficient (Vs) is used to define capacity of frames. Vs is defined herein

as the base shear at a roof drift angle of 0.005rad. divided by weight of the frame.
Nonlinear fiber Finite Element Method (FEM) analysis (Kawano, 1998) is conducted, where an updated Lagrangian formulation is used. Three ground motions (El Centro NS, Hachinohe EW, Tohoku University NS) are used. These ground motions are scaled such that the maximum velocity equals to 50cm/sec. Layley damping of 2% was applied in dynamic analysis.



a) Model CW b) Model UF c) Model UFRB d) Model CWUF

Figure 3 Analytical Models

STATIC AND DYNAMIC RESPONSE OF COMPOSITE WALL

In this chapter, seismic performance of Composite Walls is studied using analytical models of a building as shown in **Fig. 2**. In the model, 400mm thickness of RC Panels is assumed, which is replaced by elastic braces. Pushover analysis was carried using three models. Only the shear strength of the Shear Link Girders varies ($1.0N_L$, $1.5N_L$, $2.0N_L$) between these models (CW-10, CW-15, CW-20, respectively). Sections of elements constituting the models are summarized in **Fig. 2a**. In the analyses, horizontal force distributed along height according to a rule prescribed in the Japanese Building Code was applied, which was increased until roof drift angle reached 0.02rad.
Relation between base shear and roof drift angle (Δ_R/h) is shown in **Fig. 4**. The first kink in **Fig. 4** corresponds to the deformation level where the Shear Link Girders yield. Investigating the force acting in the Shear Link Girders, and comparing the result with **Fig. 4**, it is found that shear force acting in the Shear Link Girder increases in linear manner until both the upper and the lower Shear Link Girders yield at the same deformation level, Δ_R/h =0.0025rad., for the model CW-15. Base shear at this drift level is 0.45W (W: total weight of a Composite Wall), which is close to the shear strength calculated by the following equation.

$$H_{wcal} = Q_{sly} \cdot l_{cw} / h_{eq} \tag{1}$$

Q_{sly} is shear yield strength of Shear Link Girders, l_{cw} is span of the Composite Wall and h_{eq} is equivalent height (see **Fig. 2b**). Value for H_{wcal}/W is calculated and shown in **Fig. 4**. Analytical result is larger by 10% than the calculated strength. Vs (base shear at a deformation level of Δ_R/h =0.005rad.) is larger by another 10%, owing to the increase of moment in the Wall Base Hinge Elements.
Fig. 5 shows relation between axial force and moment induced in the Wall Base Hinge Elements of CW-15. In the figure, it is observed that compressive force acting in Wall Base Hinge Elements also increases linearly until the Shear Link Girder yields. Ultimate axial strength and moment capacity curve is also

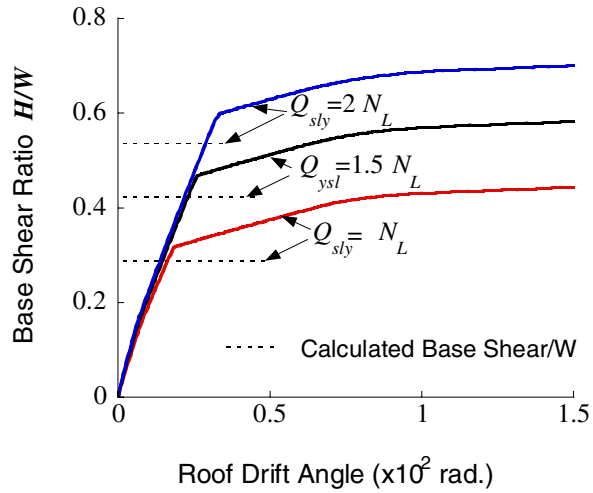


Figure 4 Pushover Curve of Composite Wall with Varied Strength

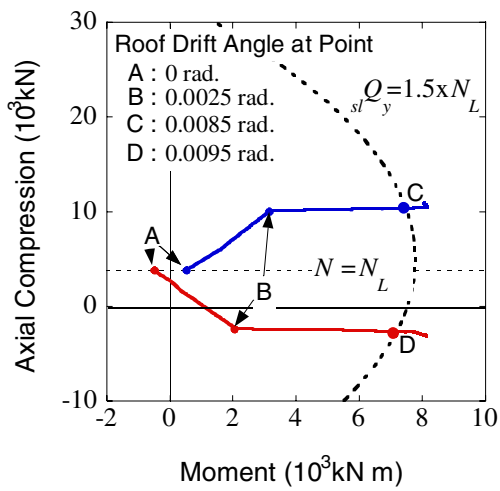


Figure 5 Axial Force – Moment Relation and Ultimate Strength of Wall Base Hinge Elements

shown in Fig. 4. The Wall Base Hinge Element reaches its ultimate state at Δ_R/h =0.0085rad. on the tension side, and at Δ_R/h =0.0085rad. on the compression side.

Deformation caused in the Composite Walls is roughly divided into flexural deformation of the Wall Base Hinge Element, shear and flexural deformation of the RC Panels. We herein define contribution to roof drift of Wall Base Hinge Element's flexural deformation (δ_{he}) as the product of node rotation at the top of the hinge element (θ_{he}) and building height. Shear deformation of a RC Panel on i^{th} floor is defined as γ_{wi} calculated by the following equation.

$$\gamma_{wi} = \left(\frac{V_{i,j+1} - V_{i,j}}{h_i} + \frac{U_{i+1,j} - U_{i,j}}{b} + \frac{V_{i+1,j+1} - V_{i+1,j}}{h_i} + \frac{U_{i+1,j+1} - U_{i,j+1}}{b} \right) \cdot \frac{1}{2} \tag{2}$$

As to the notations, h_i is height of RC panels on i^{th} floor, b is width of RC panels. Contribution of the RC Panels' shear deformation (δ_{ws}) is obtained by summing $h_i \gamma_{wi}$ along height. **Fig. 6** shows the proportion of

δ_{he} and δ_{ws} in roof drift (Δ_r). It is noted in Fig. 6 that shear deformation of the wall contributes very little to the roof drift. At roof drift angle of Δ_R/h =0.01rad., shear deformation angle γ_{wi} is less than 0.0001rad. which is smaller than the shear deformation angle of a RC wall initiating to crack (0.0025rad.). Before yielding of the Shear Link Girders, δ_{he} constitutes 35% of the roof drift. Contribution of the Wall Base Hinge Elements' flexural deformation increases drastically after the yielding of Shear Link Girders. Proportion of δ_{he} is increased to 50% at roof drift angle of Δ_R/h =0.01rad.,
In the case where deformation of RC Panels is negligibly small, shear deformation angle of the Shear Link Girders equals to θ_{he} multiplied with Shear Link Girder length to Composite Wall span ratio. Deformation of RC Panels, however, constitutes substantial portion of elastic deformation of the Composite Wall. RC Panel's deformation increases in proportion to the strength of Shear Link Girders. As a result, stiffness of Composite Walls remains almost the same as observed in **Fig. 4** as Shear Link Girder strength is increased.

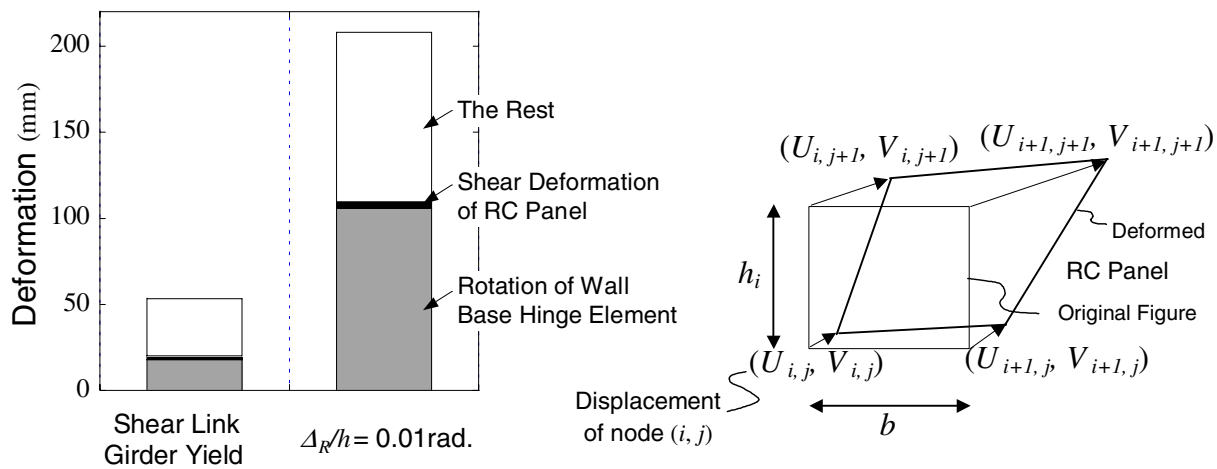


Figure 6 Deformation of Elements Consisting Composite Wall

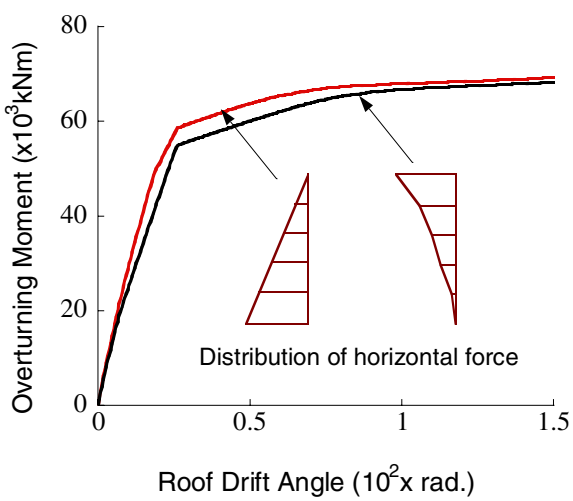


Figure 7 Pushover Curve of Composite Wall Exposed to Horizontal Force of Ai and Triangular Distribution Pattern

In real buildings, Composite Walls will be combined with moment frames. Distribution of story shear force that is carried by the Composite Walls may differ those used for building design, e.g. Ai distribution rule prescribed by Japanese Building Code. Another set of pushover analysis is carried out, in which horizontal force is distributed in triangular manner along height. **Fig. 7** shows the relation of overturning moment and roof drift angle. Elastic behavior of the Composite Wall is affected a little, e.g. yielding of Shear Link Girders takes place at roof drift angle of Δ_R/h =0.0022rad. for the 2nd floor level Girder and Δ_R/h =0.0027rad. for the 6th floor level Girder. However, the overturning moment carrying capacity is roughly the same for both cases. This result indicates seismic performance of Composite Walls is fairly insensitive to configuration frames, with which the Composite Wall is combined.
Vertical displacement of nodes on outer ends of RC Panels is 3 to 6% of horizontal displacement for any models, which confirms that three-dimensional analysis is not necessary in applying the Composite Wall. A set of time-history analysis results reveals that maximum story angle is less than 0.005rad. If the weight carried by CW-15 is increased to 2.5 times its weight, maximum story angle remains about 0.01 rad. at the largest.

SEISMIC RESPONSE OF UNIFORM FRAMES AND COMPOSITE WALL – UNIFORM FRAMES

The objective in this chapter is to investigate effect of Composite Walls in improving seismic performance of Uniform moment frames. First, dynamic response of Uniform frames (fishbone frames as shown in Fig. 3b) is investigated. The result is compared to the response of Uniform frames connected to a pin-supported rigid bar, which has the same shear and flexural stiffness as the Composite Wall. The comparison will verify the effect of stiff Composite Wall in leveling story deformation along height. Finally, results of dynamic analysis using models composed of uniform frames and a Composite Wall is studied to verify effect of hysteretic damping of the Shear Link Girders.
Base shear capacity of Composite Walls is substantially larger than the value typically required for wall-frame buildings. Composite Walls are therefore combined with frames with smaller base shear capacity. Assuming that base shear capacity of a Composite Wall - Uniform frame is obtained by superposing the base shear capacity of Composite Walls and Uniform frames, and in the case where the weight carried by the column of a Uniform frame is as large as the weight carried by a Wall Base Hinge Element, following equation is obtained.

$$\left(2W_f + nW_f\right)\cdot V_{fw} = 2W_f \cdot V_w + nW_f \cdot V_f \tag{3}$$

W_f: weight carried by the column of a fishbone frame, V_{fw}: base shear capacity coefficient of the wall-frame, V_w and V_f: base shear capacity coefficient of the wall system and fishbone, respectively, n: number of fishbone frames consisting the wall-frame. The equation (3) is transformed into the following equation.

$$V_f = \left\{1 - \frac{2}{n}\cdot\left(\frac{V_w}{V_{fw}} - 1\right)\right\} \tag{4}$$

For the analyses in this chapter, we use CW-15 as the Composite Wall. Base shear capacity of this model obtained by calculating (1) is 0.45. If a Composite Wall - Uniform frame is to be designed such that the base shear capacity, V_{fw}, exceeds 0.25, V_w/V_{fw} in (4), substituting V_w =0.45, equals to 1.8. In this discussion, we consider two cases where a Composite Wall is combined with two and ten Uniform frames. Substituting n=2 and 10, as well as V_w/V_{fw} =1.8 in (4), we get V_f =0.05 and 0.21, respectively.

Uniform fishbone frames of V_f =0.05 and 0.25 are used in the analyses. For the analytical models composed of a Composite Wall and Uniform frames, two V_f =0.05 frames or ten V_f =0.25 frames are combined with a Composite Wall.

Uniform frames

Seismic response of two Uniform fishbone frame models, UF-05 and UF-25, is investigated using results of a series of time-history analysis. Parameter for the analytical models is Vs. Column section is the same for two models. Column overstrength factor (COF) not considering axial force in the column is 0.45 for UF-25 and 12.35 for UF-05. Maximum story angle (Δ/h_{max}) is presented for each story in **Fig. 7**. Δ/h_{max} is large for model UF-05, as its base shear capacity is small. Maximum roof drift angle ranges between 0.013 to 0.015rad. Maximum roof drift angle is smaller for UF-25 (0.067 to 0.011rad.) as the base shear capacity is larger for this model. Maximum drift angle is the largest in the first story, which is larger by 39 to 138% than maximum roof drift angle. This is larger than UF-05 (29 to 44%) by large margin. While maximum roof drift angle is smaller for UF-25 owing to its large base shear capacity, the largest value of Δ/h_{max} is roughly the same for the two models, because in UF-25, hinges are formed in columns, which causes deformation to concentrate in the lower stories.

Uniform frames connected to a Pin-supported Rigid Bar

Story deformation of a Uniform fishbone frame may be distributed along height by connecting the frame to a pin-supported stiff bar, as schematically shown in **Fig. 3b**. As the pin-supported rigid bar, we utilize a Composite Wall without Shear Link Girders. Its Wall Base and Top Hinge Elements are replaced by pins. Such Composite Wall (Pin-supported Rigid Bar, hereafter) does not resist to horizontal load on its own. However, connected to frames, it will prevent deformation to concentrate in few stories. In one analytical model, UFRB-2x05, two UF-05 frames are connected to one Pin-supported Rigid Bar. Another model, UFRB-10x25, is composed of ten UF-25 frames and one Pin-supported Rigid Bar.

Maximum story angle that caused during seismic excitation is shown in **Fig. 8** for each story. Comparing the result with **Fig. 7**, it is observed that Δ/h_{max} is leveled along height owing to the Pin-supported Rigid Bar. Gap between maximum and maximum roof drift angle is reduced to 1/4 to 1/3 for both models. Maximum roof drift angle is also reduced by 15% in the case of UF-05.

Composite Wall - Uniform frames

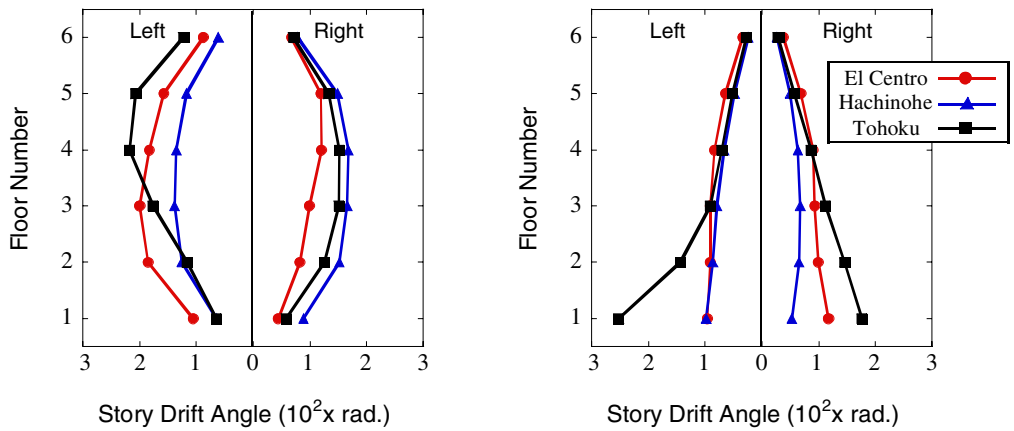
Generally, large amount of energy is absorbed by shear yielding of steel. Hysteretic damping of Shear Link Girders is expected to reduce the overall story drift of building frames. Effect of hysteretic damping is investigated using two models CWUF-2x05 and CWUF-10x25. CWUF-2x05 is composed of one Composite Wall and two UF-05 frames. Base shear coefficient at a roof drift angle of Δ_R/h =0.005, Vs, obtained from pushover analysis, is 0.28 for CWUF-2x05 and 0.30 for CWUF-10x25. Seismic load is resisted mainly by the Composite Wall in CWUF-2x05. Overturning moment carried by the Composite Wall at Δ_R/h =0.005 constitutes 95% of the total. Effect of hysteretic damping is significant in this case. Comparing the result with that for UFRB-2x05, maximum roof drift angle is reduced to 1/3. Hysteretic damping effect is small for CWUF-2x25, because in this model, the Composite Wall takes only as much as 1/3 of overturning moment.

Summarizing the results described in this chapter;

1) Maximum story angle induced by ground motions is large for Uniform frames if the base shear capacity or column overstrength is small.

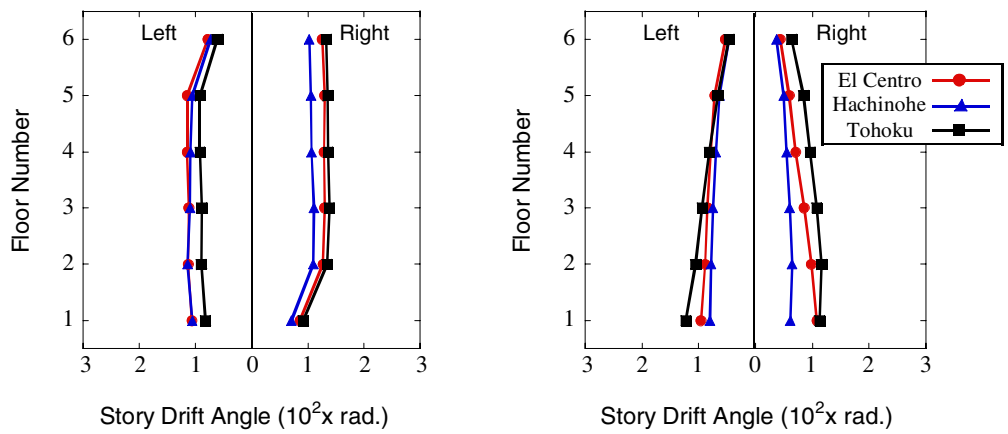
2) Behavior of the Composite Wall as Pin-supported Rigid Bar is useful in leveling story deformation along height.

3) In the case where the Composite Wall carries most of overturning moment, effect of hysteretic damping of the Shear Link Girders is significant. Story angle is reduced overall.



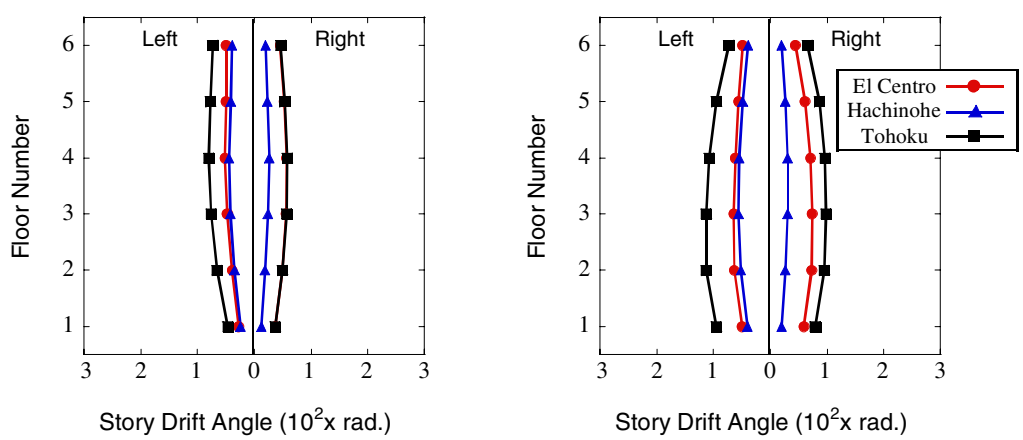
a) Vs=0.05 b)Vs=0.25

Figure 8 Maximum Story Angle of Uniform Frame



a) Vs=0.05, n=2 b)Vs=0.25, n=10

Figure 9 Maximum Story Angle of Uniform Frame Connected to Pin-Supported Stiff Bar



a) Vs=0.05, n=2 b)Vs=0.25, n=10

Figure 10 Maximum Story Angle of Composite Wall Frame – Uniform Frame

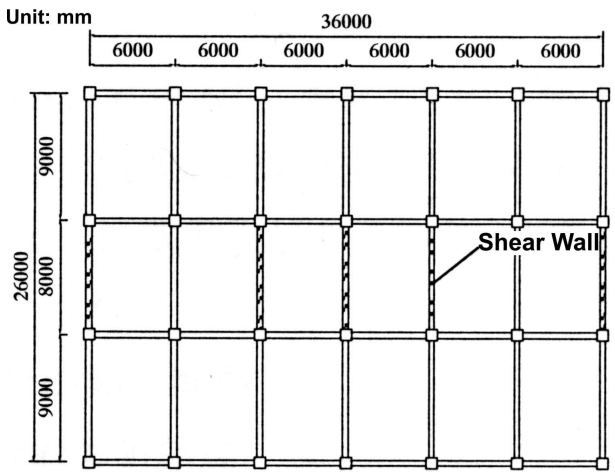


Figure 11 Plan of Design Example (AIJ)

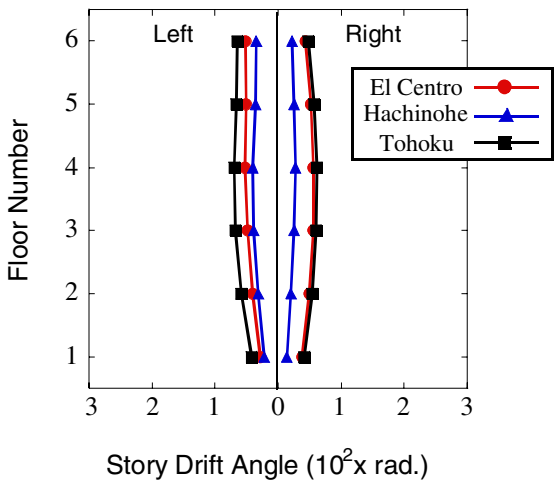


Figure 12 Maximum Story Angle of Redesigned Building with Composite Walls and Uniform Frame

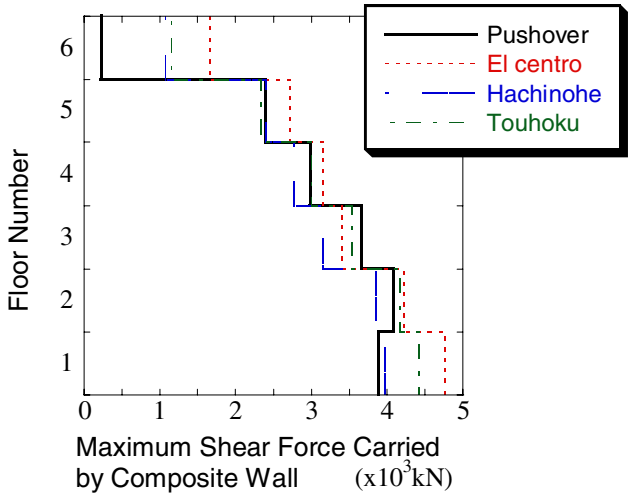


Figure 13 Maximum Shear Force Carried by Composite Wall Frame of Redesigned Building

REDESIGNED BUILDING DESIGN EXAMPLE

A design example of a six-story RC wall-frame is shown in Design Guidelines for RC Buildings Based on Ultimate Strength Concept (AIJ, 1990). Plan of the building is shown in Fig. 11. The building is redesigned here replacing RC walls with Composite Walls. Total shear strength of Shear Link Girders is 1.5 times the weight carried by one Wall Base Hinge Element. Beam sections are adjusted so that base shear coefficient at roof drift angle of 0.01rad. is 0.3. Beam and column sections are shown in Fig. 11. Dynamic analysis results are shown in Fig. 12. Maximum roof drift angle ranges between 0.0034 and 0.0062rad. It is observed in Fig. 12 that story deformation is fairly spread along height. Fig. 13 shows maximum shear force carried by one Composite Wall. Results are compared with that obtained by pushover analysis. The largest shear force is caused in the first floor, which is larger than pushover analysis results by 20%.

CONCLUSION

A new-concept RC wall system, Composite Wall, was proposed which consists of RC Panels, Shear Link Girders and Wall Base Hinge Elements. Strength of a Composite Wall is controlled by shear strength of Shear Link Girders. Four series of numerical analysis suggested that the proposed wall system is effective in spreading story deformation of building frames along height. It was also found that the hysteretic damping of the Shear Link Girders effectively reduced overall story angle of the building. These effects of Composite Walls is summarized in Fig. 14, where maximum roof drift angle and story angle is compared between all analysis results. Uniform frames of low base shear capacity or small column overstrength behave poorly when exposed to strong ground motions. Maximum story angle is reduced to less than 0.01rad. from larger than 0.02rad. if such frames are combined with Composite Walls, such that the building frame's base shear capacity coefficient is close to 0.3.

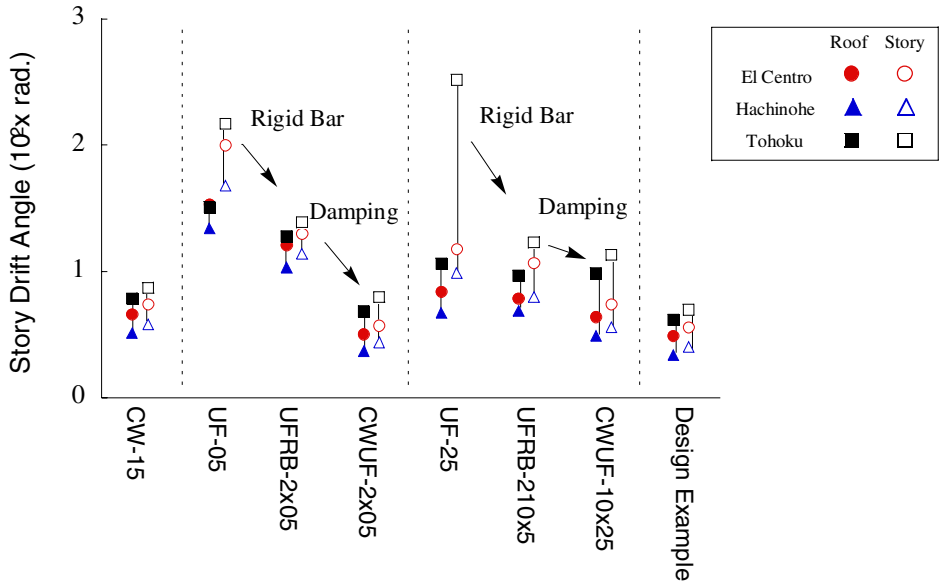


Figure 14 Comparison of Maximum Roof Drift Angle and Maximum Story Angle

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